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New Design Principles for Estimation over Fading Channels in Mobile Sensor Networks

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***Abstract*— In this paper we provide new design principles for estimation over wireless fading channels in mobile sensor networks. We show how to optimize receiver and transmitter designs to improve estimation performance in the application layer. On the receiver side, we show that the optimum packet drop mechanism is the one that provides a balance between information loss and communication noise. On the transmitter side, we show how to optimize and adapt the transmission rate for performance improvement in the application layer. We further provide stability conditions for different design strategies. The work confirms that delay-sensitive mobile sensor applications require new design paradigms and applying the same design principles of data networks can lead to performance degradation. The work also highlights the importance of cross-layer feedback and provides alternative designs if such feedbacks are not available.**

I. Introduction

There has recently been considerable interest in sensor networks [1], [2]. Such networks have a wide range of applications such as environmental monitoring, surveillance, security, smart homes and factories, target tracking and military. Communication plays a key role in the overall performance of sensor networks as both sensor measurements and control commands are transmitted over wireless links.

Considering the impact of communication channels on wireless estimation/control is an emerging area of research. Estimation/control of a rapidly-changing dynamic system is a delay-sensitive application. Therefore, the communication protocols and designs suitable for other already-existing applications like data networks may not be entirely applicable to sensor networks. Data networks are not as sensitive to delays since the application is not real time. The receiver, therefore, can afford to drop erroneous packets and wait for retransmission. Control applications, on the other hand, are typically delay sensitive as we are racing against the dynamics of the system. Therefore, new design strategies are required for such applications.

Authors in [3] have looked at the impact of packet loss on wireless Kalman filtering. They found a maximum tolerable packet loss probability beyond which the Kalman filtering process would go unstable. Authors in [4] extended the work of [3] to multiple sensors. Both of these works assumed that the receiver is dropping the erroneous packets. Furthermore, they assumed that the packets are noise-free if the receiver keeps them. For such delay-sensitive wireless applications, however, the receiver can not wait to receive noise-free packets. Therefore, it is essential to consider the impact of communication noise on estimation.

We proved in [5] that keeping all the packets results in stability and optimizes the performance if a cross-layer feedback is available in the receiver. We showed this for one class of channel noise characteristics and exponentially distributed channel. A cross-layer feedback refers to a feedback from physical layer of the receiver to the application layer. Such a feedback provides the application layer with information on the quality of the link. In this paper we will extend our work in [5] to establish fundamentals of design strategies for delay-sensitive estimation applications. We provide new paradigms for designing communication protocols in these systems. The main questions this paper addresses are the followings:

1) What is the optimum receiver design if a cross-layer feedback is not available in the receiver?
2) What is the optimum receiver design if such a feedback is available in the receiver?
3) What is the optimum transmitter design if the receiver design can not be modified?

We will answer these questions both in terms of stability and minimum asymptotic estimation error. Furthermore, our results will be general as we do not make any assumption on the shape of communication noise profile or channel distribution.

II. System Model

Consider a mobile sensor observing a system with the following linear dynamics:

$$\begin{aligned} x_{k+1} &= Ax_k + w_k \\ y_k &= Cx_k + v_k, \end{aligned} \tag{1}$$

where x_k and y_k represent the state and observation respectively. w_k and v_k represent zero-mean process and observation noises with variances of Q and R respectively. We are interested in estimating unstable dynamics and therefore we consider those cases where A has at least one eigenvalue outside the unit circle. The mobile sensor then transmits its observation over a wireless fading channel to a remote node, which is in charge of estimation. In practice, this can happen in the cases where the remote node gathers the observations of a few sensors and makes an estimate based on the gathered information. Also in some applications, the mobile sensors are cheap nodes with low computational complexity and their job is to sense and transmit their observations to a more powerful unit that would perform the estimation.

A. Physical Layer: Wireless Communication

Elements of the observation vector, y_k, are quantized, transformed into a packet of bits and transmitted over a mobile fading channel:

$$\hat{y}_k = y_k + n_k, \tag{2}$$

where $\hat{y}_k$ is the receiver version of the observation and n_k represents zero-mean communication noise. Let G_k represent the variance of n_k:

$$G_k = \overline{n_k n_k^t} = \sigma_n^2(k) \times I, \tag{3}$$

where I represents the unit matrix. In writing Eq. 3 we assumed that the communication noises of different elements of y_k are uncorrelated and have the same statistics. $\sigma_n^2(k)$ is a function of SNR_k, the instantaneous received Signal to Noise Ratio at k^{th} transmission:

$$\sigma_n^2(k) = f(SNR_k). \tag{4}$$

Function f is a decreasing function that depends on the transmitter/receiver design principles as well as the transmission environment. Fig. 1a shows one example of a noise profile, $\sigma_n^2(k)$. We can see that at as SNR goes to ∞, the communication noise variance reaches quantization noise error floor.

Depending on the receiver design, there can be a packet drop mechanism deployed in the receiver. Let $P_{drop}(k)$ represent the probability that the receiver drops the k^{th} packet. $P_{drop}(k)$ can be presented as a function of SNR_k as well:

$$P_{drop}(k) = g(SNR_k). \tag{5}$$

Fig. 1b shows a sample P_{drop} as a function of SNR (solid line). It should be noted that the receiver may not decide on dropping packets directly based on SNR_k. Since any other used measure is a function of SNR_k, we find it useful to express P_{drop} as a function of this fundamental quantity.

f and g are functions of receiver and transmitter technologies like modulation, quantization, noise figure and channel coding. SNR_k is a stochastic process and its distribution is a function of the environment and level of the mobility of the sensor. In a narrowband fading environment, we will have [6]

$$SNR_k = \frac{|h_k|^2 \sigma_x^2}{\sigma_d^2}, \tag{6}$$

where σ_x^2 and σ_d^2 represent the transmitted signal power and receiver noise power respectively. h_k is the value of the channel at k^{th} transmission. We take h_k and therefore SNR_k to be independent from one transmission to the next. This will be the case as long as the time interval between consecutive transmissions is bigger than channel coherence time [6]. To ease mathematical derivations, in this paper, we will approximate the g function with the following:

$$P_{drop}(SNR_k) = \begin{cases} 0 & SNR_k \geq SNR_{Thresh.} \\ 1 & else \end{cases} \tag{7}$$

This approximation is shown in Fig. 1b (start line).

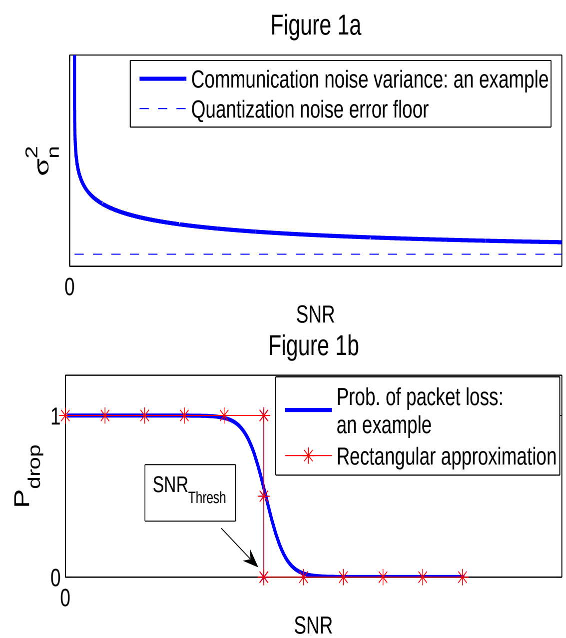

Fig. 1. Examples of communication noise variance and probability of packet loss as functions of SNR

B. Application Layer: Estimation

The distant node estimates the state based on the received observation using a Kalman filter [7]. Let $\hat{x}_k$ represent the estimate of x_k at the receiver. Let E_k represent variance of the estimation error of the k^{th} transmission:

$$E_k = \overline{(x_k - \hat{x}_k)(x_k - \hat{x}_k)^t}. \tag{8}$$

There will be different forms of recursions for E_k depending on the availability of a cross-layer feedback in the receiver. Furthermore, the average variance of the estimation error will be a function of the statistics of SNR through σ_n^2 and P_{drop}.

III. Receiver Design Optimization

In this section we will consider different receiver design strategies for estimation over mobile links. Authors in [3] considered a scenario in which packets are noise-free once the receiver decides to keep them. For non real-time applications like data networks, the receiver can afford to drop erroneous packets and wait for retransmission. Considering packets to be noise-free once they are kept in the receiver, therefore, is a reasonable model for these applications. However, estimation of a rapidly changing dynamical system is delay sensitive. Dropping erroneous packets results in loss of information, reduces the useful transmission rate of the system and can result in instability. Therefore, the receiver can not afford to wait for receiving noise-free packets. Low level of transmission power in sensor networks further makes reception of noise-free packets more difficult. Therefore, it is important to take impact of communication noise on the estimation process into account. Then the main issue is to find the right strategy for dropping received packets. The optimum receiver design can also change depending on the availability of a cross-layer feedback, which should be taken into account in the analysis. The stability conditions for the cases that the packets are not noise-free once kept in the receiver have not been derived before. In this section, we will also provide stability conditions for such scenarios. We have shown in [5] that keeping all the packets and using a cross-layer feedback in the receiver can prevent the instability that is introduced if erroneous packets are dropped. Furthermore, an analytical expression for estimation error was derived for one class of communication noise profiles and channel distributions. In this paper, we investigate the impact of cross-layer feedbacks when considering a general communication noise profile and channel distribution. We will analyze the performance and stability conditions for the following cases:

1) The receiver can keep all the packets but can not provide a cross-layer feedback

2) The receiver can not keep all the packets but can provide a cross-layer feedback for those packets that are kept

3) The receiver can not keep all the packets and can not provide a cross-layer feedback for those packets that are kept

4) Finally, the receiver can keep all the packets and is equipped with a cross-layer feedback.

Furthermore, we will not make the assumption that the received packets are noise-free once kept in the receiver. Considering the aforementioned cases will provide insight into receiver design strategies for wireless estimation applications. Table I summarizes different possible scenarios in more details. The first row, "PACKET DROP", refers to the case where the receiver deploys a packet drop mechanism. The second row, "KEEP ALL", refers to the case in which the receiver is keeping all the packets. "IDEAL NOISE PROFILE" refers to the case where those packets that are kept in the receiver are noise-free. Scenario#3 of Table I refers to the case where the receiver has a packet drop mechanism but once the packets are kept, they are noise-free. This scenario is what is considered in [3]. If all the packets are kept in the receiver, then packets can not be considered noise-free. Therefore, this possibility is crossed out in Table I. When considering non-ideal noise profiles, there will be four possibilities as shown in Table I. In both scenario#1 and 2, the receiver has a packet drop mechanism. However, in scenario#1, a cross-layer feedback is available in the receiver for those packets that are kept. Case of keeping all the packets and cross-layer feedback was studied for one class of channel noise profiles in [5]. The second row of Table I, case of keeping all the packets in the receiver, can be considered as a special case of the first row with probability of packet drop of zero. The goal of this section is to study different scenarios of Table I for a general noise profile and channel distribution. The question marks indicate the scenarios that have not been considered before. We will study scenario#1 and 2, which will allow us to study the corresponding cases for "KEEP ALL". While most of our results are derived for non-scalar cases, we will show our derivations for scalar state and observation in this paper to focus on the impact of communication link. In later sections we will discuss the extension of our results to a non-scalar case.

	NON-IDEAL NOISE PROFILE		IDEAL NOISE PROFILE
	Cross-Layer	**No Cross-Layer**	
PACKET DROP	*Scenario#1* ?	*Scenario#2* ?	*Scenario#3* Studied by Sinopoli et. al.
KEEP ALL	Studied for one class of channels and exp. dist. SNR by Mostofi et. al.	?	Not Possible

TABLE I

DIFFERENT RECEIVER DESIGNS

A. Scenario#3

In this part we briefly summarize the result of [3] for scenario#3. We will be using this result in the subsequent sections. Considering an ideal noise profile will result in the following:

$$\sigma_n^2(k) = \begin{cases} 0 & SNR_k \geq SNR_{Thresh.} \\ \infty & else \end{cases} \tag{9}$$

This results in the following condition for stability [3]:

$$\overline{P}_{drop,scenario\#3} < A^{-2}, \tag{10}$$

which imposes the following constraint on average Signal to Noise Ratio for an exp. distributed[1] SNR:

$$SNR_{ave} > \frac{SNR_{Thresh.}}{ln(\frac{A^2}{A^2-1})} \tag{11}$$

B. Scenario#2

In this case the receiver has a packet drop mechanism but the received packets are not necessarily noise-free. Furthermore, due to the lack of a cross-layer feedback in this case, the application layer does not have any knowledge of the quality of the communication link. To ease mathematical derivation of this scenario, we assume that the observation noise is negligible compared to the communication noise.

[1]In an environment with no LOS path, it is reasonable to assume that $|h_k|$ has Rayleigh fading distribution which results in an exponential distribution for SNR_k [6].

Therefore, the estimation using a Kalman filter will change as follows:

$$\hat{x}_{k+1} = \begin{cases} A\hat{x}_k & if \quad k^{th} \quad packet \quad is \quad dropped \\ AC^{-1}y_k & if \quad k^{th} \quad packet \quad is \quad kept \end{cases} \tag{12}$$

This will result in the following recursion for estimation error:

$$E_{k+1} = A^2E_k + Q - \frac{A^2E_k - A^2C^{-2}\sigma_n^2(SNR_k)}{S_k} \tag{13}$$

where,

$$S_k = \begin{cases} 1 & SNR_k \geq SNR_{Thresh.} \\ \infty & else \end{cases} \tag{14}$$

It should be noted that a Kalman filter is no longer the optimum estimator since the application layer does not have any knowledge of the quality of the link. Averaging over SNR_k will result in the following recursion for average estimation error:

$$\overline{E}_{k+1} = A^2P_L\overline{E}_k + Q + A^2C^{-2}P_N, \tag{15}$$

where P_L and P_N represent average probability of packet loss and average communication noise that entered the estimation process respectively:

$$P_L = \overline{P}_{drop} = \int_0^{SNR_{Thresh.}} pdf(SNR)dSNR \tag{16}$$

and

$$P_N = \int_{SNR_{Thresh.}}^{\infty} \sigma_n^2(SNR)pdf(SNR)dSNR. \tag{17}$$

1) Stability: For stability we will have,

$$\overline{E}_{k+1} = (A^2P_L)^{k+1}E_0 + (Q + A^2C^{-2}P_N)\frac{1-(A^2P_L)^{k+1}}{1-A^2P_L}. \tag{18}$$

Therefore, the stability condition will be as follows:

$$\overline{P}_{drop,scenario\#2} < A^{-2} \tag{19}$$

It can be seen that although the communication noise is not ideal in this case, we still have the same stability condition. Eq. 19 also suggests that for maximizing the stability range, all the packets should be kept in the receiver despite lack of a cross-layer feedback in this scenario. While keeping all the packets results in stability, it will not optimize the performance for this case. Next we show how to optimize the performance.

2) Optimum Performance: The asymptotic estimation error will be as follows for those cases that the stability condition of Eq. 19 holds:

$$\overline{E}_{\infty} = \frac{A^2C^{-2}P_N + Q}{1 - A^2P_L}. \tag{20}$$

Due to the lack of a cross-layer feedback in this case, there is an optimum $SNR_{Thresh.}$ that will minimize the asymptotic estimation error. If $SNR_{Thresh.}$ is too low, the estimation process will be too noisy. On the other hand, if $SNR_{Thresh.}$ is too high, information loss rate will be too high since most of the packets will be dropped. Minimizing Eq. 20 with respect to $SNR_{Thresh.}$ will result in the following equality for the optimum $SNR_{Thresh.}$:

$$P_L + P_{N,normalized} + \frac{C^2Q}{A^2\sigma_n^2(SNR_{Thresh.})} = A^{-2}, \tag{21}$$

where $P_{N,normalized}$ refers to the normalized communication noise that entered the estimation process: $P_{N,normalized} = \frac{P_N}{\sigma_n^2(SNR_{Thresh.})}$. P_L and P_N are functions of $SNR_{Thresh.}$ through Eq. 16 and 17. Therefore, the optimum $SNR_{Thresh.}$ is the one that provides a balance between the amount of communication noise that enters the estimation process (P_N) and information loss (P_L), as shown in Eq. 21. This suggests that depending on the environment, transmission protocols and dynamics of the state, there is an optimum way of dropping packets.

To see the impact of operating at the optimum $SNR_{Thresh.}$, Fig. 2 shows $\overline{E}_{\infty}$ as a function of $SNR_{Thresh.}$ and for different levels of average Signal to Noise Ratio, SNR_{ave}. The communication noise profile for this example is as follows: $\sigma_n^2(SNR) = \alpha + \beta \times \zeta(\sqrt{SNR})$, where $\zeta(z) = \frac{1}{\sqrt{2\pi}}\int_z^{\infty} e^{-t^2/2}dt$ for an arbitrary z. This is the variance of the communication noise for a BPSK modulation with no channel coding [8]. The following parameters are chosen for this example: $A = 2$, $N_b = 10$ and $\delta = .0391$, where N_b and δ represent the number of transmitted bits per packet and the quantization step size respectively. This results in the followings, $\alpha = 1.27 \times 10^{-4}$ and $\beta = 533.3$. Furthermore, SNR is considered to have an exponential distribution for this example. It can be seen from Fig. 2 that operating at the optimum $SNR_{Thresh.}$ will improve the performance considerably. Furthermore, it can be seen that stability ranges are as predicted by Eq. 19.

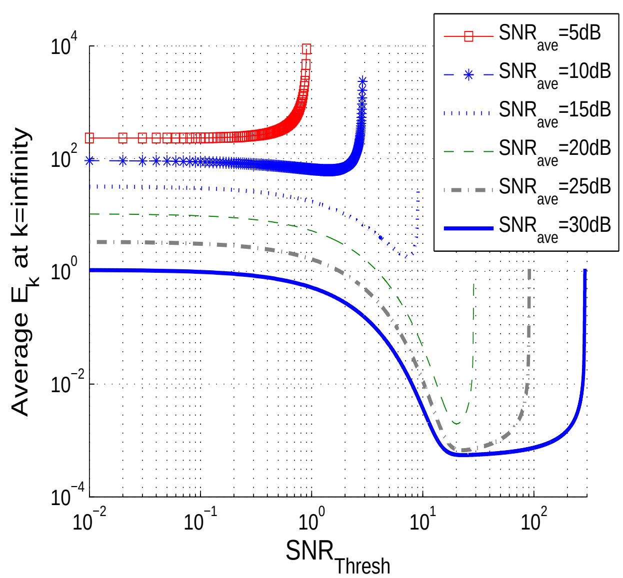

Fig. 2. Scenario#2: optimum packet drop mechanism

C. Scenario#1

In this case the receiver has a packet drop mechanism but a cross-layer feedback is available for those packets that are kept in the receiver. Then we will have the following recursion for the estimation error:

$$E_{k+1} = A^2 E_k + Q - \frac{A^2 E_k^2 C^2}{C^2 E_k + \sigma_z^2(SNR_k) + R} \quad (22)$$

where

$$\sigma_z^2(SNR_k) = \begin{cases} \sigma_n^2(SNR_k) & SNR_k \geq SNR_{Thresh.} \\ \infty & else \end{cases} \quad (23)$$

After much algebraic manipulation, we will have the following for[2] $C = 1$, $Q = 0$ and $R = 0$:

$$E_k = \frac{A^{2k} E_0}{1 + E_0 \sum_{i\geq 1}^{k} A^{2(i-1)} \eta(SNR_{i-1})}, \quad (24)$$

where $\eta(SNR) = \frac{1}{\sigma_z^2(SNR)}$.

1) Stability: While Eq. 24 can be used to evaluate the performance analytically when the information on channel distribution and noise profile is available, it does not provide insight into the stability condition in general. Therefore, we will take a different approach to evaluate stability condition of this scenario. First we compare scenario#1 with scenario#2. In both cases, there exists a packet drop mechanism in the receiver. However, in scenario#1 a cross-layer feedback is available for those packets that are kept in the receiver whereas scenario#2 is not equipped with such a feedback. This suggests that the stability range of scenario#1 should contain that of scenario#2. Let $P_{drop,critical}$ represent the maximum tolerable average probability of packet loss for stability. Then we will have,

$$P_{drop,critical,scenario\#1} \geq P_{drop,critical,scenario\#2} \quad (25)$$

Similarly, we compare scenario#1 with scenario#3. In both cases, there is a packet drop mechanism. However, in scenario#3 the packets are noise-free when they are kept in the receiver. Therefore, the stability range of scenario#3 should contain that of scenario#1. This translates into the following:

$$P_{drop,critical,scenario\#3} \geq P_{drop,critical,scenario\#1} \quad (26)$$

In previous sub-sections we saw that $P_{drop,critical,scenario\#2} = P_{drop,critical,scenario\#3}$ (Eq. 10 and 19). Therefore, we have the same condition for stability of scenario#1:

$$\overline{P}_{drop,scenario\#1} < A^{-2} \quad (27)$$

We can see that keeping all the packets will prevent instability as was the case for the previous scenarios[3].

[2] A similar expression can be easily derived for general C, R and Q.

[3] It should be noted that this result is for a general C, R and Q.

2) Optimum Performance: For this scenario, keeping all the packets not only prevent instability but will result in the minimum estimation error due to the presence of a cross-layer feedback. This can be easily confirmed by writing an expression for the average of Eq. 24 as a function of a general distribution for SNR.

To see the effect of a cross-layer feedback, Fig. 3 shows the performance for the system parameters of Fig. 2 and for both scenario#1 and 2. Comparing the corresponding cases for these scenarios, it can be seen that a cross-layer feedback can improve the performance considerably even when compared to operating at the optimum $SNR_{Thresh.}$ for scenario#2. Furthermore, it can be seen that keeping all the packets will result in minimum estimation error for scenario#1. Finally, the stability condition is confirmed to be the same as proved for both scenarios.

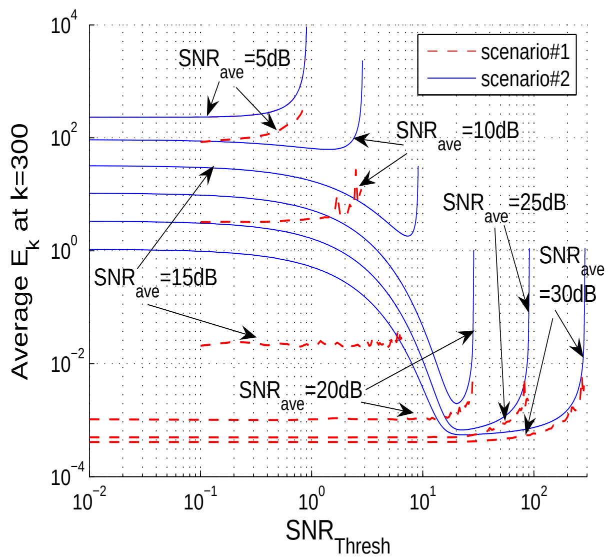

Fig. 3. Effect of cross-layer feedback: compare scenario#1 and 2

D. Receiver Design: Results Summary

In this section, we have proved the followings:

1) Keeping all the packets will result in stability independent of the shape of the communication noise profile, channel distribution and presence of a cross-layer feedback.

2) Keeping all the packets will minimize the average estimation error if a cross-layer feedback is available in the receiver.

3) Keeping all the packets will not optimize the performance if a cross-layer feedback is not available in the receiver. Instead, there is an optimum way of dropping packets (optimum $SNR_{Thresh.}$) that provides a balance between information loss and the amount of communication noise that enters the estimation process and results in minimum asymptotic estimation error.

IV. Transmitter Design Optimization

The previous section focused on optimizing the receiver design to achieve stability and better performance for Kalman filtering over wireless links. For those cases that the receiver design can not be modified, we will provide similar design strategies on the transmitter side. For instance, consider cases that the packet drop mechanism of the receiver can not be modified (i.e. $SNR_{Thresh.}$ can not be changed) or cross-layer feedback is not available in the receiver. We will show in this section how to mimic such effects in the transmitter side.

A. Optimum Transmission Rate

Consider a transmitter that is given bandwidth of B_w and time duration of T for transmission of each packet. Let b represent the number of bits transmitted in each packet. We will then have $1 \leq b \leq B_w T$. The communication noise consists of two parts: link noise, $\sigma_{n,L}^2(k)$, and quantization noise, $\sigma_{n,Q}^2(k)$,

$$\sigma_n^2(k) = \sigma_{n,Q}^2(k) + \sigma_{n,L}^2(k). \tag{28}$$

As b gets smaller, the link noise gets smaller while the quantization noise increases. On the other hand, choosing a large b will result in smaller quantization noise while it increases the link noise.

Consider scenario#2, the case in which there is a packet drop mechanism in the receiver but no cross-layer feedback is available. Furthermore, in this section consider the case in which $SNR_{Thresh.}$ of the receiver can not be changed. We are interested in finding the optimum transmission rate. SNR of Eq. 6 will be as follows as a function of b:

$$SNR_k = \frac{c|h_k|^2}{b}, \tag{29}$$

where $c = \frac{\sigma_x^2 T}{N_h}$ with N_h representing noise per Hertz of the receiver. If b is chosen small, receiver average Signal to Noise ratio, SNR, increases reducing the probability of packet drop. However, the quantization noise will increase resulting in noisy estimates. On the other hand, choosing a large b will reduce the amount of quantization noise at the price of an increase in the probability of packet loss and therefore information loss. This suggests that there should be an optimum transmission rate that provides a balance between information loss and the amount of communication noise that enters the estimation process.

1) Stability: In this case we will have the same recursion of Eq. 13 and 18 for instantaneous estimation error and average estimation error respectively. P_L of Eq. 16 will be as follows as a function of b:

$$P_L = prob\left\{|h|^2 < \tfrac{b \times SNR_{Thresh.}}{c}\right\} = \psi\left(\tfrac{b \times SNR_{Thresh.}}{c}\right) \tag{30}$$

where $\psi(d) = prob\left\{|h|^2 < d\right\}$ for an arbitrary d. Therefore the stability condition of Eq. 19 will be as follows:

$$A^2 \psi\left(\tfrac{b \times SNR_{Thresh.}}{c}\right) < 1 \rightarrow b < \frac{c \times \gamma(A^{-2})}{SNR_{Thresh.}} \tag{31}$$

where $z = \gamma(d)$ if $d = \psi(z)$ for arbitrary d and z. Eq. 31 suggests that decreasing the transmission rate will increase the range of stability. Therefore, if stability was the only concern, the transmission rate should be as low as possible. This is equivalent of reducing packet loss rate in the previous section.

2) Optimum Performance: Similar to scenario#2, minimizing the rate, while increasing chance of stability, will not provide minimum estimation error. Instead there is an optimum transmission rate that would minimize average asymptotic estimation error. The optimum rate is the solution to the following equation:

$$C^{-2}\frac{\partial P_N}{\partial b}(1 - A^2 P_L) + \frac{\partial P_L}{\partial b}(A^2 C^{-2} P_N + Q) = 0 \tag{32}$$

where

$$\frac{\partial P_L}{\partial b} = \frac{SNR_{Thresh.}}{c} pdf_{|h|^2}\left(\frac{b \times SNR_{Thresh.}}{c}\right) \tag{33}$$

where $pdf_{|h|^2}(.)$ represents the probability density function of $|h|^2$ and

$$\frac{\partial P_N}{\partial b} = -\frac{SNR_{Thresh.}}{c} pdf_{|h|^2}\left(\tfrac{b \times SNR_{Thresh.}}{c}\right)\sigma_n^2(SNR_{Thresh.}) + \int_{\frac{b \times SNR_{Thresh.}}{c}}^{\infty} pdf_{|h|^2}(u)\frac{\partial \sigma_n^2(\frac{cu}{b})}{\partial b} du \tag{34}$$

To see the performance when operating at the optimum transmission rate, Fig. 4 shows the asymptotic average estimation error as a function of number of bits per packet, for the simulation setup of Fig. 2 and at $SNR_{Thresh.} = 0$ and $10dB$. $coeff$ in Fig. 4 refers to $= c\overline{|h|^2}$. It can be seen that operating at the optimum transmission rate can improve the performance considerably.

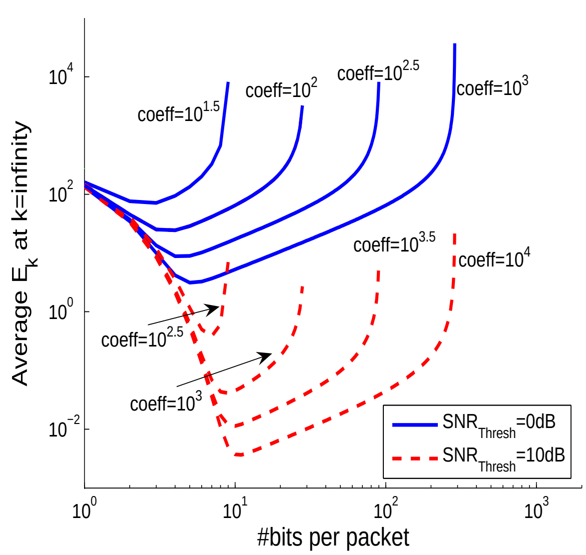

Fig. 4. Optimum transmission rate

B. Transmitter Design: Results Summary

In this section, we have shown the followings:

1) Reducing the transmission rate will increase the chance of stability independent of the shape of the link and quantization noise profiles or channel distribution.

2) Minimizing the transmission rate will not optimize the performance. Instead, there is an optimum rate that would balance quantization and link noises and hence information loss and estimation noise resulting in the minimum asymptotic estimation error.

V. Summary

In this paper we showed that new design paradigms are required for delay sensitive estimation applications in mobile sensor networks. We showed how to optimize receiver and transmitter designs to improve estimation performance in the application layer. We proved that keeping all the packets will result in stability independent of the shape of the communication noise profile, channel distribution and presence of a cross-layer feedback. Furthermore, we showed that keeping all the packets will result in minimum asymptotic estimation error if a cross layer feedback is available in the receiver. In the absence of a cross-layer feedback, we proved that the optimum packet drop mechanism is the one that provides a balance between information loss and the amount of communication noise that enters the estimation process. Similarly, on the transmitter side, we showed how to optimize the transmission rate. We showed that reducing the transmission rate will increase the chance of stability independent of the shape of quantization and link noise profiles or channel distribution. We also showed that there is an optimum rate that would balance information loss and communication noise resulting in the minimum asymptotic estimation error.

VI. Ongoing Work

The results can be easily extended to the vector case for an invertible C. We are currently working on extending the results for a general C matrix, which is challenging due to the presence of a non-ideal communication noise profile. We are also working on mimicking the functionality of a cross-layer feedback by adapting the instantaneous transmission rate to the communication link quality.

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