2003 American Control Conference
http://www.cds.caltech.edu/~murray/papers/2002j_jm03-acc.html

Stability and Performance Analysis with Double-graph Model of Vehicle Formations

Zhipu Jin, Richard M. Murray
Division of Engineering and Applied Science
California Institute of Technology
Pasadena, CA 91125
E-mail: jzp@caltech.edu

Abstract

In this paper, we treat the string stability as a kind of performance of a linear multi-vehicle system with acyclic formation structures. By using a double-graph model, we can describe information flows and formulate the string stability. This paper provides a systematic way to design the local controller and the system control strategy with the performance constrains. We also present the connection failure tolerance problem and give some essential conclusions.

Keywords: string stability, formation performance, double-graph model, information flows, connection failure tolerance

1 Introduction

The development of powerful control theories and techniques for large scale, decentralized and coupled systems of multiple vehicles is a problem with considerable interests in the control community. People are increasingly interested in how to use multiple vehicles to accomplish, in the presence of uncertainty and adversity, more challenging tasks beyond the ability of individual vehicle. The applications of multi-vehicles systems include coordination of micro-satellite clusters, formation flight of unmanned aerial vehicles (UAVs), autonomous underwater vehicles and automated highway systems. These theories can also be useful to understand the movement of flocks of birds, schools of fish, and other group motions in the nature.

In most of the applications, vehicles are coupled with each other according to the feedback control laws. Barbieri [1] has studied the interconnected systems by using z-transform. Swaroop and Hedrick [11, 12] studied the string stability and compared the different control strategies used in the Automated Highway System (AHS). Eyre [4] discussed string stability of automated vehicles by a mass-spring-damper framework. Stankovic [10] proposed a decentralized overlapping control strategy of a platoon of vehicles. Shladover [9] gave a good review of the development of advanced vehicle control systems. It has been shown that the leader's and neighbors' information play the key roles in the formation control problem. However, those results only concern a single string platoon. In many applications, the vehicles need to form a more complicated formation. The formation structure depends on the inter-vehicle communication channels and sensing, and is subject to change. Pant, Seiler and Hedrick began to study the *mesh* stability in [8] and give some useful results.

In this paper, we consider the string stability as the disturbance resistance performance of an acyclic formation and study the relationships among the information flows in the formation, local controllers and control strategy with which we can achieve the string stability. The paper is organized as follows: In Section 2, we provide some useful preliminaries about the graph theory and formation stability, and we put the string stability problems into the formation performance area. We introduce the double-graph model and a formation control strategy in Section 3 and formulate the string stability in the next section. Section 5 is used to describe the connection failure tolerance. Finally, we give out conclusions based on our researches.

2 Preliminaries

In this section, we provide some useful preliminaries about the graph theory and the stability of vehicle formations, and also give some assumptions to simply our questions.

2.1 Laplacian and Weighted Adjacency Matrix

It is well known that the *adjacency* matrix $A = \{a_{ij}\}$ of a directed graph (digraph) $G = (V, E)$ of order n is a $n \times n$ matrix defined as:

$$a_{ij} = \begin{cases} 1, & (v_i, v_j) \in E \\ 0, & otherwise \end{cases}$$

where V is the set of vertices and E is the set of edges. If let D be the diagonal matrix with the out-degree of each vertex along the diagonal, then *Laplacian* matrix is defined as:

$$L = D^{-1}(D - A).$$

Let Φ_i is the out-degree of vertex i. According to this definition, the Laplacian will have such properties.

- The i-th diagonal element equals 1 if $\Phi_i \neq 0$, otherwise, equals 0.
- Any other element of i-th row is 0 or $1/\Phi_i$.
- 0 is one of the eigenvalues, and the corresponding eigenvector is $[1, 1, \ldots, 1]^T$.

The weighted adjacency matrix is defined as:

$$\Theta = I - L = D^{-1}A.$$

For a multi-vehicle system, it's natural to describe the inter-vehicle connections by digraphs. In this paper, we only study these cases that the graph G is a connected, acyclic digraph. We call such a formation as a *look-ahead* system [8] and most of the leader-follower systems belong to this class with only one leader whose out-degree equals zero. There always exists an indexing method so that the leader is the vertex 1, and if an edge $(v_i, v_j) \in E$, then $i > j$. The resulting Laplacian matrix L has the following properties:

- L is a lower triangular matrix.
- The first row is zero.
- The eigenvalues of L are $0^{(1)}$ and $1^{(n-1)}$.

2.2 Formation stability

Suppose in a leader-follower system, there are n vehicles, whose identical linear dynamics are denoted by

$$\dot{x}_i = Ax_i + Bu_i,$$

where $i \in [1, n]$ is the index of the vehicles. Each vehicle can get information of itself and from its neighbors as

$$\begin{aligned} y_i &= C_1 x_i \\ z_{ij} &= C_2(x_i - x_j) \end{aligned}$$

where $j \in J_i$ and $J_i \subset [1, n]$ represents the set of vehicles which vehicle i can sense. Note that a single vehicle cannot drive all the z_{ij} terms to zero simultaneously, the errors must be synthesized into a single signal. We assume that all relative state measurements are weighted equally to form one error measurement as:

$$z_i = \frac{1}{\Phi_i} \sum z_{ij},\ j \in J_i.$$

According to [5], a local controller stabilizes the whole formation dynamics if and only if it simultaneously stabilizes the set of n systems

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= C_1 x \\ z &= \lambda_i C_2 x \end{aligned}$$

where λ_i are the eigenvalues of the Laplacian L.

Assume that $C_1 = C_2$, i.e., the sensed information is the same kind of information as we can measured. For a look-ahead system, the eigenvalues are only 0 and 1s and the local controller stabilizes the formation dynamics if and only if it stabilizes the individual vehicle:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= z = C_1 x. \end{aligned}$$

2.3 String stability and formation performance

For a look-ahead system, we need a local controller so that all vehicles will eventually reach their equilibriums and we also hope that the whole formation has good disturbance resistance performance. One interesting phenomena in the multi-vehicle system is that a disturbance of one single vehicle may affect other vehicles and propagate through out the formation. When a bounded disturbance is transferred, it may be become bigger and bigger and result in vehicle collisions if the scale of the formation (the number of vehicles) is large enough. This means a bad disturbance resistance performance, and we say this formation is string unstable. If the local controller can make sure that this disturbance will be attenuated as it is transferring, then we say the formation is string stable.

For a look-ahead system with n vehicles, we treat it as a n-input n-output system. A disturbance can be introduced at every single vehicle and may result relative position errors at every other vehicle. If we use a n-by-n transfer function matrix to describe these relationships, then the disturbance performance can be evaluated by these transfer functions. Obviously, this transfer function matrix is a lower triangular matrix and every diagonal element equals 1.

3 Double-graph model

We can find some perfect examples in [11, 12] and get an instinct that in order to keep the string stability of a leader-follower system, it is important to design a local controller that can adjust its behavior based on the leader as well as its neighbors. The *double-graph model* is specifically used for this idea.

Definition 1 (Double-graph model) Suppose there is a look-ahead system with only one leader.

Each vehicle can get the neighbor vehicles' and leader's information by onboard sensors or by communication channels.

- A digraph G_1 is used to describe the *leader information flow*. Every edge $(v_i, v_j) \in G_1$ represents a directed communication channel from v_j to v_i. All of the edges in this graph are only used to distribute the leader's information.
- A digraph G_2 is used to describe the *sensor information flow*. If there is a edge $(v_i, v_j) \in G_2$, it means vehicle i can sense vehicle j.
- A *double-graph model* Ω is a combination of the leader information flow graph and the sensor information flow graph. Since those two graphes have same vertices, we can write this look-ahead system as $\Omega = (G_1, G_2)$.

Here is an example of the double-graph model. The solid lines represent sensor information, and the dashed lines represent the leader information.

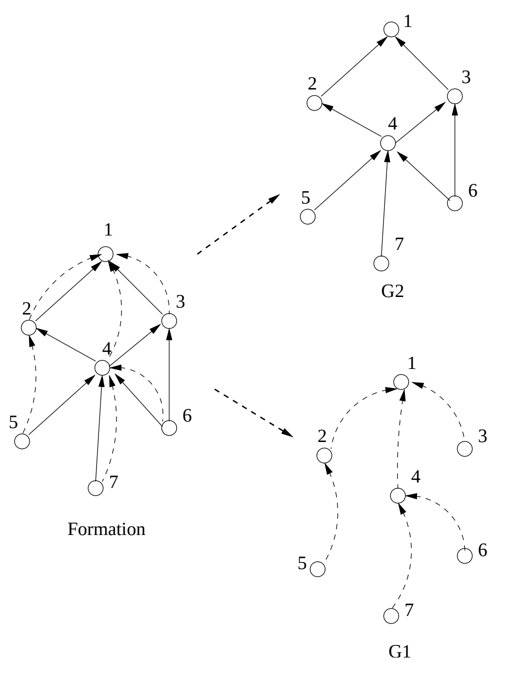

Figure 1: Double-graph model of a formation

The Laplacian of G_2 are

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1/2 & -1/2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1/2 & -1/2 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}.$$

We introduce a control strategy as

$$\begin{cases} Y_1(s) &= H(s)U(s) \\ Y_i(s) &= H(s)\big(\alpha Y_1(s) + \frac{1-\alpha}{\Phi_i}\sum_j Y_{ij}(s)\big),\ i \neq 1 \end{cases}$$

where $U(s)$ is the reference of the leader, $H(s)$ is the stable transfer function of individual vehicle, $Y_{ij}(s)$ is the neighbors of vehicle i so that $j \in J_i$, and $0 < \alpha < 1$. According to Section 2, the whole formation is stable. By this way, we separate the contributions of G_1 and G_2 by the weight coefficients α and $1-\alpha$. In the next section, we assume G_1 is perfect to distribute the leader's information and focus on how G_2 affects the formation's performance.

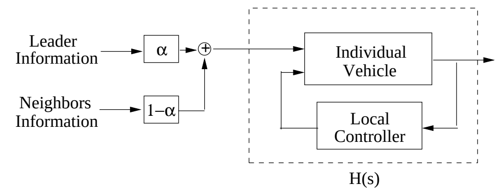

Figure 2: Diagram of the control strategy

4 Performance of a look-ahead formation

In this section, we will find the transfer function matrix and formulate the performance of a look-ahead system. Let us begin with a simple case, string formation, then generalize to arbitrary look-ahead systems.

4.1 String formation performance

In a string formation, each follower i can only sense the previous vehicle $(i-1)$. So G_2 is like a string whose root is the leader. In this case, we have

$$\begin{cases} Y_1(s) &= H(s)U(s) \\ Y_i(s) &= H(s)\Big(\alpha Y_1(s) + (1-\alpha)Y_{i-1}(s)\Big),\ i \neq 1. \end{cases}$$

In order to avoid collisions, it is important to study the relative position error as $\epsilon_i = y_i - y_{i-1} - L_{i,i-1}$ where $L_{i,i-1}$ is the offset to achieve the desired inter-vehicle spacing. We can get

$$\epsilon_i(s) = Y_i(s) - Y_{i-1}(s) = (1-\alpha)H(s)\epsilon_{i-1}(s)$$

and

$$\epsilon_{i+m}(s) = (1-\alpha)^m H(s)^m \epsilon_i(s).$$

So the transfer function matrix is

$$\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ N(s) & 1 & 0 & \dots & 0 \\ N^2(s) & N(s) & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ N^n(s) & N^{n-1}(s) & \dots & N(s) & 1 \end{bmatrix}$$

where $N(s) = (1-\alpha)H(s)$.

Now we can analysis the string formation's performance.

- It is clear that if $\alpha = 1$, then the relative position errors cannot propagate among the formation. This is the best string stability we can achieve [11]. But the cost is that each vehicle cannot sense any neighbors except the leader. It is unsafe and the connection failure tolerance is very bad. We will discuss this issue in the next section.
- If $\alpha = 0$, then graph G_1 doesn't affect the performance. If $\|H(s)\|_\infty > 1$, there always exists some disturbance at specific frequencies that will be amplified along G_2 and cause string instability. If $\|H(s)\|_\infty \leq 1$, then the formation is string stable, but this is a very strong constrain on the local controller design.
- The necessary and sufficient condition of the string stability is that

$$\|(1-\alpha)H(s)\|_\infty < 1.$$

We just need to choose a proper α so that $1 - \frac{1}{\|H(s)\|_\infty} < \alpha < 1$. Moreover, we can even design a rational, stable transfer function $\alpha(s)$ so that

$$\|(1-\alpha(s)) \cdot H(s)\|_\infty \leq \beta < 1.$$

By this way, not only can we control the converging speed of the disturbance with respect to the index number, but also adjust the weight coefficients of G_1 and G_2 with respect to the frequency.

4.2 Arbitrary look-ahead system

For an arbitrary look-ahead system, it may be hard to find out some sort of simple recursive equations to describe the propagation of the disturbance. We also need to redefine the relative position error ϵ_i since vehicle i may sense multiple neighbors simultaneously. From now on, let

$$\epsilon_i(t) = y_i(t) - \frac{1}{\Phi_i}\sum_j y_{ij}(t) - \frac{1}{\Phi_i}\sum_j L_{ij}$$

to be the relative position errors where $y_{ij}(t)$ is the output of vehicle j so that $j \in J_i$. Before giving out the main theorem, we list 3 lemmas about the digraph G_2.

Lemma 1 The relative error $\epsilon_i(s)$ of vehicle i is

$$\epsilon_i(s) = (1-\alpha)H(s)\sum_j \big(\theta_{ij} \cdot \epsilon_{ij}(s)\big)$$

where $j \in J_i$, $\epsilon_{ij}(t)$ is the error of vehicle j, and θ_{ij} are the elements of the weighted adjacency matrix Θ of G_2.

Proof: According to the definition of ϵ_i, we have

$$\begin{array}{rcl}
\epsilon_i(s) & = & Y_i(s) - \frac{1}{\Phi_i}\sum_j Y_{ij}(s) \\
 & = & H(s)\big(\frac{1-\alpha}{\Phi_i}\sum_j Y_{ij}(s) + \alpha Y_1(s)\big) \\
 & & -H(s)\frac{1}{\Phi_i}\sum_j\big(\frac{1-\alpha}{\Phi_{ij}}\sum_k Y_{ijk}(s) + \alpha Y_1(s)\big) \\
 & = & \frac{1-\alpha}{\Phi_i}\cdot H(s)\sum_j\Big(Y_{ij}(s) - \frac{\sum_k Y_{ijk}(s)}{\Phi_{ij}}\Big) \\
 & = & \frac{1-\alpha}{\Phi_i}H(s)\sum_j \epsilon_{ij}(s) \\
 & = & (1-\alpha)H(s)\sum_j\big(\theta_{ij}\cdot\epsilon_{ij}(s)\big)
\end{array}$$

where Φ_{ij} is the out-degree of vehicle $j \in J_i$, and $Y_{ijk}(s)$ is the output of vehicle k so that $j \in J_i$ and $k \in J_j$.

This lemma says that the error of an individual vehicle depends on the errors of its neighbors. Consider any two distinct vertexes m and n in the formation with $m < n$, the disturbance of m will propagate to n by any possible directed paths exist in G_2. Of course, the number of the paths, the length of each path, and weighted edges in these paths are the factors we must concern about.

Lemma 2 Elements of Θ^k represent the paths in G_2 with length k.

- If $(\Theta^k)_{ij} = 0$, then there is no path between i and j with length k.
- If $(\Theta^k)_{ij} \neq 0$, the there is at least one path between i and j with length k.
- For any $1 \leq i, j \leq n$, $0 \leq (\Theta^k)_{ij} \leq 1$ and

$$(\Theta^k)_{ij} = \sum_{m=1}^{M}\big(\prod_{l=1}^{k}\theta_{ml}\big)$$

where M is the number of the paths between i and j with length k, θ_{ml} is the weight factor of edge l in path m. Let's call $\prod_{l=1}^{k}\theta_{ml}$ the weight of path m.
- $\Theta^n = 0$ where n is the number of vertexes in G_2.

Proof: It is obvious for $k = 1$ since Θ is the weighted adjacency matrix of G_2. For $k = 2$, the i-th row of Θ represents the weights of all edges from the vertex i, the j-th column of Θ represents all edges connected to the vertex j. So $(\Theta^2)_{ij}$ is the sum of all paths' weights from i to j with length 2. By induction, it is easy to prove for any $1 < k < n$. Since n is the number of vertexes, there is no path with length n in G_2 if G_2 is an acyclic digraph, so $\Theta^n = 0$.

Lemma 3 For a look-ahead system, we define the path matrix Q as

$$Q = \Theta + \Theta^2 + \cdots + \Theta^{n-1}.$$

The matrix $Q = q_{ij}$ is a lower triangular matrix with zero diagonal elements, and $0 \le q_{ij} \le 1$ for any $1 \le j < i \le n$.

Proof: Since Θ is a lower triangular matrix with zero diagonal elements, so is Q. Actually, Q describe any possible paths between any two vertexes in graph G_2. For example, q_{ij} is the sum of any possible path's weight with length from 1 to $n-1$. It is obviously true when there is only one path between i and j. If there exists multiple paths, by induction we get

$$\begin{array}{rcl} q_{ij} & = & \frac{1}{\Phi_i}\sum_l q_{il} \\ & \le & \frac{1}{\Phi_i}\cdot\Phi_i = 1 \end{array}$$

where $l \in J_i$.

By using these lemmas, we get an upper bound of the infinity norm of the disturbance transfer functions between any two vehicles.

Theorem 1 In the double-graph model of a look-ahead system, if $\|(1-\alpha)H(s)\|_\infty < 1$, then the disturbance transfer function between any two vehicles is bounded by

$$\|H_{ij}(s)\|_\infty = \|\frac{\epsilon_i(s)}{\epsilon_j(s)}\|_\infty \le \|(1-\alpha)H(s)\|_\infty.$$

Proof: According to lemma 1, we can easily get that

$$\begin{array}{rcl} H_{ij}(s) & = & (1-\alpha)H(s)\theta_{ij} + (1-\alpha)^2H(s)^2(\Theta^2)_{ij} \\ & & + \cdots + (1-\alpha)^{n-1}H(s)^{n-1}(\Theta^{n-1})_{ij}, \end{array}$$

since the disturbance can transfer from j to i by any possible path. So

$$\begin{array}{rcl} \|H_{ij}(s)\|_\infty & \le & \|(1-\alpha)H(s)\theta_{ij}\|_\infty \\ & & +\|(1-\alpha)^2H(s)^2(\Theta)^2_{ij}\|_\infty + \ldots \\ & & +\|(1-\alpha)^{n-1}H(s)^{n-1}(\Theta)^{n-1}_{ij}\|_\infty \\ & \le & \|(1-\alpha)H(s)\|_\infty\big(\theta_{ij} + (\Theta)^2_{ij} \\ & & + \cdots + (\Theta)^{n-1}_{ij}\big) \\ & = & \|(1-\alpha)H(s)\|_\infty \cdot q_{ij} \\ & \le & \|(1-\alpha)H(s)\|_\infty \end{array}$$

We notice that this conclusion is totally independent on n, i.e., the scale of the formation. The transfer function matrix is

$$\begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ H_{21}(s) & 1 & 0 & \ldots & 0 \\ H_{31}(s) & H_{22}(s) & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ H_{n1}(s) & H_{n2}(s) & \ldots & H_{n,n-1}(s) & 1 \end{bmatrix}$$

where $\|H_{ij}(s)\|_\infty \le \|(1-\alpha)H(s)\|_\infty < 1$. It is clear that $\|(1-\alpha)H(s)\|_\infty < 1$ is a sufficient condition for string stability. Also we can tell that the performance of disturbance resistance of the whole formation will be no worse than $\|(1-\alpha)H(s)\|_\infty$.

So for a look-ahead system, we can first design the local controller without considering the formation, then select a proper α to achieve the desired disturbance resistance performance. We use the formation in figure 1 as an example and get the path matrix

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 1 & 0.5 & 0.5 & 1 & 0 & 0 & 0 \\ 1 & 0.25 & 0.75 & 0.5 & 0 & 0 & 0 \\ 1 & 0.5 & 0.5 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

Suppose the transfer function of the individual vehicle system is a typical second order system as

$$H(s) = \frac{2s+3}{s^2+2s+3}.$$

We select $\alpha = 0.5$ and can get the transfer function matrix. For instance, the transfer function between vehicle 2 and 7 is

$$H_{72} = \frac{0.5s^2+1.5s+1.125}{s^4+4s^3+10s^2+12s+9}.$$

The magnitude bode plot is shown below:

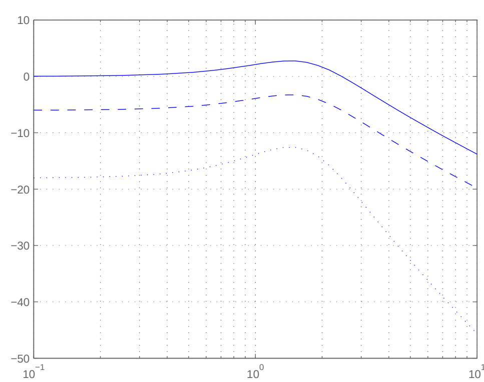

Figure 3: Magnitude bode plots

The solid curve is for $H(s)$, the dashed curve is $(1-\alpha)H(s)$, and the dotted is $H_{72}(s)$. It is clear that the disturbance is attenuated by at least -12.6 dB, and easy to verify that any other element in the transfer function matrix satisfies $\|H_{ij}(s)\|_\infty \le \|(1-\alpha)H(s)\|_\infty$ if $0 < i < j \le n$.

5 Connection failure tolerance

We have mentioned that if $\alpha = 1$, then the disturbance will not propagate among the formation. This is the best string stability performance we can achieve. But this strategy also has its fatal disadvantages:

- It cannot avoid the internal collisions naturally since every vehicle does not sense its neighbors.

- It cannot tolerate any connection failure. We focus on this issue in this section.

The discussions in the previous section are based on one assumption: every follower can get the leader's information through the digraph G_1. But in the real system, the communication channels of G_1 may be broken due to the interferences, moving obstacles, or devices failures. We use *connection failure tolerance* to describe how those communication channels' failures will affect the formation performance.

When $\alpha = 1$, G_2 will not affect the formation any more. Each individual vehicle only try to follow the leader through G_1. Any connection failure in G_1 will cause at least one vehicle lose its target. This will result that the formation becomes instable since this vehicle will be separated from the formation.

When $\alpha \neq 1$, each vehicle has two references: one is the leader vehicle, the other is the equilibrium point among its neighbors. The coefficients α and $1-\alpha$ indicate how these two references contribute to the vehicle's movement. Any connection failure in G_1 or in G_2 will impair one reference of at least one vehicle. The formation still can keep stable since every vehicle still has references to follow. The only problem is that each vehicle need to update the value of its out-degree.

Since the probability of the connection failures in G_1 and G_2 occur simultaneously is relative very small, we can say the double-graph strategy has good connection failure tolerance. Of course, part of the formation will suffer a reduction in the string stability performance when a connection failure happens.

6 Conclusion and future work

In this paper, we use digraphs to describe the connections between vehicles in a look-ahead system. The double-graph model is a new method to describe the different information flows. By using this model, we introduce a local control structure so that the whole formation can has good disturbance resistance performance, i.e., be string stable. By giving an example, we verify the validity of this control strategy.

We also describe another formation performance: connection failure tolerance. This performance concerns about if the formation can still be stable with inter-vehicle connection failures. A look-ahead system with double-graph strategy has good connection failure tolerance.

There are still some interesting issues need to study. For example, if there exists loops in G_2, how we can keep the whole system still has good performance. This may result more constrains on the local controller design since individual stable vehicle will not guarantee the whole formation stable. Another future work may focus on how to formulate the trade-off between disturbance resistance and connection failure tolerance.

Acknowledgements

Work described in this paper was supported in part by the DARPA MICA program under contract F30602-01-2-0577.

References

[1] E. Barbieri. Stability analysis of a class of interconnected systems. *Journal of Dynamic Systems, Measurement and Control*, 115(3):546–551, 1993.

[2] K. C. Chu. Decentralized control of high speed vehicle strings. *Transportation Research*, pages 361–383, 1974.

[3] J. P. Desai, J. P. Ostrowski, and V. Kumar. Modeling and control of formations of nonholonomic mobile robots. *IEEE Transactions on Robotics and Automation*, 17(6):905–908, 2001.

[4] J. Eyre, D. Yanakiev, and I. Kanellakopoulos. A simplified framework for string stability analysis of automated vehicles. *Vehicle System Dynamics*, 30:375–405, 1998.

[5] J. A. Fax. *Optimal and Cooperative Control of Vehicle Formations*. PhD thesis, California Institute of Technology, 2002.

[6] M. Mesbahi and F. Y. Hadaegh. Formation flying control of multiple spacecraft via graphs, matrix, inequalities, and switching. *Journal of Guidance, Control, and Dynamics*, 24(2):369–377, 2001.

[7] R. Olfati-Saber and R.M. Murray. Consensus protocols for networks of dynamic agents. *Submitted, American Control Conference*, 2003.

[8] A. Pant, P. Seiler, and J.K. Hedrick. Mesh stability of look_ahead interconnected systems. *IEEE Transactions on Automatic Control*, 47(2):403–407, 2002.

[9] S. E. Shladover. Review of the state of development of advanced vehicle control systems. *Vehicle System Dynamics*, 24:551–595, 1995.

[10] S. S. Stankovic, M. J. Stanojevic, and D. D. Siljak. Decentralized overlapping control of a platoon of vehicles. *IEEE Transactions on Control Systems Technology*, 8(5):816–832, 2000.

[11] D. Swaroop and J.K. Hedrick. String stability of interconnected systems. *IEEE Transactions on Automatic Control*, 41(3):349–357, 1996.

[12] D. Swaroop and J.K. Hedrick. Constant spacing strategies for platooning in automated highway systems. *Journal of Dynamic Systems Mesasurement and Control-Transactions*, 121(3):462–470, 1999.