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Approximation of Subnetwork Models using Frequency-Domain Data

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***Abstract*— In this paper, we approximate models of interconnected systems that are to be used for decentralized control design. The suggested approach is based on approximation of so-called subnetwork models. A subnetwork model is a model of the interconnected system, as seen from one specific position in the network. The simplification is done by using weighted model reduction, and several approximation criteria are given. A new method for weighted model reduction is used. The method is based on a combination of known techniques that use semidefinite programming and frequency-data samples of transfer functions. The method is guaranteed to preserve stability and does not depend strongly on the order of the original model. This is particularly important for large interconnected systems. Two examples are given to illustrate the technique.**

I. Introduction

Control of interconnected systems has become a large research area in the last couple of years. Applications for this research are, for example, automated highway systems, vehicle formation control, power systems, sensor/actuator networks, and cross-directional control in paper machines. In an interconnected system, there are many systems and they are influenced both by neighboring systems and by external signals. In Fig. 1, a small interconnected system is shown with three subsystems $P, P^{(1)}, P^{(2)}$. In this small-scale case, standard control techniques can be used to control various properties of the systems.

If we let the number of interconnected systems grow, the standard control-design techniques become computationally infeasible. The reason is that these techniques often are centralized. A good way to beat the complexity is often to decentralize the control structure. A decentralized controller is often easier to implement, less complex, and more robust to model/sensor faults. Distributed H_∞-control has been considered in, for example, [1], [2], and distributed receding horizon control in, for example, [3], [4].

Before designing a controller for a high-order system, it is often a good idea to simplify the plant dynamics as much as possible, while retaining its important features. Again, if the interconnected system is of small scale, as in Fig. 1, the problem is easily solved by using standard model reduction techniques, such as Hankel-norm approximation or balanced truncation, see, for example [5], [6]. But typically the computational complexity of these methods grows as $O(n^3)$, where n is the total number of states in the model. This makes them hard to use for systems with more than a few hundred states. Furthermore, these methods were not designed to preserve the special structure of an interconnected system. It is, however, possible to enforce structure by some modifications, see, for example, [7] and [8].

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Just as for control design, it has been suggested to decentralize system approximation to deal with the computational complexity, and to maintain the interconnection structure. In [9], subnetwork models were simplified for studies of fault sequences in large power systems. In [10], subsystems were simplified separately in an iterative fashion. In both [9] and [10], time-domain data from simulations of nonlinear models were used together with principal orthogonal decomposition (POD) type of simplification, see [11]. A similar philosophy is used in the work in this paper, but we instead use frequency-domain data and only linear models. Another difference is the focus on modeling for control design here.

The first contribution of this paper is to derive a framework for simplification of subnetwork models for control design. A subnetwork model is a model of the network, as seen from one specific position in the network. To simplify the overall network, one can repeatedly simplify subnetworks, in a fashion similar to [10]. The details of such a scheme are not studied here, though. The simplification boils down to weighted model reduction problems, where the weights depend on the structure of the decentralized controller. The second contribution of this paper is to solve the weighted model reduction problem for multi-input–multi-output (MIMO) models using a novel approximation method that combines the ideas of [12] and [13]. In particular, the method preserves stability and does not suffer from the $O(n^3)$ complexity mentioned above.

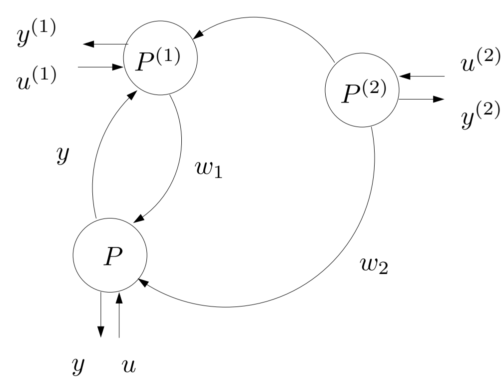

Fig. 1. An interconnected system. The three systems $P, P^{(1)}, P^{(2)}$ have interconnected dynamics. To each system there are local external inputs and outputs.

II. Problem formulation

We shall focus on one (arbitrary) subsystem in an interconnected structure, and denote it by P. The rest of the network, $P^{(1)}$ and $P^{(2)}$ in Fig. 1, is called the *subnetwork* and is denoted by N. The subsystem P is modeled by two (continuous time) transfer function matrices (P_1, P_2) and the dynamics is described by

$$y = P_1 u + P_2 w \tag{1}$$

$$w = -Ny + l. \tag{2}$$

The output $y(t) \in \mathbb{R}^p$ is available for feedback control of P and may influence the subnetwork. The local control signal is $u(t) \in \mathbb{R}^m$. The signal $w(t) \in \mathbb{R}^o$ represents the influence from the subnetwork, and $l(t) \in \mathbb{R}^o$ is an additional external disturbance. The subnetwork is assumed to be stable and is modeled by the (continuous time) transfer function matrix $N \in H_\infty$. It influences P through the feedback (2). We assume throughout that the feedback connection of P_2 and N is internally stable, and hence $(I + P_2 N)^{-1} \in H_\infty$. The interconnected system is assumed to be large, and N may be a transfer function of high McMillan degree. In the method described in Section III, knowledge of frequency samples of the subnetwork,

$$N_k = N(j\omega_k), \quad k = 1 \dots K, \tag{3}$$

is assumed. This is an assumption that works to our advantage because it may be hard to compute an exact model of N. Next, we give two examples of interconnected systems, and how they fit into the framework.

Example 1 (Power systems): Power systems are an example of interconnected systems that can be modeled with $P_1 = P_2 =: P$, when the so-called swing equations are used, see [9]. The output y is the phase angle at the bus under consideration. We have

$$y = \frac{1}{ms^2 + ds}(u + w) = P(u + w), \tag{4}$$

where u is the local power input, m is the local "mass", and d the local "damping". w represents the power exchange with the subnetwork. It depends on the phase differences and can be modeled by

$$w = \sum_j k_j \sin(y - \delta_j) \approx \sum_j k_j (y - \delta_j),$$

for small phase differences (constant bus voltages are assumed). The index j runs over all buses in the subnetwork that are connected to P, and δ_j is the phase angle at bus j. Since the dynamics for each phase angle δ_j is in the form (4), we can solve for N, given that we know the local masses, local damping, and interconnection constants k_j.

Example 2 (Vehicle formations): An interconnected system is not necessarily physically connected. An example is vehicle formations, see [4]. Consider two vehicles in formation. They may be modeled as

$$y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} P \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ I \end{bmatrix} w = P_1 u + P_2 w,$$

where x_1, x_2 are positions and velocities of vehicles 1 and 2. P is the dynamics of vehicle 1, and for control it may use the position and velocity of vehicle 2, that is $w = x_2$. The subnetwork model N here models how vehicle 2 reacts to changes in x_1, x_2. This reaction depends on the possible controller in vehicle 2. The disturbance could here be modeled as $l = Pu_2$, if the vehicles have identical dynamics and u_2 is the input to vehicle 2.

The *internal model principle* suggests that a good decentralized feedback controller should contain a model of the controlled process. Here that means P_1, P_2, and N. It is unrealistic (and unnecessary) to have an exact model of the large subnetwork N in the feedback controller, and we will discuss how simplified subnetwork models $\hat{N}$ can be obtained and included in the feedback. The approach we use has many similarities to controller reduction, see, for example, [14]. The criteria we use for the simplification of N are:

(A) How is the robust stability of the interconnected system influenced by the choice of $\hat{N}$?

(B) How is the performance of the subsystem influenced by the choice of $\hat{N}$?

(C) How does the choice of $\hat{N}$ influence how the subsystem is seen from the subnetwork?

Before discussing these criteria in detail, we introduce the approximation method that is used.

III. Weighted MIMO Transfer Function Approximation

The problems we end up solving are weighted model reduction problems. There are many methods available for solving such problems, see [5], [14], [15]. Typically they require a state-space model of N, does not always preserve stability, and need $O(n^3)$ operations. For models given by frequency samples (3), we could potentially use techniques for robust system identification, see, for example the survey [16]. But instead we use a novel method utilizing semidefinite programming, based on ideas from [12], [13], [17], [18].

A. Approximation problem

The approximation method consists of two steps. The first step is inspired by [12], and the second step by [13]. The poles of $\hat{N}$ are determined in the first step, and the zeros are determined in the second step. The problem we would like to solve can be formulated as

$$\text{minimize} \quad \|W_o(N - \hat{N})W_i\|_\infty \tag{5}$$

$$\text{subject to} \quad \deg \hat{N} \leq r, \quad \hat{N} \in H_\infty, \tag{6}$$

where $N \in H_\infty$ is given together with frequency-dependent weights $W_i, W_o \in L_\infty$ and $r \in \mathbb{Z}_+$. Here $\|\cdot\|_\infty$ denotes the L_∞-norm and $\deg$ the McMillan degree. To the best knowledge of the authors, no polynomial time algorithm is available to solve (5)–(6), and the suboptimal methods mentioned above are frequently used. We will relax the problem (5)–(6) and obtain thereby a convex optimization problem.

B. Step 1: Fix pole locations

The first step is to replace $\hat{N}$ in (5) with a polynomial fraction description

$$\tilde{N}(s) = \frac{1}{\tilde{a}(s)}\tilde{B}(s), \quad \tilde{a}(-s) = \tilde{a}(s), \quad \tilde{a}(j\omega) > 0 \quad \forall\omega, \tag{7}$$

where $\tilde{B}(s)$ is a polynomial matrix of degree $2r$ and with dimension $o \times p$, and $\tilde{a}(s)$ is a scalar polynomial of degree $2r$ with no odd monomials. Notice in particular that $\tilde{N} \notin H_\infty$. The problem is now to fix the coefficients of $\tilde{a}$ and $\tilde{B}$. This is done by solving a sampled version of (5):

$$\begin{aligned} &\text{minimize} \quad \gamma \\ &\text{subject to} \quad \overline{\sigma}(W_{o,k}(N_k - \tilde{N}_k)W_{i,k}) < \gamma, \quad k = 1 \dots K, \end{aligned}$$

where subscript k means evaluation of the corresponding transfer function at $j\omega_k$, and $\overline{\sigma}$ is the largest singular value. This problem can equivalently be formulated as

$$\begin{aligned} &\text{minimize} \quad \gamma \\ &\text{subject to} \quad \begin{bmatrix} \gamma\tilde{a}_k I & W_{o,k}(N_k\tilde{a}_k - \tilde{B}_k)W_{i,k} \\ * & \gamma\tilde{a}_k I \end{bmatrix} > 0, \quad (8) \\ &\qquad\qquad\qquad\qquad\qquad k = 1 \dots K, \\ &\qquad \tilde{a}(j\omega) > 0, \quad \forall\omega, \quad (9) \end{aligned}$$

using Schur complements, and $*$ denoting the complex conjugate transpose of the upper right entry. Feasible coefficients of $\tilde{a}, \tilde{B}$ can be found for fixed γ if they exist because (8)–(9) can be turned into a semidefinite program. Since $\tilde{a}, \tilde{B}$ are linear in their unknown coefficients, (8) are simply K linear matrix inequalities (LMIs). Inequality (9) can also be turned into LMIs using, for example, the Kalman-Yakubovich-Popov lemma, see [17], [18]. The minimization of γ is then achieved via bisection over γ (the problem is quasiconvex).

Once a solution $\tilde{N}$ is found, we make a stable-antistable decomposition

$$\tilde{N}(s) = \frac{1}{\hat{a}(s)}R_1(s) + \frac{1}{\hat{a}(-s)}R_2(s) \tag{10}$$

where $\hat{a}(s)$ is a Hurwitz polynomial of degree r such that $\hat{a}(s)\hat{a}(-s) = \tilde{a}(s)$. We obtain $\hat{a}(s)$ from a spectral factorization of $\tilde{a}$. Because of (9), such a factorization always exists. The polynomial $\hat{a}$ contains the poles of $\hat{N}$.

Remark 1: It may seem odd to approximate $N \in H_\infty$ with an unstable $\tilde{N}$ in L_∞-norm and then only retain the stable part. However, this is also done in optimal Hankel-norm approximation, see [6]. Here we look for poles that lie symmetrically around the origin. This idea was introduced in [12] (in the discrete-time single-input–single-output (SISO) case) and worked very well there.

C. Step 2: Fix zero locations

From Step 1, the stable poles $\{p_i\}_{i=1}^r$ of $\hat{N}$ are fixed from the roots of $\hat{a}$. The problem is now to find the zeros of $\hat{N}$, such that the McMillan degree of $\hat{N}$ is as small as possible while the weighted error $\overline{\sigma}(W_{o,k}(N_k - \hat{N}_k)W_{i,k})$ is small. Assuming that p_i are distinct[1], we use the parametrization

$$\hat{N}(s) = \hat{N}_0 + \sum_{i=1}^{r} \frac{1}{s - p_i}\hat{N}_i, \tag{11}$$

and shall fix $\hat{N}_i \in \mathbb{C}^{o\times p}$, where $\overline{\hat{N}_i} = \hat{N}_j$ when $\bar{p}_i = p_j$. The McMillan degree of (11) is given by

$$\deg \hat{N} = \sum_{i=1}^{r} \text{rank}\, \hat{N}_i.$$

Minimization of the rank of a matrix subject to LMIs is known as a difficult and nonconvex problem. However, there exist simple and effective heuristics, such as the one in [13]. There the trace-class norm of $\hat{N}_i$,

$$\|\hat{N}_i\|_1 = \sum_{k=1}^{\min\{o,p\}} \sigma_k(\hat{N}_i),$$

where σ_i are the singular values, is minimized instead of the rank. The minimization problem we solve is the following: Fix a desired approximation accuracy $\gamma > 0$. Then solve

$$\begin{aligned} &\text{minimize} \quad \sum_{i=1}^{r} \|\hat{N}_i\|_1 \\ &\text{subject to} \quad \overline{\sigma}(W_{o,k}(N_k - \hat{N}_k)W_{i,k}) < \gamma, \quad k = 1 \dots K, \end{aligned} \tag{12}$$

where $\hat{N}$ is given by (11). How to solve this problem by means of LMIs is shown in [13]. What typically happens is that as γ is decreased to obtain a better approximation, the McMillan degree of $\hat{N}$ increases until there no longer is a feasible solution to (12). There is a trade off between approximation accuracy and complexity. Notice that an upper bound on $\deg \hat{N}$ is $r \cdot \min\{o, p\}$.

Remark 2 (Implementation): We are currently solving the LMIs in Step 1 and Step 2 with SeDuMi [19] and YALMIP [20].

IV. Control Structure 1: $P_1 = P_2$

In this section, we study the problem of finding subnetwork models $\hat{N}$ based on the criteria (A)–(C) when $P_1 = P_2 =: P$. That is, the influence from the subnetwork, w, and from the control signal, u, enter the process in the same way. One example where this is the case is in power systems, see Example 1.

We will make the analysis based on the simple feedback

$$u = K(y_{ref} - y) + \hat{N}y, \tag{13}$$

where K is a transfer function matrix, y_{ref} a reference signal, and $\hat{N}$ a subnetwork model. The controlled system is shown in Fig. 2. The idea behind the control law is easily understood by looking at the closed-loop transfer function

$$y = (I + P(K + \Delta))^{-1}P(Ky_{ref} + l + (\hat{N} - K)n),$$

[1] Generically p_i are distinct. If not, we have to modify the parametrization in (11) slightly.

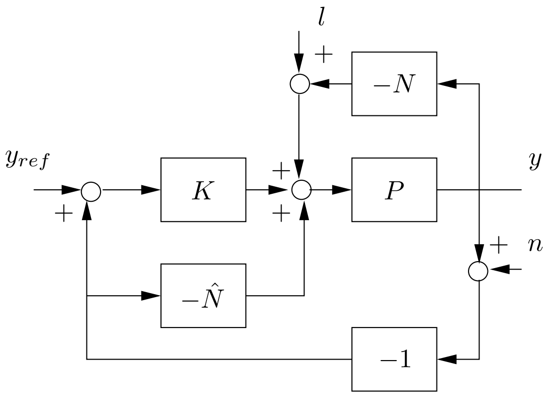

Fig. 2. The closed-loop system when $P_1 = P_2 = P$ and the feedback (13) is used.

where $\Delta = N - \hat{N}$ is the subnetwork model error. If $\Delta = 0$, then the response of the loop to references, y_{ref}, and to disturbances, l, is the same as when P is not connected to the subnetwork N. Notice, however, that the response to measurement noise, n, depends on $\hat{N}$.

A simple control design methodology can then be as follows: First, design K for P using standard methods so that the local performance is good, assuming that $N = 0$. Second, find an approximation $\hat{N}$ and add to the feedback (13) to compensate for the subnetwork. This is discussed next. Notice that $\hat{N}$ and K may need to be modified to attenuate the measurement noise n.

Robustness (A): Assuming that the closed-loop system in Fig. 2 is stable when $\Delta = 0$, we have that $(I + PK)^{-1}P \in H_\infty$. According to the small-gain theorem [15], applied to this loop, a sufficient condition for the loop to be stable is

$$\|(I + PK)^{-1}P(N - \hat{N})\|_\infty < 1.$$

Hence, from robustness concerns, it makes sense to choose $W_o = (I + PK)^{-1}P$ and $W_i = I$ in the weighted model reduction problem. Here, W_o is chosen as the load sensitivity function. A fine approximation is thus recommended at frequencies where the sensitivity is high to load disturbances l. For other frequencies, a good model of the subnetwork N is not necessary.

Performance (B): Another rationale for choosing $\hat{N}$ is to match closed-loop transfer functions from y_{ref} to y. Since Δ appears in the inverse, we make a Taylor expansions to obtain a problem in the form (5). We have

$$\begin{aligned}(I + PK)^{-1}PK - (I + PK + P\Delta)^{-1}PK \\ \approx (I + PK)^{-1}P\Delta(I + PK)^{-1}PK\end{aligned}$$

for small $P\Delta$. Hence we choose $W_o = (I + PK)^{-1}P$ and $W_i = (I + PK)^{-1}PK$.

Subnetwork view (C): An important issue is how the choice of controller affects the subnetwork. Assuming that controllers already have been designed for the other subsystems $P^{(i)}$, we do not want $(K, \hat{N})$ to change the conditions too much. Here we model this by looking at the transfer function from disturbances in the subnetwork, l, to the signal

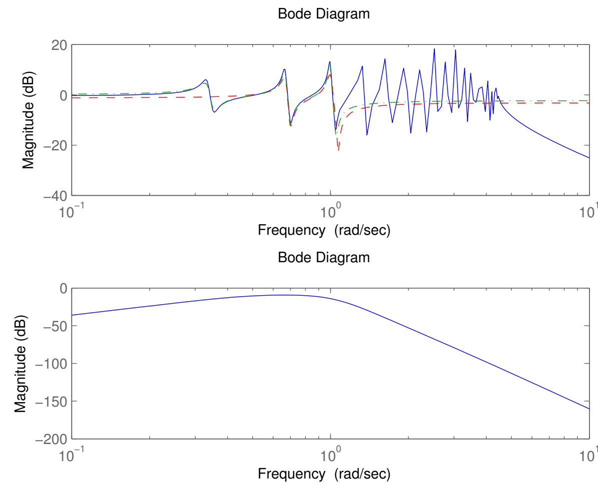

Fig. 3. The upper plot shows magnitude data from the subnetwork N (solid), a fourth-order approximation $\hat{N}$ (dashed), and a sixth-order approximation $\hat{N}$ (dash-dotted) from Example 3. The lower plot shows the weight $P^2/(1+PK)^2$ (from criteria (C)) used in the approximation. There is good agreement between the models for the relevant frequencies.

y that goes back into the subnetwork. If K has been chosen to fulfill requirements from the subnetwork, we then should choose $\hat{N}$ so that the error

$$\begin{aligned}(I + PK)^{-1}P - (I + PK + P\Delta)^{-1}P \\ \approx (I + PK)^{-1}P\Delta(I + PK)^{-1}P\end{aligned}$$

is small. This leads to $W_o = W_i = (I + PK)^{-1}P$.

In all of the cases (A)–(C), it is seen that frequencies for which $(I + PK)^{-1}P$ is large, it is essential that $\hat{N}$ is accurate.

Example 3: In this example, we assume that the subsystem is given by $P(s) = 1/(s + 1)^4$ and we choose K as a PID-controller

$$K(s) = 2\left(1 + \frac{1}{2.5s} + \frac{s}{1 + 0.05s}\right).$$

A Bode diagram of the subnetwork N is shown in Fig. 3. It is seen to be a highly resonant system with many poles and zeros close to the imaginary axis. One could think of N as a lightly damped power system, see Example 1. We choose criteria (C), and use $W_i = W_o = (1 + PK)^{-1}P$, which is also shown in Fig. 3. Furthermore, we assume knowledge of frequency samples on a grid $\{\omega_k\} = \{0.2, 0.21, \dots, 1.2\}$. To obtain $\hat{N}$, we use the method in Section III with $r = 4$ and $r = 6$. In Step 2 we use $\gamma = 0.04$, in both cases. The result is shown in Fig. 3. Notice, that since this example is SISO, the McMillan degree of $\hat{N}$ is equal to r. A load step response test is shown in Fig. 4, with and without the subnetwork model $\hat{N}$ in (13). As can be seen, adding just a low-order model $\hat{N}$ almost brings the behavior back to nominal, even though N is very complex.

V. Control Structure 2: IMC

When $P_1 \neq P_2$, it is no longer a good idea to apply the feedback (13), and a more general structure is needed. We

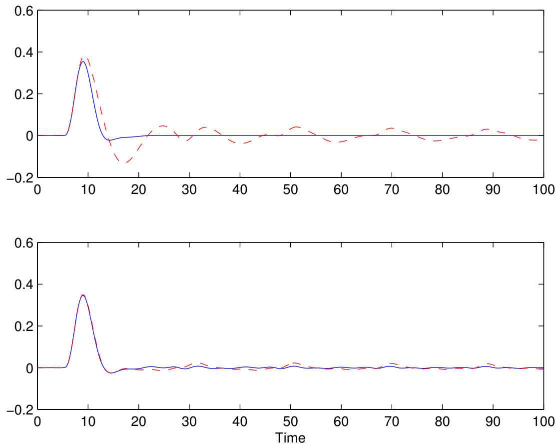

Fig. 4. The load step response test in Example 3. In the upper plot, P is just controlled by K. In the nominal case (solid) it is not connected to the subnetwork N, and in the other case (dashed) it is connected to N. The resonant subnetwork N introduces oscillations in the system. In the lower plot, fourth- and sixth-order models $\hat{N}$ (dashed and solid, respectively) are added to the feedback (13), and are seen to almost bring the performance back to nominal.

choose to use an internal model controller (IMC), see, for example [21]. The IMC structure is seen in Fig. 5. To use IMC we need to assume that $G = (I + P_2 N)^{-1} P_1 \in H_\infty$. Under ideal conditions, when the internal model $\hat{G}$ is exact, $G = \hat{G}$, it is well know that the closed loop in Fig. 5 is internally stable if and only if the controller parameter $Q \in H_\infty$, see [21]. But ideal models are not a realistic assumption here, since the subnetwork N is of high order. If we have a simplified subnetwork model $\hat{N}$, we can use

$$\hat{G} = (I + P_2 \hat{N})^{-1} P_1, \tag{14}$$

in the IMC. Generally it makes more sense to directly match $\hat{G}$ to G, however. This will be the approach taken here. Next, we discuss the criteria (A)–(C) from Section II.

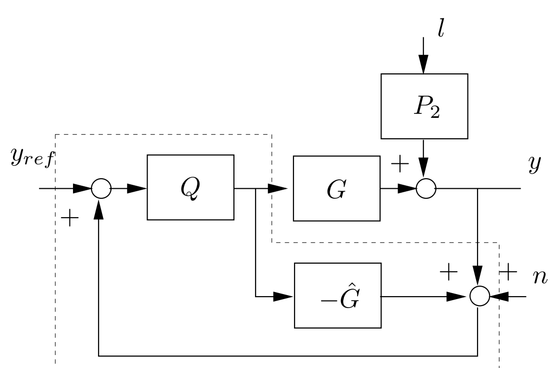

Fig. 5. An internal model controller.

Robustness (A): To study robust in the presence of model uncertainty, we again use the small gain theorem. We have robust stability in the closed loop if

$$\|(G - \hat{G})Q\|_\infty < 1.$$

It is clear that the necessary approximation accuracy depends on the choice of controller parameter Q. A common recommendation is to choose $Q \approx G^{-1}$, at least for low frequencies. Hence, we suggest the approximation problem

$$\text{minimize} \quad \|(G - \hat{G})G^{-1}\|_\infty. \tag{15}$$

Because P_1 and P_2 are known by assumption, we can compute $G(j\omega_k)$, $k = 1 \dots K$, and estimate $\hat{G}$ using the method in Section III with $W_o = I$ and $W_i = G^{-1}$. If $\hat{N}$ is of direct interest, it may be estimated using the Taylor expansion

$$(G - \hat{G})G^{-1} \approx (I + P_2 N)^{-1} P_2 (N - \hat{N})$$

which is valid for small $P_2(N - \hat{N})$.

Performance (B): Here we would like to preserve the response to reference signals. By comparing the nominal ($G = \hat{G}$) system to the perturbed system, we obtain the transfer function

$$GQ - GQ(I + (G - \hat{G})Q)^{-1} \approx GQ(G - \hat{G})Q.$$

Hence we have $W_o = GQ$ and $W_i = Q$. Notice that if the recommendation $Q = G^{-1}$ is followed, we again obtain (15).

Subnetwork view (C): Here we would like to preserve the view from the subnetwork, i.e., the map $l \mapsto y$. This gives the transfer function

$$(I - GQ)P_2 - (I - \hat{G}Q)(I + (G - \hat{G})Q)^{-1} P_2 \approx -GQ(G - \hat{G})QP_2,$$

and we get $W_o = GQ$ and $W_i = QP_2$.

Example 4: In this example, we consider a 2×2 MIMO subsystem P, that is connected to a complex subnetwork. The transfer function $G = (I + P_2 N)^{-1} P_1$ has McMillan degree 208 and is shown in Fig. 6. We assume knowledge of $G(j\omega)$ on a 75-point frequency grid in the interval $[0.1, 3]$ rad/s. We choose to use criterion (A) and thus we solve (15) using the method in Section III.

Step 1 of the approximation procedure turns out to be numerically sensitive in the current implementation in the MIMO case, and a good fit is not obtained in this example. On the other hand, in the weighted SISO case, Step 1 works much more reliably. Hence, to find the poles of $\hat{G}$, we make a relative fit to each entry of G separately:

$$\text{minimize} \quad \|(G_{ij} - \tilde{G}_{ij})/G_{ij}\|_\infty, \quad i, j = 1, 2,$$

using $r = 2$ or $r = 4$ depending on the entry. This gives us a set of 10 stable poles $\{p_i\}_{i=1}^{10}$ to be used in Step 2.

In Step 2, we have a trade off between accuracy and order of $\hat{G}$. In Table I, this trade off is shown. We choose $\gamma = 0.34$, which gives a 13th-order approximation. $\hat{G}$ is plotted together with G in Fig. 6, and there is seen to be a reasonable fit over the interval $[0.1, 3]$ rad/s.

For the control design, we choose $Q \approx \hat{G}^{-1}$, up to about 1 rad/s. Since there are unstable zeros in $\hat{G}$, the inverse cannot be computed directly (Q must be stable). Therefore, we again use Step 1 and Step 2 to obtain a stable approximate

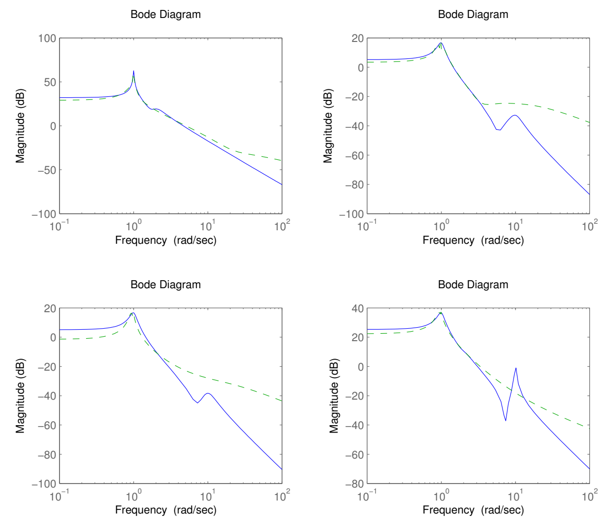

Fig. 6. The 208th-order model G (solid) and the 13th-order approximation $\hat{G}$ (dashed) from Example 4. The relative approximation criteria (15) has been used.

TABLE I
ACCURACY γ AND CORRESPONDING MCMILLAN DEGREE OF $\hat{G}$.

γ	Degree
0.24	17
0.29	17
0.34	13
0.39	10

inverse. In Fig. 7, the sensitivity functions for G connected to the resulting IMC are shown. The closed loop is seen to behave well, with a bandwidth of about 0.6 rad/s. Again, a relatively low-order model of the interconnected system is enough to control the subsystem.

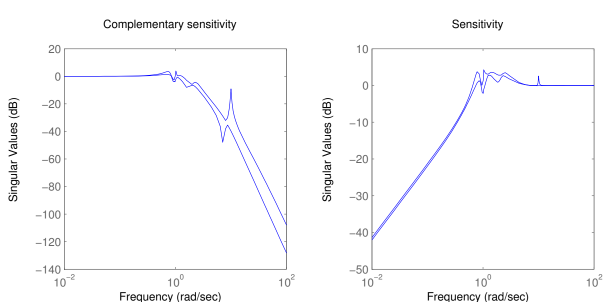

Fig. 7. Sensitivity functions from Example 4.

VI. CONCLUSION AND FUTURE WORK

We have formulated a framework for simplification of subnetwork models using weighted model reduction. The method was illustrated by two examples that showed that a decentralized controller can do very well with only a low-order model of the network, given it is accurate for the important frequencies. The important frequencies are characterized by the weights. We used a two-step method for weighted model reduction of MIMO systems. Step 1 turned out to be numerically sensitive in the MIMO case, but with the slight modifications outlined in Example 4, the method worked well.

Here we have assumed that the subnetwork is given and that it already has working controllers. This assumption is fine if we are connecting a new subsystem to a large working system like the power grid. However, future work should include development of schemes for the simultaneous design of controllers in different parts of the network. The method used here should then be a valuable tool. Also it would be interesting to explore the possibilities of incorporating additional knowledge about the subnetwork, such as topological information and node dynamics. Regarding the proposed model reduction method, one can consider other parameterizations of the MIMO transfer function $\tilde{N}$.

VII. ACKNOWLEDGMENTS

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