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On a Cooperative Pursuit Strategy

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***Abstract*— We study a simple pursuit scenario in which the pursuer has potential access to an additional off-board global sensor. However, the global sensor can be used for either of two purposes: to improve the state estimate of the pursuer, or to obtain more data about the trajectory being tracked. The problem is to determine the variation in the performance of the system as the global sensor changes its behavior. We use a stochastic strategy to optimize over the transmission pattern of the global sensor.**

I. Introduction

We consider the following cooperative pursuit problem. A road needs to be monitored for threats emanating along it. There are two types of monitoring vehicles. There are vehicles on the ground that can move fast and can pursue a threat. However, their sensing radius is limited. In addition, there are air-borne vehicles that can sense the entire region; however they cannot actually pursue the target. There are communication constraints that prevent passage of arbitrary amounts of information (essentially a map of the entire road) from the air-borne vehicle to the vehicles on the road. The problem is to obtain the best possible estimate of the situation on the road by using the two types of vehicles in a cooperative fashion.

II. Analysis

In this paper, we consider the case when only one air-borne vehicle and one ground vehicle are present. The ground vehicle has dynamics given by

$$x_{k+1} = Ax_k + Bu_k + w_k, \quad (1)$$

where $x_k \in \mathbf{R}^n$ is the process state, $u_k \in \mathbf{R}^m$ is the control input and w_k is zero-mean white Gaussian noise with covariance R_w. We assume that the target has a random initial state and then evolves according to the dynamics

$$t_{k+1} = A_t t_k + v_k, \quad (2)$$

where v_k is white Gaussian noise with covariance R_v. We also assume that it is not possible to observe the control inputs being fed to the target and hence the control input is also clubbed with the noise v_k. For simplicity we consider the mean of v_k to be 0, although in general, the effect of the control input might be to make the mean equal to μ, to represent, e.g., the general tendency of the target to move in a particular direction.

The ground vehicle has on-board sensors detecting its own state and is capable of following a trajectory through the control input u_k. The sensor equation is

$$y_k^1 = C^1 x_k + n_k^1, \quad (3)$$

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where n_k^1 is zero-mean white Gaussian noise with covariance R^1. The noises w_k, v_k and n_k^1 are all assumed independent of each other. Thus on its own, the ground vehicle is incapable of identifying the state (or location) of the threat and hence of pursuing the threat. To gain knowledge about the threat, it depends on the air-borne vehicle. The air-borne vehicle can sense the entire region of interest and can send either of two measurements

$$y_k^2 = C^2 x_k + n_k^2 \qquad y_k^3 = C^3 t_k + n_k^3.$$

The communication between the air-borne vehicle and the ground vehicle is over a wireless channel and is thus subject to stochastic losses. For now, we model this loss as occurring either in an i.i.d. fashion with probability λ. We can also consider the loss as according to a Markov chain as described later in the paper.

The object of the two pursuers is to pursue the threat by reducing the error $e_k = x_k - t_k$. We consider the quadratic cost function

$$J = \lim_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K} E\left[e_k^T Q e_k + u_k^T R u_k\right]. \quad (4)$$

The expectation in the cost function is taken over the various noises, the random initial conditions of the dynamics and over the probability of message drop λ. The expectation over v_k is needed since the future values of t_k are not known when the optimal control law is designed. We can consider the minimization of J by the ground vehicle through design of control input u_k for $k = 0, 1, 2, \cdots$. However, the minimization depends on what information the controller has access to at each time step. The ground vehicle can minimize the cost better either by knowing its own state x_k more accurately (hence if the air-borne vehicle communicates y_k^2) or by knowing the target state t_k with more accuracy (hence if the air-borne vehicle transmits y_k^3). For any schedule of transmission S of y_k^2 and y_k^3, we define $J^\star = \min_{u_k}(J|S)$. We aim at further minimizing $J^\star$ over the schedule of transmission of y_k^2 and y_k^3, i.e., to find $J_{opt} = \min_S J^\star$.

To solve the problem, we proceed as follows. We first note that we can easily calculate $J^\star$ for any given S. To this end, define $z_k = \begin{bmatrix} x_k^T & t_k^T \end{bmatrix}^T$. Then we can combine equations (1) and (2) into the joint equation

$$\begin{aligned} z_{k+1} &= \begin{bmatrix} A & 0 \\ 0 & A_t \end{bmatrix} z_k + \begin{bmatrix} B \\ 0 \end{bmatrix} u_k + \begin{bmatrix} w_k \\ v_k \end{bmatrix} \\ &= \mathcal{A} z_k + \mathcal{B} u_k + \mathcal{W}_k, \end{aligned} \quad (5)$$

with the covariance of the noise $\mathcal{W}_k$ being denoted by $\mathcal{R}_w$. For this system, the aim is to minimize the cost function (4) which can be re-written as

$$J = \lim_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K} E\left[z_k^T \begin{bmatrix} Q & -Q \\ -Q & Q \end{bmatrix} z_k + u_k^T R u_k \right] \tag{6}$$

At each time step, the controller has access to one of three sensor measurements given by

$$\begin{aligned} Y_k^1 &= \begin{bmatrix} C^1 & 0 \\ C^3 & 0 \end{bmatrix} z_k + \begin{bmatrix} n_k^1 \\ n_k^2 \end{bmatrix} = \mathcal{C}^1 z_k + \mathcal{N}_k^1 \\ Y_k^2 &= \begin{bmatrix} C^1 & 0 \\ 0 & C^2 \end{bmatrix} z_k + \begin{bmatrix} n_k^1 \\ n_k^3 \end{bmatrix} = \mathcal{C}^2 z_k + \mathcal{N}_k^2 \\ Y_k^3 &= \begin{bmatrix} C^1 & 0 \\ 0 & 0 \end{bmatrix} z_k + \begin{bmatrix} n_k^1 \\ 0 \end{bmatrix} = \mathcal{C}^3 z_k + \mathcal{N}_k^3 \end{aligned}$$

Denote the covariance matrix of noise $\mathcal{N}_k^i$ by R_{N^i}. For any given schedule, the problem is to minimize the quadratic cost function (6) for the linear plant (5) with a time-varying sensor of the form

$$Y_k = C_k z_k + n_k,$$

Using the standard method, e.g., described in [2] Chapter 9, we can prove that a separation principle exists between the optimal control input and the optimal estimate given the previous measurements. Thus the optimal control law is designed assuming that the state could directly be measured and the gain matrix for it is given by

$$\begin{aligned} F &= -\left(R + \mathcal{B}^T S \mathcal{B}\right)^{-1} \mathcal{B}^T S \mathcal{A}^T \\ S &= \mathcal{A}^T S \mathcal{A} + \mathcal{Q} - F^T \left(R + \mathcal{B}^T S \mathcal{B}\right) F. \end{aligned}$$

The optimal control input u_k at any point is formed by calculating the optimal state estimate $\hat{z}_{k|k-1}$ based on the measurements $\{Y_j\}_{j=0}^{k-1}$ and then using the optimal control law calculated above to yield $u_k = F\hat{z}_{k|k-1}$. Furthermore, the cost $J^\star$ can be evaluated to be

$$J^\star = \text{trace}\left((\mathcal{A}^T S \mathcal{A} + \mathcal{Q} - S)P\right) + \text{trace}(S\mathcal{R}_w),$$

where P is defined as

$$P = \lim_{k\to\infty} E\left[(z_k - \hat{z}_{k|k-1})^2\right].$$

The expectation in the above equation is again taken over all the noises in the system and the initial conditions. If the schedule involved the use of only one sensor, P would have been given by the usual Ricatti equation, however for a time-varying schedule, this is no longer the case.

Thus to minimize $J^\star$ over all possible choices of the schedule S, we need to minimize P. This is the classical sensor scheduling problem. We need to optimize the schedule of use of sensors Y^1, Y^2 and Y^3 to minimize the estimate error covariance P. Since the sensor Y^3 is chosen stochastically, the usual tree-search based methods do not work in this case. We use the stochastic sensor selection strategy proposed in [1]. We assume that the airborne vehicle transmits the message y_k^3 at every time step with probability ν and y_k^4 with probability $1-\nu$. Initially, we assume that the choice of the message to be sent is done in an i.i.d. fashion. Thus the sensors Y_k^1, Y_k^2 and Y_k^3 are chosen with probabilities $\nu(1-\lambda)$, $(1-\nu)(1-\lambda)$ and λ respectively. We aim at minimizing P by varying the parameter ν. Because of the additional randomness in the system introduced by the probability of choosing y^2 or y^3, the error covariance P (and the cost $J^\star$) now becomes stochastic. We consider its average value $E[P]$ where the expectation is taken over the probabilities ν and λ. We can now use the results from [1] to obtain an upper bound on the cost as follows.

Proposition 1: An upper bound for $E[J_{opt}]$ is given by

$$E[J_{opt}] < \text{trace}\left((\mathcal{A}^T S \mathcal{A} + \mathcal{Q} - S)X\right) + \text{trace}(S\mathcal{R}_w),$$

where X satisfies

$$\begin{aligned} X &= \mathcal{A} X \mathcal{A}^T + \mathcal{R}_w \\ &\quad - \sum_i q_i \mathcal{A} X (\mathcal{C}^i)^T \left(\mathcal{C}^i X (\mathcal{C}^i)^T + R_{N^i}\right)^{-1} \mathcal{C}^i X \mathcal{A}^T, \end{aligned}$$

where $q_1 = \nu(1-\lambda)$, $q_2 = (1-\nu)(1-\lambda)$ and $q_3 = \lambda$. Further the upper bound converges if there exists a positive definite matrix Δ and matrices K_1, K_2 and K_3, such that

$$\begin{aligned} \Delta &> \mathcal{R}_w \\ &\quad - \sum_i q_i \left((\mathcal{A} + K_i\mathcal{C}^i)\Delta(\mathcal{A} + K_i\mathcal{C}^i)^T + K_i R_{N^i} K_i^T\right). \end{aligned}$$

This is thus a sufficient condition for convergence of $E[P]$. A necessary condition is

$$q_i|\lambda_{\max}(\bar{\mathcal{A}}_i)|^2 \le 1,$$

where $\lambda_{\max}(\bar{\mathcal{A}}_i)$ is the eigenvalue with the maximum magnitude of the unobservable part of $\mathcal{A}$ when the pair $(\mathcal{A}, \mathcal{C}^i)$ is put in an observable cannonical form.

Proof: Proof follows readily from the bounds on $E[P]$ presented in [1] and the fact that $\text{trace}\left((\mathcal{A}^T S \mathcal{A} + \mathcal{Q} - S)P\right)$ can be rewritten as $\text{trace}\left((\mathcal{A}^T S \mathcal{A} + \mathcal{Q} - S)^{1/2} P (\mathcal{A}^T S \mathcal{A} + \mathcal{Q} - S)^{T/2}\right)$. ■

Since exact calculation of $E[P]$ appears intractable, we minimize the upper bound presented in Proposition 1 instead. Thus the optimization problem is $\min_\nu X$ where

$$\begin{aligned} X &= \mathcal{A} X \mathcal{A}^T + \mathcal{R}_w \\ &\quad - \sum_i q_i \mathcal{A} X (\mathcal{C}^i)^T \left(\mathcal{C}^i X (\mathcal{C}^i)^T + R_{N^i}\right)^{-1} \mathcal{C}^i X \mathcal{A}^T. \end{aligned}$$

We can solve this problem to obtain the optimal value of ν and implement the corresponding schedule.

The case where either the message loss occurs in a Markovian fashion or the choice of which measurement to transmit is done according to a Markov chain can also be handled by using the results from [1]. Suppose the message loss occurs according to a Markov chain with transition matrix Q_1 and the messages are chosen according to transition matrix Q_2. We need to consider 4 sensor states represented by Y^1, Y^2, $Y^{3,1}$ and $Y^{3,2}$. The sensor $Y^{3,1}$ and $Y^{3,2}$ are of the same form as Y^3, but correspond to the message being lost when y^2 and y^3 respectively was transmitted. The sensors switch according to a Markov chain with transition matrix

$$Q_3 = Q_1 \otimes Q_2,$$

where $A \otimes B$ represents the Kronecker product of matrices A and B. Let q_{ij} denote the elements of Q_3 and π_i be the steady-state probabilities of the various sensors being used. Now we can write the upper bound to be minimized.

Proposition 2: An upper bound for J_{opt} is given by

$$J_{opt} < \text{trace}\left((\mathcal{A}^T S \mathcal{A} + \mathcal{Q} - S)X\right) + \text{trace}(S\mathcal{R}_w),$$

where $X = \sum_j \pi_j X^j$ and

$$\pi_j X^j = \sum_{i=1}^{4} q_{ij}\pi_i\big(\mathcal{A}X^i\mathcal{A}^T + \mathcal{R}_w - \mathcal{A}X^i(\mathcal{C}^j)^T\left(\mathcal{C}^j X^i (\mathcal{C}^j)^T + R_{N^j}\right)^{-1}\mathcal{C}^j X^i \mathcal{A}^T\big).$$

We can write down conditions for the convergence of $E[P]$ similar to the i.i.d. case.

We now consider two examples to illustrate the algorithm discussed above. For the first example, we consider the ground vehicle to be double integrators with a step size of 0.2. Thus

$$A = \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0.02 \\ 0.2 \end{bmatrix}.$$

For the target vehicle, we assume $A_t = 0.8I$, where I is the identity matrix. We consider the sensing matrices C^i, all the noise covariance matrices except R^1 and the cost matrices Q and R to be all identity. R^1 is taken to be 0.2 times the identity matrix. Figure 1 shows the optimum value of the cost calculated using the above algorithm as the packet loss probability λ is varied. It can be seen that the message loss probability has a great impact on the information pattern transmitted by the air-borne vehicle. In the second example, we show how the algorithm is useful in modeling many similar situations. Consider the case when there is only one air-borne vehicle but N ground vehicles. The air-borne vehicle has to provide the measurements to many ground vehicles, which reduces the time it can spend on a particular vehicle. We can study the decline in performance as N increases. For simplicity, we assume that there are no message losses and all vehicles are served with equal priority. Further for each vehicle, the measurement y_k^2 and y_k^3 is transmitted with the same probability. Thus any given vehicle obtains measurements Y_k^1 and Y_k^2 with probability $1/(2N)$ and the measurement Y_k^3 with probability $(N-1)/N$. We assume the same system parameters as in the above example except for $R_w = R_v = 0.1I$ and $R^1 = 10I$. Figure 2 shows the variation in performance as a function of N. It can be seen that initially the addition of more vehicles causes a large decrease in performance. However, after some time the plot flattens considerably showing that the control part of the cost function dominates the estimation error.

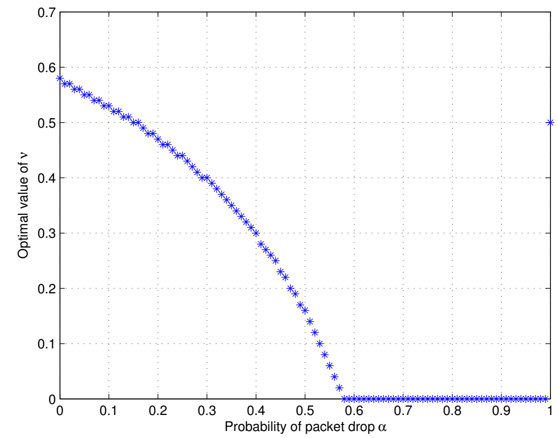

Fig. 1. Optimal value of parameter ν as the packet loss probability λ is varied.

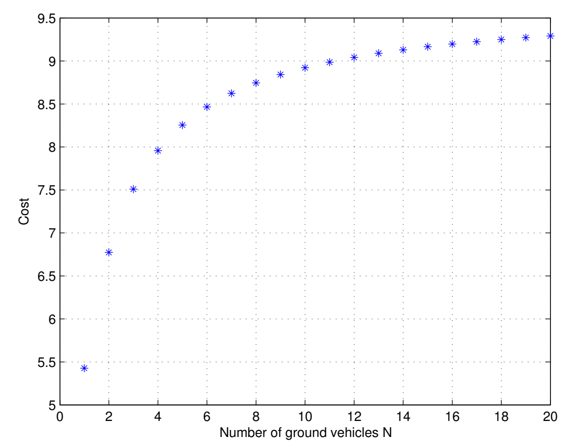

Fig. 2. Performance of the system across a packet dropping channel.

III. Conclusions and Future Work

We considered a simple cooperative pursuit scenario in this paper and applied some known results in sensor scheduling to obtain the optimal information pattern. Future work would concentrate on multiple air-borne vehicles and the case when the pursuers have partial information about the control inputs of the target.

References

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