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Submitted, 2006 American Control Conference

# Robust Control Over a Packet-based Network

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***Abstract*— In this paper, we consider a robust network control problem. We consider linear unstable and uncertain discrete time plants with a network between the sensors and controller and the controller and plant. We investigate two defining characteristics of network controlled systems and the impact of uncertainty on these. Namely, the minimum data rates required for the two networks and the tolerable data drop out in the form of packet losses. We are able to derive sufficient conditions in terms of the minimum data rate and minimum packet arrival rate to ensure stability of the closed loop system.**

## I. Introduction

Recently, networked control systems (NCS) have gained great attention from both the control community and the network and communication community. When compared with classical feedback control system, networked control systems have many advantages. For example, they can reduce the system wiring, make the system easy to operate and maintain and later diagnose in case of malfunctioning, and increase system agility [16]. In spite of the great advantages that the networked control architecture brings, inserting a network in between the plant and the controller introduces many problems as well. For instance, zero-delayed sensing and actuation, perfect information and synchronization are no longer guaranteed in the new system architecture as only finite bandwidth is available and packet drops and delays may occur due to network traffic conditions. These must be revisited and analyzed before any practical networked control systems are built.

In the past decade, many researchers have spent effort on those issues and a number of significant results were obtained and many are in progress. Many of the aforementioned issues are studied separately. Tatikonda [15] and Sahai [11] have presented some interesting results in the area of control under communication constraints. Specifically, Tatikonda gave a necessary and sufficient condition on the channel data rate such that a noiseless LTI system in the closed loop is asymptotically stable. He also gave rate results for stabilizing a noisy LTI system over the digital channel. Sahai proposed the notion of anytime capacity to deal with real time estimation and control for a networked control system. In our companion paper [13], the authors have considered various rate issues under finite bandwidth, packet drops and finite controls. The effect of pacekt loss and delay on state estimation was studied by the work of Sinopoli, et. al. in [2]. It has further been investigated by many researchers including the present authors in [12] and [5].

One of the hallmarks of a good control system design is that the closed loop remain stable in the presence of uncertainty [3], [4]. While the researchers in [7] studied the problem of LQG control across a packet dropping networks, not many have considered the norm bounded uncertainty investigated in the present paper. We examine the impact of a norm bounded uncertainty on the network control system and provide sufficient conditions for stability in terms of the minimum data rates and packet arrival rates for the networks.

The paper is organized as follows. In Section II, we present the mathematical model of the closed loop system and state our assumptions. In Section III, we state the sufficient conditions for closed loop stability for the case where a network connects the sensors to the controller. In Section IV, we state the sufficient stability conditions where in addition there is a network between the controller and the plant. For both sections we obtain results for scalar and general vector cases. Conclusions and future work are given in the last section.

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Work supported in part by NSF ITR grant CCR-0326554.

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## II. Problem Set Up

We consider linear discrete time systems with a norm bounded uncertainty in the $A$ matrix. We will investigate two NCS that we will define by the type of networks embedded in the control loop. The first NCS considered has a network between the measurement sensors and the controller, with the controller then directly connected to the actuators/plant. The second NCS will also include a network between the controller and the actuators/plant. These two network types and depicted in Figures 1 and 2. The networks are defined in terms of their data rates and probability of dropping packets. We would consider any packet delays as losses, *i.e.*, we do not use delayed packets for estimation or control.

The following equations represent the closed loop system for NCS I (Figure 1).

$$x_{k+1} = (A+\Delta_k)x_k + Bu_k \quad (1)$$

$$y_k = \lambda_k C x_k \quad (2)$$

where $x_k \in \mathbb{R}^n$ is the state of the system, $u_k \in \mathbb{R}^m$ is the control input, $y_k \in \mathbb{R}^p$ is the output of the system, and $\lambda_k$ are Bernoulli i.i.d random variable with parameter $\lambda$, *i.e.*, $E[\lambda_k] = \lambda$ for all $k$. $\Delta_k$ satisfies $\Delta_k^T \Delta_k \leq K^2 I$ for all $k$. We also assume the initial condition $x_0 \in \mathbb{R}^n$ is bounded. The matrix $A$ is assumed to be unstable without loss of generality as for any matrix $A$, we can always do some state transformation to decompose the states into stable ones and

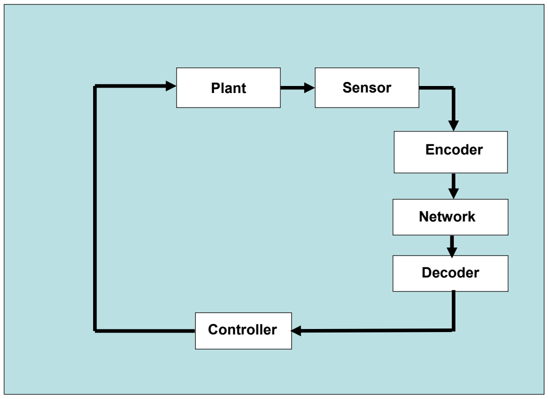


Fig. 1. Networked Control System I

unstable ones. The stables ones converges to the origin for any given initial condition even without any control. The network has data rate $R+n$, *i.e.*, the network can deliver a packet of $R+n$ bits of information per discrete time step, which can be dropped depending on what value $\lambda$ is. The bits of the packet are allocated such that $R$ bits are reserved for the magnitude of the state signals and $n$ bits are to indicate the sign of each of the state signals.

The corresponding scalar system is represented by

$$x_{k+1} = (a+\Delta_k)x_k + u_k, \quad (3)$$
$$y_k = \lambda_k x_k, \quad (4)$$

where $a > 1$ and $|\Delta_k| \leq K$ for all $k$.

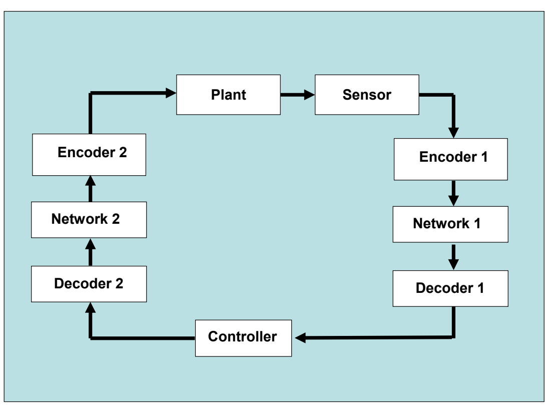


Fig. 2. Networked Control System II

For NCS II (Figure 2), the closed loop system are represented by

$$x_{k+1} = (A+\Delta_k)x_k + \gamma_k B u_k, \quad (5)$$
$$y_k = \lambda_k C x_k, \quad (6)$$

where the parameters are exactly the same as in (1) except that $\gamma_k$ are Bernoulli i.i.d random variable with parameter $\gamma$, *i.e.*, $E[\gamma_k] = \lambda$ for all $k$. The network one has data rate $R_1+n$ and the network two has data rate $R_2+n$.

The scalar version is

$$x_{k+1} = (a+\Delta_k)x_k + \gamma_k u_k, \quad (7)$$
$$y_k = \lambda_k x_k. \quad (8)$$

For all the rest of the paper, if $x \in \mathbb{R}^n$, $|x|$ means the Euclidian norm of it. If $X \in \mathbb{R}^{n\times n}$, $|X|$ means the induced matrix norm of it. $\log$ is assumed to have base 2.

As packet drops introduce unavoidable randomness into the system, the classical notion of stability for deterministic systems in the sense of Lyapunov [14] is not adequate. The definition of stability in a probabilistic setting is not new. It is usually considered when there is inherent randomness in the system, for example, in the jump linear systems [6] or in stochastic hybrid systems [1]. In [6], the authors have given the most frequently seen definitions of stochastic stability. We use almost sure stability in our problem formulation which is defined below.

*Definition 1:* System (1) is called almost sure stable if

$$P\{\lim_{k\to\infty} |x_k(x_0,\omega)| = 0\} = 1,$$

where $\omega$ is the underlying randomness for the closed loop system.

Stability in the sense of Lyapunov requires that for any $\varepsilon > 0$, there exists a time $T$, such that for all $k \geq T$, $|x_k| \leq \varepsilon$. For almost sure stability, however, it is allowed that $x_k > \varepsilon$ for any $k > 0$ and for any $\varepsilon > 0$ which may occur with arbitrary low probability .

## III. Gurantees for Closed Loop Stability for Network Type I

### A. Scalar Systems

All the lemmas in this section are for the scalar system (3) in NCS I (Figure 1).

*Lemma 2:* Assume $\lambda = 1$, *i.e.*, there is no packet drop and $R = \infty$. Then the closed loop system is exponentially stable if $K < 1$.

**Proof:** At time 1, the controller knows $x_0$ but not $\Delta_0$. Let $u_0 = -ax_0$ and so $x_1 = \Delta_0 x_0$. Similarly, let

$$u_1 = -ax_1 = -a\Delta_0 x_0,$$

we have

$$x_2 = \Delta_1\Delta_0 x_0.$$

Following this procedure, by letting $u_k = -ax_k$, we have

$$x_n = \Delta_n\Delta_{n-1}\cdots\Delta_0 x_0.$$

Hence if $K < 1$, then

$$|x_n| \leq K^n|x_0|,$$

which converges to zero exponentially fast.

QED

*Lemma 3:* Assume $\lambda = 1$ and $K < 1$. Then the closed loop system is exponentially stable if $R$ satisfies

$$R > R_{\min} = \log a - \log(1-K).$$

**Proof:** Encode $\bar{x}_k$ to be the most $R$ significant bits of $x_k$, hence if $|x_k| \le 2^M$, then

$$|\epsilon_k| = |x_k - \bar{x}_k| \le 2^{M-R}.$$

Now let $u_k = -a\bar{x}_k$, we have that

$$|x_{k+1}| = |a\epsilon_k + \Delta_k x_k| \le a2^{M-R} + K2^M.$$

Hence if $|x_{k+1}| < 2^M$, *i.e.*, the upper bound on the state norm is shrinking, or

$$R > R_{\min} = \log \frac{a}{1-K} = \log a - \log(1-K),$$

the closed loop system will be exponentially stable.

QED

*Lemma 4:* Assume $\lambda < 1$ and $R = \infty$. Then the closed loop system is almost sure stable if $K < 1$ and

$$\lambda > \lambda_{\min} = \frac{\log(a+K)}{\log(a+K) - \log K}. \quad (9)$$

**Proof:** Suppose $\lambda_0 = 1$, *i.e.*, $x_0$ is received. Let $u_0 = -ax_0$, then $x_1 = \Delta_0 x_0$. Now assume $x_1$ is not received. Applying no control we get

$$x_2 = (a + \Delta_1)\Delta_0 x_0.$$

Suppose $x_2$ is received, then we can let

$$u_2 = -ax_2 = -a(a+\Delta_1)\Delta_0 x_0.$$

Hence

$$x_3 = \Delta_2(a+\Delta_1)\Delta_0 x_0.$$

Following this procedure, whenever $x_k$ is received, $u_k = -ax_k$ and whenever $x_k$ is not received, $u_k = 0$. Then we can write

$$x_n = \prod_i \Delta_i \prod_j (a+\Delta_j) x_0,$$

where $i$ indicates that packet at time $i$ is received and $j$ indicates that packet at time $j$ is dropped. For $n$ sufficiently large, from the weak law of large numbers [10], with arbitrary high probability that $n\lambda$ packets are received and $n(1-\lambda)$ packets are dropped. Hence

$$|x_n| \le K^{n\lambda}(a+K)^{n(1-\lambda)}|x_0| = [K^\lambda (a+K)^{1-\lambda}]^n |x_0|,$$

is true almost surely for $n$ sufficiently large. Therefore, if

$$K^\lambda (a+K)^{1-\lambda} < 1,$$

or

$$\lambda > \frac{\log(a+K)}{\log(a+K) - \log K},$$

the closed loop system is almost surely stable.

QED

The lower bound on $\lambda$ is shown in Figure 3 for different values of $(a, K)$. Note that $\lambda_{min} \to 1$ when $K \to 1$ meaning that zero drop rate is required and $\lambda_{min} \to 0$ when $K \to 0$ meaning if there is no uncertainty we only require a nonzero arrival rate to be almost surely stable. In addition $\lambda_{min} \to 1$ as $a \to \infty$, meaning all packets must be received. This also means that all the contour lines approach the y-axis in Figure 3.

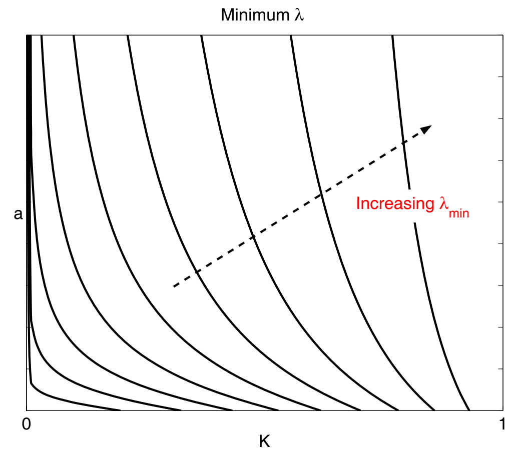


Fig. 3. Minimum packet arrival rate for NCS I. The contour lines are constant values of $\lambda_{\min}$ given in Eqn. 10. The closed loop system is almost surely stable for all $(a, K)$ pairs to the left of the contour lines.

*Lemma 5:* In proving Lemma 2 and Lemma 4, the proposed control law is optimal in the sense that it minimizes the upper bound of the state variable each time step.

**Proof:** Let's first revisit the proof in Lemma 2. At the first time step, $u_0 = -ax_0$ is indeed the best control law as $|x_1| \le |\Delta_0||x_0| \le K|x_0|$. For any other control law, it is not guaranteed that

$$|x_1| \le K|x_0|$$

as there always exists allowable $\Delta_0$ such that

$$|x_1| > K|x_0|.$$

For example, if we let

$$u_0 = -ax_0 + 0.5K,$$

then

$$\Delta_0 = \mathrm{sgn}(x_0)K$$

leads to the fact that

$$x_1 = K(0.5 + |x_0|) > K|x_0|.$$

Therefore setting the control such that it cancels only the known state is the best choice in the sense that it minimizes the upper bound of the state.

Now consider the control strategy in proving Lemma 4. Consider the same scenario that $x_1$ is dropped. The control law $u_1 = F(x_0)$, as $x_0$ is the only known factor to the controller. Then

$$x_2 = (a\Delta_0 + \Delta_0\Delta_1)x_0 + F(x_0).$$

If $F(x_0) = 0$, then we know for sure

$$|x_2| \leq (aK + K^2)|x_0|.$$

For any other $F(x_0) \neq 0$, there always exists allowable $\Delta_0$ and $\Delta_1$ such that

$$|x_2| > (aK + K^2)|x_0|.$$

Hence the proposed control law is optimal.

QED

*Remark 6:* The above lemma basically tells that if we only know the upper bound but not the distribution of the uncertainties, the best control law is that it either compensate the known state or does nothing.

*Lemma 7:* Assume $K < 1$ and

$$\lambda > \frac{\log(a+K)}{\log(a+K) - \log K}.$$

Then the closed loop system is almost sure stable if $R$ satisfies

$$R > R_{\min} = \log a - \log(2^{\frac{\lambda-1}{\lambda}\log(a+K)} - K).$$

**Proof:** Use the same encoding and decoding strategy in proving Lemma 3. When there is a packet drop, applying no control so that the norm of the state expands at most $a+K$ times. When there is no packet drop, the norm shrinks at least $a2^{-R} + K$ times. Hence by the weak law of large numbers, the critical value of $R$ for the closed loop almost sure stability satisfies

$$(a+K)^{1-\lambda}(a2^{-R} + K)^{\lambda} < 1,$$

which after simplification gives

$$R > R_{\min} = \log a - \log(2^{\frac{\lambda-1}{\lambda}\log(a+K)} - K).$$

QED

*Remark 8:* Notice that the condition

$$\lambda > \frac{\log(a+K)}{\log(a+K) - \log K},$$

guarantees that

$$2^{\frac{\lambda-1}{\lambda}\log(a+K)} - K > 0.$$

And $\lambda < 1$ guarantees that

$$2^{\frac{\lambda-1}{\lambda}\log(a+K)} - K < 1 - K,$$

hence the required bandwidth is bigger than that in Lemma 3 which is as expected.

### B. General Systems

The results for the scalar case are extended to the general vector case and combined into the theorem below.

*Theorem 9:* Assume $B, C$ are invertible and the system dimension is $n$. Then a sufficient condition for the closed loop almost sure stability (if there are no packet drops, *i.e.*, $\lambda = 1$, change this notion to exponential stability) is that the network parameters and system parameters satisfy the inequality below

$$(|A| + K)^{1-\lambda}(|A|\, 2^{-\frac{R}{n}} + K)^{\lambda} < 1.$$

**Proof:** The proof to this theorem is similar to the proofs for Lemma 2, 3, 4 and 7 and is omitted here.

QED

*Remark 10:* The assumptions are in general very conservative as we need both the matrices $B$ and $C$ to be invertible. We need this assumption because if $B$ is not invertible, we may not be able to compensate for the known state in one time step and hence may make the uncertainties grow out of control. If $C$ is not invertible, we need to wait for long enough consecutive packets to get complete state information and hence make the probabilistic analysis difficult. The authors are working towards relaxing these conditions.

It is interesting to make a comparison between our rate results with those in the literature, especially in Taktionda's work [15], where various rate results were given for noiseless LTI systems over digital communication channels. We briefly state one of his main results below.

*Theorem 11:* (Tatikonda [15]) Consider the discrete time system (1) in Figure 1. Assume $\lambda = 1$, *i.e.*, there is no packet drop and $K = 0$, *i.e.*, there is no uncertainty about the plant. Further assume that $(A, B)$ is controllable and $(C, A)$ is observable. Then a sufficient and necessary condition for the overall closed loop system to be asymptotically stable is that $R$ satisfies

$$R > R_{\min} = \sum_i \log|\lambda_i(A)|,$$

where $\lambda_i(A)$ are the unstable eigenvalues of $A$.

One of the many reasons that we cannot directly apply his result to system (1) is that $(A, B)$ being controllable does not imply $(A + \Delta_k, B)$ is controllable for all $k$. If we apply his result to the scalar case (3), we might be able to say that

$$R > R_{\min} = \log(a + K),$$

as $R_{\min} \geq \log(a + \Delta_k)$ for all $k$. However as we see from Lemma 2, $K < 1$ is needed for closed loop stability and directly applying Tatikonda's result reveals this fact. The correct sufficient condition is given by Lemma 3.

## IV. Gurantees For Closed Loop Stability For NCS II

### A. Scalar Systems

*Lemma 12:* Assume network one has data rate $R_1$ and network two has data rate $R_2$. Further assume that $\lambda = 1$,

$\gamma = 1$ and $K = 0$. Then a sufficient condition on $R_1$ and $R_2$ such that the closed loop system (7) is exponentially stable is that

$$a(2^{-R_1} + 2^{-R_2}) < 1$$

**Proof:** Use the same encoding and decoding strategy in proving Lemma 3. Without loss of generality, let $0 < x_0 < 2^M$ and we write the binary expansion of $x_1$ as

$$x_1 = \sum_{i=-\infty}^{M-1} \alpha_i 2^i,$$

where $\alpha_i$ s are either 1 or 0 depending on the value of $x_0$. At time 1, let $\bar{x}_0$ denote the most $R_1$ significant bits of $x_0$, *i.e.*,

$$\bar{x}_0 = \sum_{i=M-R_1-1}^{M-1} .$$

Hence

$$|\epsilon_0| = |x_0 - \bar{x_0}| \le 2^{M-R_1}.$$

Let $u_0 = -a\bar{x}_0$, then $|u_0| \le a2^M$. Let $\bar{u}_0$ denote the most $R_2$ bits of it, hence

$$|u_0 - \bar{u_0}| \le a2^{M-R_2}.$$

Then

$$\begin{aligned} |x_1| &= |ax_0 + \bar{u}_0| \\ &= |a\bar{x}_0 + a\epsilon_0 + \bar{u}_0| \\ &= |-u_0 + \bar{u}_0 + a\epsilon_0| \\ &\le |u_0 - \bar{u}_0| + a|\epsilon_0| \\ &\le a2^{M-R_2} + a2^{M-R_1}. \end{aligned}$$

In order that the norm of $x_1$ is shrinking, a sufficient condition is then

$$a2^{M-R_2} + a2^{M-R_1} < 2^M,$$

which after simplification gives

$$a(2^{-R_1} + 2^{-R_2}) < 1.$$

Similarly if the above condition holds, for all later time steps, the state norm shrinks by a factor of at least

$$\frac{1}{a2^{M-R_2} + a2^{M-R_1}}$$

which guarantees the closed loop system is exponentially stable.

QED

*Remark 13:* In the above lemma, it is required that $R_1 > \log a$ and $R_2 > \log a$ to make (10) hold. Furthermore, if we set either $R_1 = \infty$ or $R_2 = \infty$, from (10), we get $R_1 > \log a$ or $R_2 > \log a$ which is the same as in Tatikonda's result (Theorem 11). Figure 4 plots the contour lines of $2^{-R_1} + 2^{-R_2}$. Stability regions are above the lines for fixed $a$ and $K$.

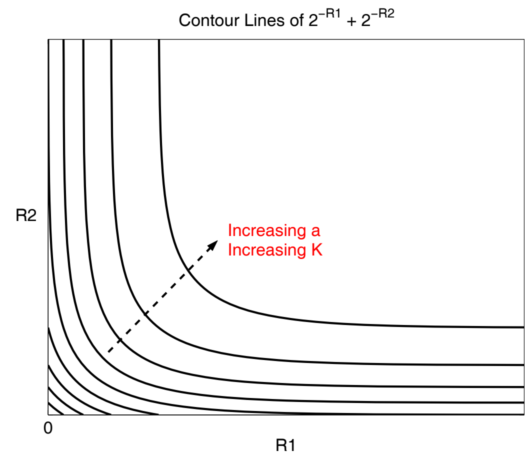


Fig. 4. Contour plot of $2^{-R_1} + 2^{-R_2}$. The lines depict fixed values of $a$ and $K$. Regions above the lines are stable for these fixed values.

*Lemma 14:* Assume network one has data rate $R_1$ and network two has data rate $R_2$. Further assume that $\lambda = 1$, $\gamma = 1$ and $K < 1$. Then a sufficient condition on $R_1$ and $R_2$ such that the closed loop system (7) is exponentially stable is that

$$a(2^{-R_1} + 2^{-R_2}) < 1 - K.$$

**Proof:** The proof is similar to the proves for Lemma 4 and Lemma 12 and is omitted here.

QED

*Lemma 15:* Assume $\lambda < 1$, $\gamma < 1$, $R_1 = \infty$ and $R_2 = \infty$. Then the closed loop system is almost sure stable if $K < 1$ and

$$\lambda\gamma > \frac{\log(a+K)}{\log(a+K) - \log K}. \tag{10}$$

**Proof:** The proof is similar to the proves for Lemma 4 and is omitted here.

QED

It is interesting to note the form of Eqn. 10 involves the product of the two network arrival rates. This means the stabliity conditions result in a plot similar to that in Figure 5.

*Theorem 16:* Assume $\lambda < 1$, $\gamma < 1$, $R_1 < \infty$ and $R_2 < \infty$ and $K < 1$. Then the closed loop system is almost sure stable if the following inequality holds

$$(a+K)^{1-\lambda\gamma}(a2^{-R_1} + a2^{-R_2} + K)^{\lambda\gamma} < 1$$

**Proof:** The proof is similar to the proves for Lemma 4 and Lemma 7 and is omitted here.

QED

*Remark 17:* The above theorem links all the different pieces together to produce a unified framework for closed loop almost sure stability.

*Example 18:* We wish to present a simple numerical example to illustrate our results. Consider the closed loop scalar system (7) with $a = 2.75$ and $K = 0.4$. We pick an initial

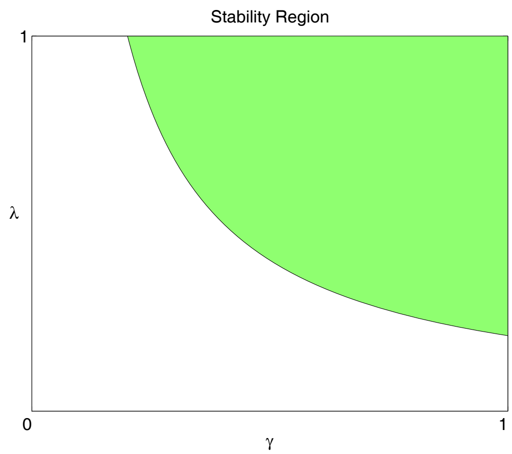


Fig. 5. Stability plot for the two network case packet arrival rates $\gamma$ and $\lambda$.

condition of $x_0 = 100$ and simulate the closed loop system for various values of the network properties defined above.

In Figure 6 we only consider packet drops, that is let $R_1 = R_2 = \infty$. We let $\lambda = 0.85$ and $\gamma = 0.8$, which satisfy the sufficient condition from Lemma 4. The plot shows that indeed the system is almost surely stable.

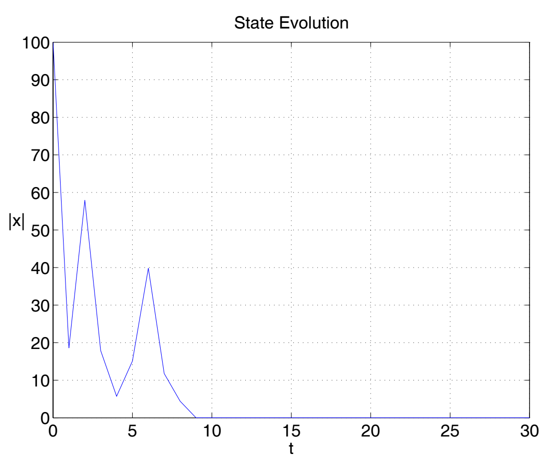


Fig. 6. System in Example 18 with $R_1 = R_2 = \infty, \lambda = 0.85, \gamma = 0.8$.

In Figure 7 we do not consider packet drops, that is let $\lambda = \gamma = 1$ and let $R_1 = 5$ and $R_2 = 5$, which satisfy the sufficient condition from Lemma 4 for exponential stability.

Next we assume both packet drops and finite bandwidth, with the network parameters satisfying the sufficient conditions for almost sure stability in Lemma 4. The results are plotted in Figure 8.

Lastly, we again assume packet drops and finite bandwidth, but set the parameters to $R_1 = 5, R_2 = 3, \lambda = 0.5$ and $\gamma = 0.6$. These values do not give sufficient conditions for almost sure stability. Figure 9 shows the norm of the state to diverge. Note that the conditions for stability in this paper are only sufficient conditions, hence violating them does not necessarily mean the state will diverge. It so happens that the parameters chosen here were degraded far enough that the state did diverge.

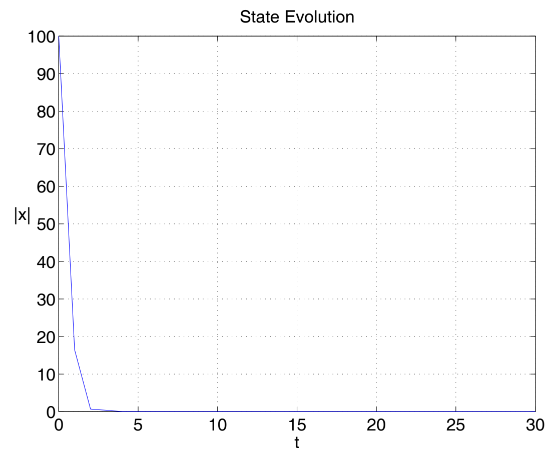


Fig. 7. System in Example 18 with $R_1 = R_2 = 5, \lambda = \gamma = 1$.

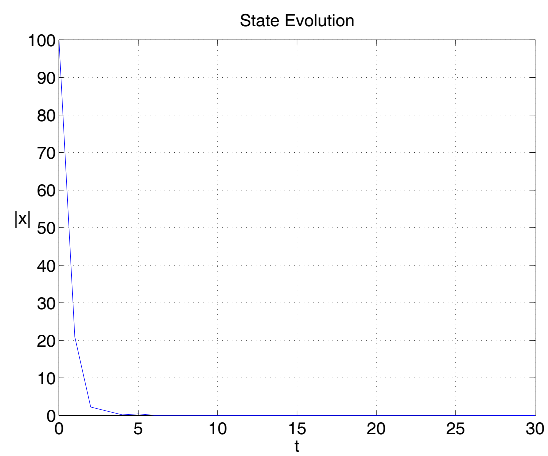


Fig. 8. System in Example 18 with $R_1 = R_2 = 5, \lambda = 0.85, \gamma = 0.8$.

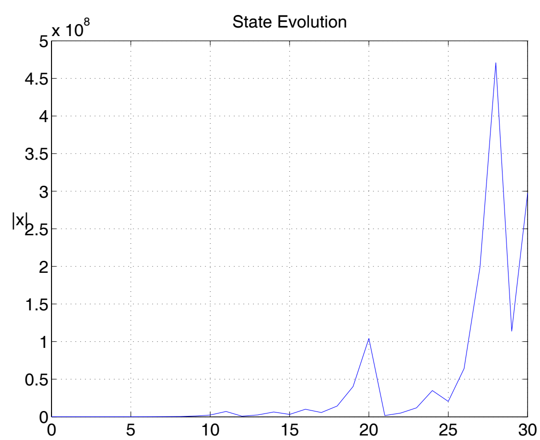


Fig. 9. System in Example 18 with $R_1 = 5, R_2 = 3, \lambda = 0.5, \gamma = 0.6$.

### B. General Systems

The results for the scalar case are extended to the general vector case and combined into the theorem below.

*Theorem 19:* Assume $B, C$ are invertible and the system dimension is $n$. Then a sufficient condition for the closed loop almost sure stability (if there are no packet drops, *i.e.*, $\lambda = 1$ and $\gamma = 1$, change this notion to exponential stability) is that the network parameters and system parameters satisfy the inequality below

$$(|A| + K)^{1-\lambda\gamma}(|A|2^{-\frac{R_1}{n}} + |B|\,|B^{-1}A|\,2^{-\frac{R_2}{n}} + K)^{\lambda\gamma} < 1.$$

**Proof:** The proof to this theorem is similar to the proofs for Lemma 2, 3, 4 and 7 and is omitted here.

QED

*Remark 20:* We can in fact recover any of the above lemmas or theorems from Theorem 19 only. For example, take $R_2 = \infty$ and $\gamma = 1$, *i.e.*, the NCS has changed from II to I. Then we get Theorem 9 which summarizes all the lemmas in section III.

## V. Conclusion and Future Work

In this paper we analyzed controlling linear discrete time systems with norm bounded uncertainty in the plant matrix over packet dropping networks. We considered the effect of finite bandwidth and packet losses on closed loop stability. Sufficient conditions for stability are given in terms of the minimum data rates and packet arrival probability as well as the norm of the uncertainty.

The most obvious extension to this work is to relax the restriction that $B$ and $C$ are invertible for the general case. Likewise to extend the NCS II results to the vector case. As most networks experience not only finite bandwidth and packet drops but delays as well [8], we would like to include this effect in the stability analysis.

Although the norm bounded uncertainty is frequently used, there are other types of uncertain models that might be more applicable in certain cases. For example, when the uncertainty is described by a convex set [9]. We would like to obtain similar results for other types of uncertainty as well.

This paper has given sufficient conditions for stability and certainly some of them are not neccesary. It is interesting to find necessary conditions for stability as well. Lastly, this paper has dealt with guarantees for stability but has not addressed the issue of performance. Investigating how the embedded networks degrade performance is a interesting area for future work.

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