

Motion Planning with Wireless Network Constraints

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Abstract—This work discusses feasibility aspects of motion planning for groups of agents connected by a range-constrained wireless network. Specifically, we address the difficulties encountered when trajectories are required to preserve the connectedness of the network. The analysis utilizes a quantity called the *connectivity robustness* of the network, which can be calculated in a distributed fashion, and thus is applicable to distributed motion planning problems arising in control of vehicle networks. Further, these results show that network constraints posed as connectivity robustness constraints have minimal impact on reachability, provided that an appropriate topology control algorithm is implemented. This contrasts with more naïve approaches to connectivity maintenance, which can significantly reduce the reachable set.

I. INTRODUCTION

Current interest in automated vehicle groups has raised several research questions regarding the mobile networking necessary to support this technology. Much recent work has addressed aspects of control over unreliable communication channels, as well as distributed mechanisms for motion control, but there are many networking problems to be resolved before mobile vehicle networks become a practical reality. In this work, we present an approach to one issue in this line of research: motion planning implications of a range-constrained multi-hop wireless network.

The essence of a mobile wireless network is that it provides connectivity among all the members of the vehicle group. For truly autonomous applications, this will be the primary (and perhaps only) communication mechanism available to the vehicles, and so it is essential that care be taken to prevent separation of this network into multiple components. While many factors will affect the availability of links between vehicles, and hence the connectedness of the overall network, we focus in this article on purely geometric aspects of the problem. That is, we ignore any of the usual problems discussed in the mobile networking research such as fading, cross-talk, and delay.

Connectedness of a wireless network poses a unique problem to motion planning research, and is characterized by three main difficulties. The first difficulty lies in the fact that connectedness of a graph is an intrinsically com-

binatorial notion, and is difficult to embed into geometric and analytical models typically used in motion control. Second, connectedness is a *global* aspect of the network, but distributed motion planning must rely on locally available information. Finally, it is not clear how to drive a group of vehicles from one connected configuration to another without disconnecting the network in the process. Indeed, it is not even clear that this is always possible, especially in the presence of obstacles that must be avoided.

This work approaches connectivity-constrained motion planning using a locally computable quantity called the *geometric connectivity robustness*, which quantifies the freedom of individual vehicles to undergo arbitrary motions without disconnecting the network. This function, though non-smooth, is “friendly” enough to be used in optimization models. This is particularly important given the availability of new receding-horizon path-planning methods. Indeed, this latter fact is the primary motivation for studying *feasibility* of connectivity-preserving motions, rather than explicit *construction* of such motions.

II. OUTLINE

The following two Sections discuss the background for motion planning with network constraints, the mathematical formulation of the problem, and some results from previous work that will be utilized in the remainder of the paper.

Sections V and VI present the main results of this work, and show that network constraints posed as connectivity robustness constraints only minimally reduce the reachable set. Section VII briefly addresses the problem of connectivity-constrained motion planning in the presence of obstacles, and gives a preliminary result in this direction.

Finally, Section VIII summarizes the main contributions of the paper, and discusses avenues for future research and applications.

III. BACKGROUND AND MOTIVATION

This paper touches on three main research areas: mobile wireless networking, path planning, and distributed motion control. The first two in particular are extremely large fields,

and for the purposes of this work we take only a very limited view of either subject.

Recently, a significant research effort has turned toward geometric and optimization-oriented ideas for mobile wireless networks, and this has formed the basis for our treatment of the subject. We refer the reader to the work of Rodoplu and Meng [6], as well as that of Li and Halpern [4] for some recent geometric efforts in wireless routing. The works of Ramanathan and Rosales-Hain [5], as well as Wattenhofer *et al.* [9] are also particularly relevant regarding optimization-based models and distributed topology control algorithms for wireless networks. We also refer the reader to Spanos and Murray [8] for an introduction to the notion of geometric connectivity robustness, and some preliminary applications in distributed motion and topology control.

Within the area of path-planning, we are primarily interested in modern optimization-based receding-horizon methods, e.g. the work of Schouwenaars, How, and Feron [7]. These methods forgo analytical construction of controls (as is common in, e.g., potential-based methods), and instead rely on algorithmic search to produce optimal controls subject to obstacle-avoidance constraints. This approach to motion control, although computationally intensive, allows for very general constraints. Thus, this is ideally suited for the constraint-based connectivity maintenance discussed in this work.

Finally, we briefly review a few approaches to distributed motion control. The work of Leonard and Fiorelli [3] approaches the problem of formation control using a potential-based virtual-leader architecture. For a graph-theoretic approach to formation control, we refer the reader to Fax and Murray [2]. This work develops distributed (classical) feedback controls for formation maintenance based on information available on a wireless network. For a distributed receding-horizon approach, which is most relevant to our constraint-based formalism, we direct the reader the recent work of Dunbar and Murray [1].

To understand the motivation for this work, consider the situations depicted in Figures 1 and 2. Here a network of vehicles begins in a densely connected configuration, but must undergo some maneuver either to reconfigure the formation, or to negotiate some obstacle. Both of these situations require that links be given up during the maneuver, due to broadcast range constraints. However, allowing complete freedom to give up links introduces the risk of disconnecting the network during the transient motion. On the other hand, requiring that *all* links be preserved greatly reduces the set of reachable configurations. This work attempts to strike a balance between these two undesirable extremes.

IV. SETUP AND NOTATION

We consider a set V of N vehicles, labeled $i = 1, 2, \dots, N$. These vehicles are modeled as point particles in the plane, and each has an associated position vector $\mathbf{q}_i \in \mathbf{R}^2$. We will make use of the Euclidean distances

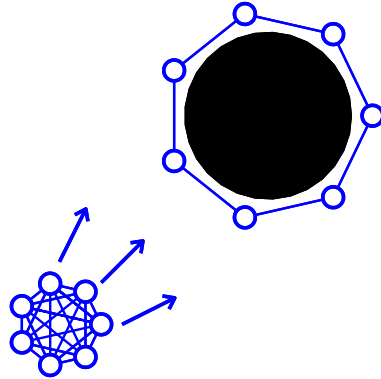


Fig. 1. A maneuver in which a vehicle network surrounds an impenetrable obstacle. The interconnections indicate wireless links between vehicles. Note that due to range constraints the topology of the network in the final state is very different from that of the initial state.

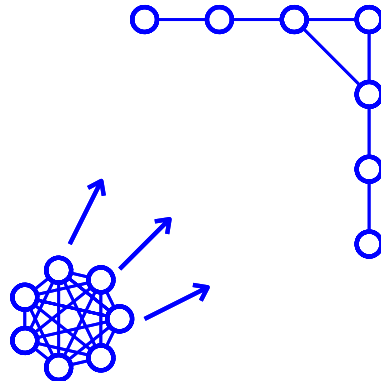


Fig. 2. A reconfiguration maneuver. Connectivity robustness provides a mechanism to constrain the vehicles' motion so that the network is not disconnected during such a maneuver, while simultaneously ensuring that all connected configurations are reachable from one another.

between the vehicles, denoted $d_{ij} = \|\mathbf{q}_i - \mathbf{q}_j\|$. We will use the symbol \mathbf{Q} to denote the N -tuple $(\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_N)$.

Our target application lies in control of vehicle groups, which are second-order systems, but for the sake of simplicity we will assume first-order dynamics of the form

$$\dot{\mathbf{q}}_i = \mathbf{u}_i, \quad \mathbf{u}_i \in \mathbf{R}^2.$$

This approximation amounts to a local controllability assumption on a sufficiently large length-scale. This assumption is reasonable given that the broadcast range for most wireless networks is at least one order of magnitude larger than the length of a vehicle. Further, many applications involving robots instead of autonomous vehicles will truly be kinematic.

Each of the vehicles has an associated (fixed) broadcast range $r_i \in \mathbf{R}$, and can thus communicate with other vehicles within a circle of radius r_i centered at \mathbf{q}_i . We will only consider bidirectional communication, and so a communication link exists between vehicles i and j if and only if they are each within the other's communication

range. This induces a graph $C = (V, E_C)$, in which

$$(i, j) \in E_C \Leftrightarrow \min\{r_i, r_j\} - d_{ij} \geq 0.$$

The graph C will be called the *communication network*. Note that C in fact depends on the \mathbf{q}_i variables, and so each configuration of the vehicles in \mathbf{R}^2 induces an associated communication network. We denote the set of vehicles to which vehicle i is connected $N_C(i)$, the *communication neighborhood*.

We will also make use of another undirected graph $I = (V, E_I)$, which we call the *information flow*. This graph represents an abstract design requirement for the network, and indicates some minimum connectivity. It could, for example, specify which vehicles must communicate in order to implement some formation control algorithm as in, e.g., Fax and Murray [2].

The notation $N_I(i)$ is used to denote the set of vehicles to which i is connected in the information flow, and we will call this set the *information neighborhood*. For the moment, we will assume that the graph I is given in advance by a designer, but we will also discuss the case where I need only be connected, and can be adapted on-line to better facilitate motion of the network.

We say that a communication network C is *I-connected* if and only if any two vehicles connected by an edge in I are also connected by a *path* of at most *two* edges in C . Thus, every connection in the information flow must be implementable in the communication network with at most two hops. This path-length limitation serves as a crude proxy for a delay-time restriction, which is essential for good control performance.¹

In order to discuss the connectivity robustness of the network, we must first quantify the robustness of two-hop paths to arbitrary displacements of the vehicles. To do so we will make use of the following quantity, the *path robustness* of the path (i, k, j) :

$$P(i, j, k) \doteq \min \{ \min\{r_i, r_k\} - d_{ik}, \min\{r_j, r_k\} - d_{jk} \}.$$

We now define the *geometric connectivity robustness* associated with vehicle i , relative to the information flow I :

$$R_I(i) \doteq \min_{k \in N_I(i) \cup N_C(i)} \left[\frac{1}{2} \max_{j \in N_C(i)} P(i, j, k) \right]$$

Note that this quantity depends on the configuration variables \mathbf{q}_i ; when we wish to show this dependence we will write $R_I(i; Q)$. Below, we recall some results from Spanos and Murray [8] pertaining to the connectivity robustness function and its relationship to I -connectedness.

Proposition 1: Let the information flow I be given, and suppose $R_I(i) \geq 0$ for all $i \in V$. Then, the communication network C is I -connected.

¹The restriction to *two* hops is, in part, arbitrary and can be modified without much consequence in the upcoming analysis. We restrict our attention to the two-hop case because it makes the upcoming robustness calculations truly distributed, in the sense that each vehicle needs only to exchange information within its *communication neighborhood*.

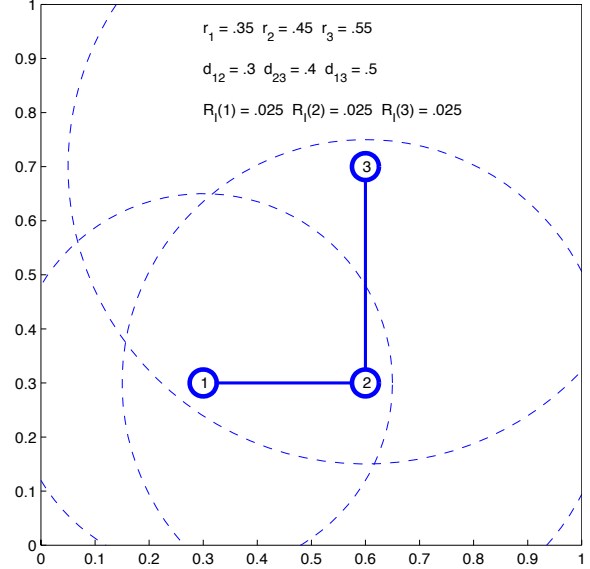


Fig. 3. A simple network and the connectivity robustness quantities (the information flow I requires all vehicles to connect to each other). Nodes 1 and 3 use 2 as an intermediary to implement a two-hop (1, 3) connection.

Corollary 1: If $R_I(i) \geq 0$ for all $i \in V$, and the graph I is connected, then the communication network C is also connected.

Proposition 2: Suppose $R_I(i) \geq 0$ for all $i \in V$, and I is connected, and consider an arbitrary set of displacements $\mathbf{q}_i \rightarrow \mathbf{q}_i + \Delta_i$, with $\|\Delta_i\| < R_I(i)$. Then, the communication network C , in the displaced configuration, is connected.

The set specified by a constraint of the form $R_I(i) \geq 0$ is typically non-convex, but it is always a star-convex polygon, and can easily be included in mixed-integer optimization models such as those used in [7].

V. CONNECTED MOTION WITH FIXED INFORMATION FLOW

We now address the problem of maintaining I -connectedness of a network while undergoing some re-configuration maneuver. Our main result in this section shows that all I -connected configurations are reachable from one another, using only trajectories that preserve I -connectedness.

First, it is useful to note that just preventing the loss of communication links is not an efficient way to approach this problem. This can remove even very simple reconfigurations from the reachable set. To see this, consider the two configurations depicted in Figure 4, which are both I -connected for any information flow I . It is impossible to drive the vehicles between these two configurations without losing some of the links, and so any such requirement will unduly constrain the motion of the vehicle group. We will now show that a connectivity robustness constraint eliminates this difficulty.

A *motion* γ between two configurations $Q =$

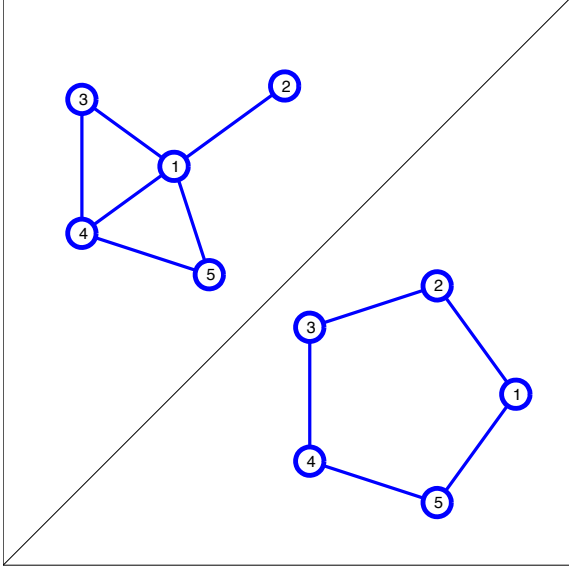


Fig. 4. Both of these configurations are I -connected for any I , but neither is reachable from the other without loss of a link. Adding a connectivity robustness constraint disallows motions that violate the connectivity requirement, and still allows transition between any two I -connected configurations.

$(\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_N)$ and $\tilde{\mathbf{Q}} = (\tilde{\mathbf{q}}_1, \tilde{\mathbf{q}}_2, \dots, \tilde{\mathbf{q}}_N)$ is a curve

$$\begin{aligned} \gamma : t \in [0, 1] &\rightarrow Q(t) \in \mathbf{R}^{2N} \\ \gamma(0) &= \mathbf{Q} \\ \gamma(1) &= \tilde{\mathbf{Q}} \end{aligned}$$

which is continuous in each of the \mathbf{q}_i slots. One can compose two motions γ_1 and γ_2 in the usual way, provided that $\gamma_1(1) = \gamma_2(0)$. The composite motion is given by

$$(\gamma_1 \circ \gamma_2)(t) \doteq \begin{cases} \gamma_1(2t) & 0 \leq t \leq \frac{1}{2} \\ \gamma_2(2t - 1) & \frac{1}{2} < t \leq 1 \end{cases}.$$

We will also use the inverse of a motion, denoted γ^{-1} , and which is given by

$$\gamma^{-1}(t) \doteq \gamma(1 - t).$$

A motion γ is said to be an I -connected motion if and only if

$$R_I(i, \gamma(t)) \geq 0 \quad \text{for all } t \in [0, 1].$$

With this definition, we can now state the main result of this section.

Proposition 3: Let I be given, and let \mathbf{Q} and $\tilde{\mathbf{Q}}$ be any two I -connected configurations. Then, there exists an I -connected motion γ such that $\gamma(0) = \mathbf{Q}$ and $\gamma(1) = \tilde{\mathbf{Q}}$.

Proof: Consider the ‘‘centroid’’ of the configuration \mathbf{Q} , i.e. $\mathbf{q}_c = \frac{1}{N} \sum_{i \in V} \mathbf{q}_i$; we will use this point to exploit the star-convexity of the set of I -connected configurations. Recall that a star-convex set S has at least one ‘‘center’’ point s_c , and that any convex combination of s_c and any other point in S is also in S .

Now, consider the following two motions, one taking \mathbf{Q} to $\mathbf{Q}_c = (\mathbf{q}_c, \mathbf{q}_c, \dots, \mathbf{q}_c)$, and one taking $\tilde{\mathbf{Q}}$ to \mathbf{Q}_c :

$$\begin{aligned} \gamma_1(t) &= ((1-t)\mathbf{q}_1 + t\mathbf{q}_c, \dots, (1-t)\mathbf{q}_N + t\mathbf{q}_c) \\ \gamma_2(t) &= ((1-t)\tilde{\mathbf{q}}_1 + t\mathbf{q}_c, \dots, (1-t)\tilde{\mathbf{q}}_N + t\mathbf{q}_c). \end{aligned}$$

Observe that both of these motions are non-expansive. That is,

$$\|\mathbf{q}_i(t_2) - \mathbf{q}_j(t_2)\| \leq \|\mathbf{q}_i(t_1) - \mathbf{q}_j(t_1)\| \quad \text{for all } t_2 \geq t_1$$

holds for either γ_1 or γ_2 . Now, the robustness function $R_I(i, \gamma(t))$ increases monotonically with decreasing distance, for each d_{ij} term. Since all the d_{ij} terms are non-increasing, we have that the robustness is non-decreasing. By hypothesis, the two configurations \mathbf{Q} and $\tilde{\mathbf{Q}}$ are I -connected, and so we have

$$\begin{aligned} R_I(i, \gamma_1(t)) &\geq 0 \quad \text{for all } t \in [0, 1], \\ R_I(i, \gamma_2(t)) &\geq 0 \quad \text{for all } t \in [0, 1]. \end{aligned}$$

From Proposition 1, this implies that the network remains I -connected throughout the course of these two motions.

To complete the proof, consider the composite motion $\gamma_c = \gamma_1 \circ \gamma_2^{-1}$, which is a motion from \mathbf{Q} to $\tilde{\mathbf{Q}}$. From the previous argument, this is an I -connected motion, and the desired result follows. ■

The construction for γ_c used in this proof is useful for theoretical purposes, but is practically very conservative in that it requires all the vehicles to coalesce at a point before expanding to the final configuration. However, this motion is a very useful starting point for optimization solvers, because it is analytically constructible and *a priori* feasible. The centroid position can also be computed using distributed methods (see e.g. [2]), and so this is also useful for distributed applications. Further, although the set of all I -connected motions between two configurations \mathbf{Q}_1 and \mathbf{Q}_2 is not a vector space, it is a convex set under the usual addition and scalar multiplication operations. Thus, having an *a priori* distributed online construction of an element of this set will be useful for receding-horizon path-planning applications.

Figure 5 shows a simple demonstration of this result with a piece-wise linear reconfiguration maneuver. Vehicle 2 stays close enough to 3 until vehicle 1 can act as an intermediary to 4. Naïve linear interpolation between the initial and final configuration fails to preserve I -connectedness in the transient motion, because vehicle 2 loses access to 4 through the two-hop path (2, 3, 4) before the path (2, 1, 4) becomes available.

VI. CONNECTED MOTION WITH VARIABLE INFORMATION FLOW

The previous section discussed the feasibility of I -connected motion where the graph I was given in advance and was fixed throughout the course of the motion. This is significantly more flexible than requiring all links to be

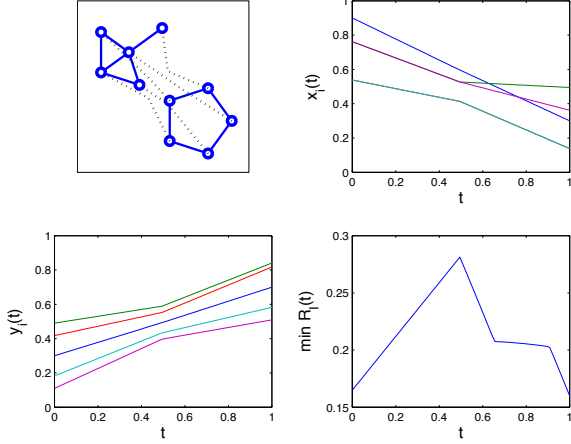


Fig. 5. A piece-wise linear reconfiguration trajectory that guarantees I -connectedness throughout the motion. The broadcast ranges were all set to $r_i = 0.4$. The required information was the complete graph, and so all the robustness quantities are equal (depicted in the bottom right).

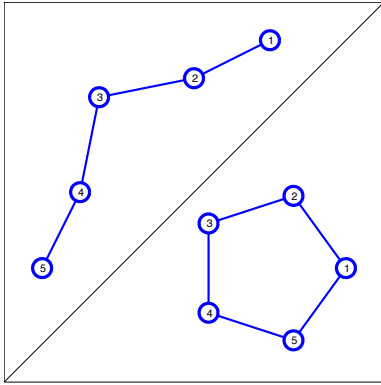


Fig. 6. For a complete graph I , the lower right configuration is I -connected, but the upper left is not. The use of an adaptable information flow algorithm allows provably connected transitions between any two connected configurations using a robustness constraint.

preserved, but it still eliminates some reconfigurations from the reachable set. Specifically, there is no reason to expect that, given two different information flows I_1 and I_2 , an I_2 -connected configuration $\tilde{\mathbf{Q}}$ will be reachable from an I_1 -connected configuration \mathbf{Q} . To see an instance of this problem, see Figure 6.

In this section, we relax the fixed information flow, and use a feedback mechanism to update the graph I based on the connectivity robustness of the network. This additional flexibility will allow us to use a constraint of the form

$$\begin{aligned} R_I(i, \mathbf{Q}) &\geq 0, \\ I &= f(\mathbf{Q}) \end{aligned}$$

to ensure connectedness of the network. Note that our goal now is not to preserve I -connectedness, but rather the general graph-theoretical notion of connectedness, and so this allows for much more general reconfiguration maneuvers. In fact, we will show that *all* connected configurations are

reachable from one another under motions which satisfy the above constraint.

We will make use of a robustness-based algorithm for constructing a sparse information flow presented in [8]. Specifically, consider the graph $I_s(\mathbf{Q}) = (V, E_s)$, with the edge set E_s defined as follows:

$$(i, j) \in E_s(\mathbf{Q}) \Leftrightarrow \min\{r_i, r_j\} - d_{ij} = R_C(i, \mathbf{Q}).$$

Note that this function is defined in terms of the robustness of the *communication network* itself, rather than some superimposed information flow.

It was shown in [8] that the graph $I_s(\mathbf{Q})$ is connected if and only if $C(\mathbf{Q})$ is connected, so a constraint that preserves the connectedness of I_s also preserves the connectedness of C . This graph is also typically much sparser than C , and so motion constraints based on this graph are much less conservative than those based directly on C . Furthermore, the fact that I_s adapts to changes in \mathbf{Q} and C makes constraints based on this graph much more flexible than those based on any fixed information flow I .

We will say that a motion γ is a *connected motion* if the communication network $C(\gamma(t))$ is connected for all t . From the previous comments regarding I_s , γ is a connected motion if and only if $I_s(\gamma(t))$ is connected.

We now state the main result of the paper, which shows that any connected configuration is reachable from any other configuration using a connected motion that respects a robustness constraint based on I_s , regardless of the connectivity structure of the initial and final configurations (i.e. without an assumption about I -connectedness).

Proposition 4: Let \mathbf{Q} and $\tilde{\mathbf{Q}}$ be two configurations, and suppose $C(\mathbf{Q})$ and $C(\tilde{\mathbf{Q}})$ are both connected. Let R_m be the minimum robustness of these two configurations. Then, there exists a motion $\gamma(t)$ from \mathbf{Q} to $\tilde{\mathbf{Q}}$ satisfying

$$R_{I_s}(i, \gamma(t)) \geq R_m \text{ for all } i \in V, t \in [0, 1].$$

Proof: We will again exploit the centroid, $\mathbf{q}_c = \frac{1}{N} \sum_i \mathbf{q}_i$. As in the previous proof, consider the following two motions, linking \mathbf{Q} and $\tilde{\mathbf{Q}}$ to \mathbf{Q}_c , the configuration in which all vehicles are at the centroid position \mathbf{q}_c :

$$\begin{aligned} \gamma_1(t) &= ((1-t)\mathbf{q}_1 + t\mathbf{q}_c, \dots, (1-t)\mathbf{q}_N + t\mathbf{q}_c) \\ \gamma_2(t) &= ((1-t)\tilde{\mathbf{q}}_1 + t\mathbf{q}_c, \dots, (1-t)\tilde{\mathbf{q}}_N + t\mathbf{q}_c). \end{aligned}$$

Recall that these are non-expansive motions, and so each d_{ij} term decreases as the vehicles approach \mathbf{Q}_c . This implies that

$$\begin{aligned} R_{C(\gamma_1(t_2))}(i, C(\gamma_1(t_2))) &\geq R_{C(\gamma_1(t_1))}(i, C(\gamma_1(t_1))) \\ R_{C(\gamma_2(t_2))}(i, C(\gamma_2(t_2))) &\geq R_{C(\gamma_2(t_1))}(i, C(\gamma_2(t_1))) \end{aligned}$$

for all $i \in V$ and $t_2 \geq t_1$. Now, I_s is a sub-graph of C and so its robustness is bounded below by the robustness of C itself. Since we always have $R_C(i) \geq R_m$, we now know that

$$R_{I_s}(i, \gamma_k(t)) \geq R_m \text{ for all } i \in V, t \in [0, 1]$$

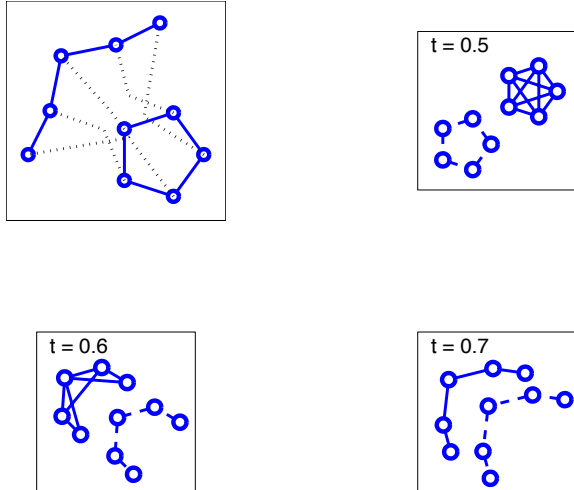


Fig. 7. A motion between two connected configurations, and snapshots of the communication network (solid) and the graph I_s (dashed) at three different times. As the network reconfigures, the graph adapts I_s and allows the (1,5) link to be broken.

along both motions γ_1 and γ_2 . Again consider the composite motion, $\gamma_c = \gamma_1 \circ \gamma_2^{-1}$. This is a motion from \mathbf{Q} to $\tilde{\mathbf{Q}}$, satisfying the robustness constraint on I_s , which was the desired result. ■

This proposition shows that it is in fact possible to use connectivity robustness as a constraint on very general motions between any two connected configurations. Figure 7 shows an example of a motion designed using this approach.

VII. DIFFICULTIES WITH OBSTACLES

Here we very briefly address the problem of obstacles. By an obstacle, we mean a closed region $F \subset \mathbf{R}^2$ which is forbidden to the vehicles, so $\mathbf{q}_i \notin F$ for all $i \in V$. An interesting asymmetry arises for reachability in this situation.

We will call a configuration \mathbf{Q} *unobstructed* if and only if there exists a contractive motion respecting the obstacle constraints which takes all vehicles to the centroid configuration, \mathbf{Q}_c . Any other configuration will be called *obstructed*. Proposition 4 fails to hold for obstructed configurations, but we can obtain the following result, which we present without proof due to length limitations.

Proposition 5: Let \mathbf{Q} be an unobstructed connected configuration, and let $\tilde{\mathbf{Q}}$ be any other connected configuration. Then, there exists a connected motion γ taking \mathbf{Q} to $\tilde{\mathbf{Q}}$, avoiding the forbidden region F and satisfying

$$R_{I_s}(i, \gamma(t)) \geq 0 \quad \text{for all } i \in V, t \in [0, 1].$$

VIII. SUMMARY, CONCLUSIONS, AND FUTURE WORK

We have explored applications of the connectivity robustness function in motion planning with wireless network connectivity constraints. This article has focused on proving that both I -connectedness and general graph-theoretical

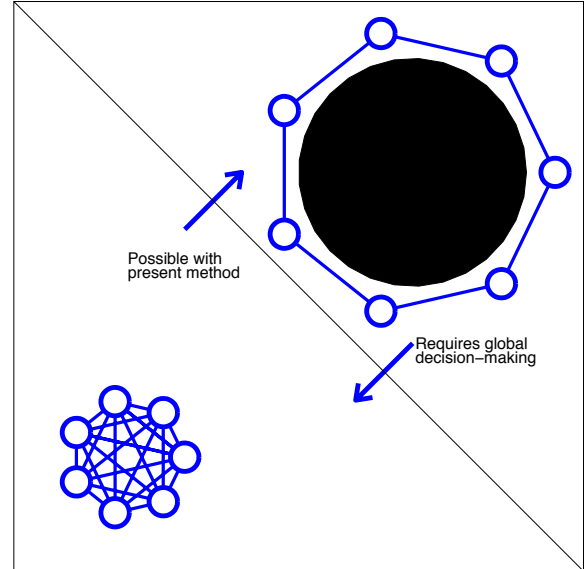


Fig. 8. Any connected configuration is reachable from an *unobstructed* configuration, but the local methods presented in this paper cannot yet handle obstructed configurations.

connectedness can be preserved by enforcing connectivity-robustness constraints, without significantly reducing the reachable set. Future work will focus on resolving problems arising in the presence of obstacles.

Although we have devoted our efforts here to analysis and proof of feasibility, the main contribution of this work is that it provides a tractable mathematical framework for ensuring connectivity based on motion constraints. Combined with a receding-horizon control approach, connectivity robustness allows one to rely on algorithmic procedures to provably preserve connectivity of the network.

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