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Model Predictive Control for an Uncertain Smart Thermal Grid

Samira S. Farahani*, Zofia Lukszo**, Tamás Keviczky***, Bart De Schutter***, Richard M. Murray*

***Abstract*— Smart Thermal Grids (STG) represents a new concept in the energy sector that involves the use of the smart grid concept in heat grids connecting several parties to each other via bidirectional transport of heat. The focus of this paper is on modeling and control of STGs in which the uncertainties in the demand and/or supply are included. To this end, we use Model Predictive Control (MPC), which is one of the most widely used advanced control design methods in the process industry. We solve the worst-case MPC optimization problem using mixed-integer-linear programming (MILP) techniques to provide a day-ahead prediction for the heat production in the grid. In an example, we show that this approach successfully keeps the supply-demand balance in the STG while satisfying the physical constraints of the network in the presence of uncertainties in the heat demand.**

I. INTRODUCTION

Realizing sustainable societies is one of the major challenges facing humanity in the 21^{st} century. This clarifies the increasing interest of both research institutes and industries in sustainable energy systems, particularly in smart energy systems. The design goal in smart grids is to improve the efficiency, reliability, and sustainability of the production and the distribution of energy. Smart Thermal Grids (STGs) can contribute to obtaining sustainable energy systems by guaranteeing a reliable heating supply to various customers by using renewable energy sources such as solar or geothermal energy. In STGs, the parties can be both producers and consumers. This concept is known as prosumer, where an entity (e.g. a greenhouse) fulfills the role of a consumer when it demands more energy than it produces with its production units (e.g. heat pumps) and fulfills the role of a producer when the demand is less than the production of its production units [13]. Because of this capability, STG implementation could contribute to a further decrease in carbon emissions, improved energy efficiency, and renewable energy implementation [7], [18].

STGs are best applicable to neighborhoods with small-scale utility companies and independent users. As about half of a neighborhood's electricity consumption is typically used for thermal purposes [2], introducing STG neighborhoods could have substantial benefits, such as: 1) less transport of energy, less energy loss, and lower transportation costs, and 2) using the produced heat at the neighborhood level as an energy source to avoid wasting heat. The convergence of these two aspects brings major efficiency improvements. Currently most of the thermal networks, for instance in the Netherlands, are mainly connected to one main heat producer and there is no explicit control strategy that considers both economic optimization as well as the network constraints. However, recently, there has been a growing interest in implementing STGs and benefiting from its features.

Considering the complexities of such systems, mainly due to uncertain demand and supply characteristics as well as their large size, smart energy systems need to be managed and controlled in an automated way in order to increase the efficiency for both producers and consumers. To this end, model predictive control (MPC) [21], [25] is a control method that has been proved to be a useful tool in both simulations and real-life applications [22], [23]. MPC uses real-time optimization in order to determine the control inputs for systems. MPC has the following features: it is applicable to multi-variable and nonlinear systems; it can handle constraints on both inputs and outputs in a systematic way; and it is capable of tracking pre-scheduled reference signals. MPC is based on a receding horizon approach to obtain an optimal control sequence that minimizes the given objective function subject to the model and operational constraints.

In this paper, we consider worst-case MPC of STGs due to the presence of uncertainties in the grid to provide a day-ahead heat production plan for the thermal grid. The uncertainties in the network can be due to the uncertainty in the demand and/or in the production because of using different resources such as solar energy or biogas. Although the control aspects of thermal energy have been studied implicitly in the context of Combined Heat and Power (CHP) systems or general smart grids, using distributed MPC and other similar agent-based control approaches [14], [24], the explicit implementation of the controller for STG systems requires careful investigation due to the structural differences between STGs and other types of smart grids such as smart electric grids. Smart thermal grids have been studied as a deterministic system by few researchers [20], [27] and uncertain/stochastic MPC has been studied by many researchers [6], [9], [10], [17]; however, in the context of smart thermal grids and MPC, to the authors' best knowledge, this paper will be the first attempt that studies worst-case MPC for STGs. To model the network, we use a mixed logical dynamical (MLD) model and we assume that the uncertainty is bounded within a polyhedral set. Hence, the worst-case MPC optimization problem can be recast as a Mixed-Integer-

This work was mainly done during the time that Samira S. Farahani was a postdoctoral researcher at the Energy and Industry Division at Delft University of Technology.

*Samira S. Farahani and Richard M. Murray are with the Department of Control and Dynamical Systems, California Institute of Technology, Pasadena, CA. farahani@caltech.edu

**Zofia Lukszo is with the Energy and Industry Division, Delft University of Technology, Delft, the Netherlands

***Tamás Keviczky and Bart De Schutter are with the Delft Center for Systems and Control, Delft University of Technology, Delft, the Netherlands

Linear Programming (MILP) problem which can be solved using the available algorithms.

II. SMART THERMAL GRIDS

In this work, we consider a regional network of greenhouses, which is a typical example of a thermal grid. Each of these greenhouses is considered as an agent and the full information of each agent, such as the production resources, the demand request for the next day, etc., is assumed to be available to the whole network. Each agent is facilitated with a Combined Heat and Power (CHP) system and a boiler and hence, is capable of local production of heat and electricity that can be used by the same agent or be exported/sold to the network. In this paper, we assume that the agents can only trade heat among each other and the electricity will be bought or sold to the electricity market only. Moreover, each agent has a buffer system to store heat and either to use it internally or to sell it to the other agents in the network.[1] In addition to the local heat generation, there are one or more external parties that can provide heat to the network. In this paper, we consider all the external parties as one single agent. The greenhouses are connected to each other and to the external suppliers by several pipes of different sizes. Moreover, to adjust the input and output heat to and from the greenhouses, there are several heat exchangers located outside the greenhouses.

To model the physical system, we discretize the system with sampling interval of one hour. The time step counter is denoted by k. For the sake of compactness, the model parameters are presented in Table I.

Parameters	Symbol	Unit
Transportation cost per MW	C_{trans}	€
The energy content of gas for CHP start up	g_{start}	MW
Electrical efficiency of the CHP unit	η_{e}	-
Thermal efficiency of the CHP unit	η_{th}	-
Thermal efficiency of the boiler	η_{Boil}	-
Turnaround efficiency of the buffer unit	η_{Buf}	-
Fuel price per MW	F_{price}	€
CHP maintenance cost per MW	C_{CHP}	€
CHP fixed start up cost	C_{fix}	€
Buffer capacity of each greenhouse	B_{C}	MWh
Minimum heat production capacity for unit u	$\underline{\text{U}}_u$	MWh
Maximum heat production capacity for unit u	$\bar{\text{U}}_u$	MWh

TABLE I

MODEL PARAMETERS AND THEIR MEASUREMENT UNITS.

To keep our model simple, we assume that the heat exchangers do not add additional costs to the heat production and hence, both can be left out from the network model. The fuel energy content (gas in our case) used by a CHP unit at greenhouse j at time step k in MW can be specified as [14], [26]

$$g_{\text{CHP}j}(k) = \frac{P_{\text{G}_{\text{CHP}}j}(k)}{\eta_{\text{e}}} = H_{\text{G}_{\text{CHP}}j}(k) \cdot \frac{1}{\eta_{\text{th}}}, \tag{1}$$

[1]In general, the greenhouses may also have access to other renewable energy sources such as solar energy collectors, geothermal energy, etc.

where $P_{\text{G}_{\text{CHP}}j}(k)$ and $H_{\text{G}_{\text{CHP}}j}(k)$ are respectively the electrical power and the heat generated by the CHP unit of greenhouse j at time step k in MW. Similarly, for a boiler, we have [15]

$$g_{\text{Boil}j}(k) = H_{\text{G}_{\text{Boil}}j}(k) \cdot \frac{1}{\eta_{\text{Boil}}}, \tag{2}$$

where $g_{\text{Boil}j}(k)$ is defined similarly to $g_{\text{CHP}j}(k)$ and $H_{\text{G}_{\text{Boil}}j}(k)$ is the heat generated by the boiler of greenhouse j at time step k in MW.

If the CHP or boiler unit are operating at greenhouse j, the thermal power can vary at each time step between a certain minimum and maximum for both the CHP and the boiler as

$$\underline{\text{U}}_{\text{CHP}j} \leq H_{\text{G}_{\text{CHP}}j}(k) \leq \bar{\text{U}}_{\text{CHP}j} \quad \forall\, k, j \tag{3}$$

$$\underline{\text{U}}_{\text{Boil}j} \leq H_{\text{G}_{\text{Boil}}j}(k) \leq \bar{\text{U}}_{\text{Boil}j} \quad \forall\, k, j. \tag{4}$$

Moreover, in the case that the production units, i.e., the boiler and the CHP, of greenhouse j produce more heat than is demanded by the greenhouse itself, the heat can be stored in a buffer to be used at other hours or to be used by other greenhouses in the network. We assume that each greenhouse j can only send or receive heat to or from its immediate neighbors, respectively. Let $H_{\text{exch}ij}$ denotes the exchanged heat between two adjacent greenhouses i and j. The buffer state of greenhouse j can then be defined as

$$B_{\text{S}j}(k) = B_{\text{S}j}(k-1) + \eta_{\text{Buf}}\Big(H_{\text{G}_{\text{CHP}}j}(k) + H_{\text{G}_{\text{Boil}}j}(k) - H_{\text{D}j}(k) + H_{\text{impEx}j}(k) + \sum_{i\in\phi_j}(1-\alpha_{ij})H_{\text{exch}ij}\Big), \tag{5}$$

where $H_{\text{D}j}(k)$ denotes the heat demand of greenhouse j at time step k, $H_{\text{impEx}j}(k)$ denotes the imported heat by greenhouse j from an external party at time step k, ϕ_j is the set of neighbors of greenhouse j, and α_{ij} denotes the percentage of heat loss due to transportation between greenhouse i and j. Moreover, we have capacity constraints for the buffer and constraint for the amount of heat imported from external parties or exchanged between two neighbors due to for instance pipe or network capacity. There is also an additional constraint for the transported heat among the greenhouses in order to make sure that the supply-demand balance is satisfied at each time step. These constraints are defined as

$$0 \leq B_{\text{S}j}(k) \leq B_{\text{C}j} \quad \forall\, k, j \tag{6}$$

$$0 \leq H_{\text{impEx}j}(k) \leq \bar{\text{U}}_{\text{impEx}j} \quad \forall\, k, j \tag{7}$$

$$H_{\text{exch}ij} = -H_{\text{exch}ji} \quad \forall\, j, i \in \phi_j \tag{8}$$

$$\underline{\text{U}}_{\text{exch}ij} \leq H_{\text{exch}ij} \leq \bar{\text{U}}_{\text{exch}ij} \tag{9}$$

where $\bar{\text{U}}_{\text{impEx}j}$ is the maximum possible heat import from external parties and $\underline{\text{U}}_{\text{exch}ij}$ and $\bar{\text{U}}_{\text{exch}ij}$ are minimum and maximum amount of heat that can be exchanged between two adjacent neighbors. Note that $H_{\text{exch}ij}$ takes both positive and negative values indicating the imported heat by greenhouse j from greenhouse i and exporting heat from greenhouse i to greenhouse j, respectively.

Figure 1 illustrates the energy flow between one greenhouse and the network of greenhouses, as well as the heat-producing external parties and the energy retailers.

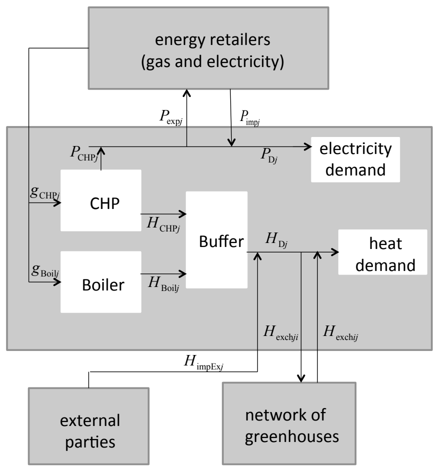

Fig. 1. Energy flow between greenhouse j, network of greenhouses, heat producing external parties, and energy retailers.

Remark 1: The connection between each greenhouse j and the external parties in Figure 1 does not reflect the physical connection and it is rather an indicator for the heat flow. In fact, the external parties are physically connected to the whole network (through a main pipe) and not to each greenhouse individually.

Now that the mathematical model of the production units is specified, we discuss our control strategy, which aims at providing a day-ahead production plan for each greenhouse.

III. MODEL PREDICTIVE CONTROL FOR STGS

Our aim is to reduce the overall production costs of the network while providing the network's required heat under different operational constraints such as the limits for the generators and the buffers. To this end, we intend to develop an advanced control approach that is suitable for practical applications. The control objective will be focused on demand response [14], [28], which is the ability of domestic net-consumption of heat to respond to real-time[2] electricity prices. In this paper by "real-time" electricity prices we mean the hourly varying supply tariff, which is equal to the hourly day-ahead prices of the electricity market.

The control strategy that is proposed here for demand response is Model Predictive Control (MPC). In MPC, at each iteration, the optimal control sequence is computed over a finite horizon, i.e., a finite period of time. MPC uses the receding horizon principle, which means that after computation of the optimal control sequence, only the first sample will be implemented in the next iteration. Subsequently, the horizon will be shifted one sample, and the optimization will be restarted with new information of the measurements.

[2]In general, the real-time electricity price is the one that varies almost every 15 minutes in the electricity market on the exact day of the electricity production.

The control objective is to minimize the total heat production costs, which includes the variable costs of the network related to the heat production as well as the earnings. Without loss of generality, we assume that the network is owned by a single owner and hence, all greenhouses cooperate with each other in order to keep the total heat generation costs of the network as low as possible. This means that they try to generate as much heat as possible in order to satisfy the heat demand of the network and buy as less as possible from the external parties. The total heat production cost function of greenhouse j at time step k can be defined as

$$\begin{aligned} &C(P_{\mathrm{G_{CHP}}j}(k), H_{\mathrm{G_{Boil}}j}(k), H_{\mathrm{impEx}j}(k), \mu^{\mathrm{start}}_{\mathrm{CHP}j}(k)) \qquad (10)\\ &\quad = C_{\mathrm{G}}(P_{\mathrm{G_{CHP}}j}(k), H_{\mathrm{G_{Boil}}j}(k)) + C_{\mathrm{O}}(P_{\mathrm{G_{CHP}}j}(k))\\ &\qquad + C_{\mathrm{imp}}(H_{\mathrm{impEx}j}(k)) + C_{\mathrm{start}}(\mu^{\mathrm{start}}_{\mathrm{CHP}j}(k)) - E_{\mathrm{P}}(P_{\mathrm{CHP},j}(k)). \end{aligned}$$

The definition of each of the functions and variables is given below.

The heat generation cost for each greenhouse depends on the amount of fuel that is used. Therefore, considering equations (1) and (2), it can be defined as

$$C_{\mathrm{G}}(P_{\mathrm{G_{CHP}}j}(k), H_{\mathrm{G_{Boil}}j}(k)) = \bigg(g_{\mathrm{CHP}j}(k) + g_{\mathrm{Boil}j}(k)\bigg) F_{\mathrm{price}}. \qquad (11)$$

For each CHP, there will also be an additional cost, namely, the operation cost, which is defined for each greenhouse j at time step k as

$$C_{\mathrm{O}}(P_{\mathrm{G_{CHP}}j}(k)) = P_{\mathrm{G_{CHP}}j}(k) \cdot C_{\mathrm{CHP}}. \qquad (12)$$

The import cost matters when the greenhouse needs to buy heat from an external party, in the case that the generated heat by the greenhouse itself and the amount that is imported from other greenhouses in the network is less than its demand. The cost of importing heat by greenhouse j at time step k can be defined as

$$C_{\mathrm{imp}}(H_{\mathrm{impEx}j}(k)) = H_{\mathrm{impEx}j}(k) \cdot H_{\mathrm{buyingEx}}(k), \qquad (13)$$

where $H_{\mathrm{buyingEx}}(k)$ is the price that greenhouse j pays for buying heat from external parties at time step k. We assume that the taxes and the transportation cost are included in $H_{\mathrm{buyingEx}}(k)$. Moreover, there are fixed start-up costs and fuel-based start-up costs for a CHP unit of greenhouse j, which can be calculated as [11]

$$C_{\mathrm{start}}(\mu^{\mathrm{start}}_{\mathrm{CHP}j}(k)) = \mu^{\mathrm{start}}_{\mathrm{CHP}j}(k) \bigg(C_{\mathrm{fix}} + g_{\mathrm{start}} \cdot F_{\mathrm{price}}\bigg), \qquad (14)$$

where $\mu^{\mathrm{start}}_{uj}$ is a binary variable such that $\mu^{\mathrm{start}}_{uj}(k) = 1$ if unit u (CHP or boiler) of greenhouse j is started for production of energy at time step k and $\mu^{\mathrm{start}}_{uj}(k) = 0$ otherwise. The second part of the production cost is related to the electricity earnings obtained from selling electricity to the electricity market. The selling price is variable and is different every hour. The electricity earnings of greenhouse j at time step k

can be written as

$$E_{\mathrm{P}}(P_{\mathrm{G_{CHP}}j}(k)) = \begin{cases} \Big(P_{\mathrm{G_{CHP}}j}(k) - P_{\mathrm{D}j}(k)\Big)P_{\mathrm{selling}}(k) \\ \qquad\qquad \text{if } P_{\mathrm{G_{CHP}}j}(k) \geq P_{\mathrm{D}j}(k) \\ 0 \qquad\qquad \text{if } P_{\mathrm{G_{CHP}}j}(k) < P_{\mathrm{D}j}(k) \end{cases} \tag{15}$$

where $P_{\mathrm{D}j}(k)$ indicates the electricity demand of greenhouse j at time step k and $P_{\mathrm{selling}}(k)$ is the selling price of electricity at time step k. Note that since we assume cooperation between the greenhouses, there are no heat earnings while the greenhouses exchange heat among each other.

Therefore, considering equation (10), the cost function $J(k)$ at time step k over the prediction horizon N_{p} is defined as

$$J = \sum_{l=0}^{N_{\mathrm{p}}-1} \sum_{j=1}^{n} C\Big(P_{\mathrm{G_{CHP}}j}(k+l), H_{\mathrm{G_{Boil}}j}(k+l), H_{\mathrm{impEx}j}(k+l), \mu^{\mathrm{start}}_{\mathrm{CHP}j}(k+l)\Big). \tag{16}$$

This cost function will be minimized subject to the constraints on different components of the systems. Some of these constraints have been presented in the previous section. In addition to those, we need extra constraints related to on-off states of the CHP and boiler [14]. We define $\mu^{\mathrm{stop}}_{\mathrm{u},j}$ as a binary variable such that $\mu^{\mathrm{stop}}_{\mathrm{u},j}(k) = 1$ if unit u (CHP or boiler) of greenhouse j is shut down at time step k and 0 otherwise. Moreover, we define the binary variable $v_{uij}(k)$ for each production unit u, i.e., the CHP and the boiler, of greenhouse j at time step k as

$$v_{\mathrm{CHP}j}(k) = \begin{cases} 1, & \text{if CHP operates} \\ 0, & \text{if CHP does not operate} \end{cases} \tag{17}$$

$$v_{\mathrm{Boil}j}(k) = \begin{cases} 1, & \text{if the boiler operates} \\ 0, & \text{if the boiler does not operate} \end{cases} \tag{18}$$

Therefore, the capacity constraints for the heat production, i.e., equations (3)-(4) can be rewritten as

$$\underline{\mathrm{U}}_{\mathrm{CHP}j} \cdot v_{\mathrm{CHP}j}(k) \leq H_{G_{\mathrm{CHP}}j}(k) \leq \bar{\mathrm{U}}_{\mathrm{CHP}j} \cdot v_{\mathrm{CHP}j}(k) \ \ \forall\, k, j \tag{19}$$

$$\underline{\mathrm{U}}_{\mathrm{Boil}j} \cdot v_{\mathrm{Boil}j}(k) \leq H_{G_{\mathrm{Boil}}j}(k) \leq \bar{\mathrm{U}}_{\mathrm{Boil}j} \cdot v_{\mathrm{Boil}j}(k) \ \ \forall\, k, j. \tag{20}$$

Moreover, the following equations link the above binary variables:

$$v_{\mathrm{CHP}j}(k) - v_{\mathrm{CHP}j}(k-1) = \mu^{\mathrm{start}}_{\mathrm{CHP},j}(k) - \mu^{\mathrm{stop}}_{\mathrm{CHP},j}(k) \ \ \forall j,k \tag{21}$$

$$v_{\mathrm{Boil}j}(k) - v_{\mathrm{Boil}j}(k-1) = \mu^{\mathrm{start}}_{\mathrm{Boil},j}(k) - \mu^{\mathrm{stop}}_{\mathrm{Boil},j}(k) \ \ \forall j,k \tag{22}$$

$$\mu^{\mathrm{start}}_{\mathrm{CHP},j}(k) + \mu^{\mathrm{stop}}_{\mathrm{CHP},j}(k) \leq 1 \ \ \forall j,k \tag{23}$$

$$\mu^{\mathrm{start}}_{\mathrm{Boil},j}(k) + \mu^{\mathrm{down}}_{\mathrm{Boil},j}(k) \leq 1 \ \ \forall j,k. \tag{24}$$

Note that this control approach is a centralized one, which means while the overall production cost of the network is minimized, each individual greenhouse may not have the optimal cost at each time step.

In order to obtain a linear system with continuous and binary variables, we apply the mixed logical dynamical (MLD) formalism [4], which allows the transformation of logical statements involving continuous variables into mixed-integer linear inequalities. Accordingly, we can rewrite equation (15) as a linear equation by introducing new binary and continuous auxiliary variables. In this way, the system dynamics and the constraints are formulated as mixed-integer linear equations and hence, we will solve a mixed-integer linear programming (MILP) problem. In the next section, we explain how to solve the worst-case MILP-MPC optimization problem.

IV. SOLVING THE WORST-CASE MPC

At the beginning of each time step k, the controller measures the system state of the previous step. In our case, the state variables are $B_{\mathrm{S}j}$, $v_{\mathrm{CHP}j}$, and $v_{\mathrm{Boil}j}$. At each control step k, we assume that the previous value of these variables is known or measured. Then, using the information regarding the demand and the energy price, the controller determines the decision variables, which are $P_{\mathrm{G_{CHP}}j}, H_{\mathrm{G_{Boil}}j}, H_{\mathrm{impEx}j}, \mu^{\mathrm{stop}}_{\mathrm{u}j}$, and $\mu^{\mathrm{start}}_{\mathrm{u}j}$. We choose the prediction horizon $N_{\mathrm{p}} = 24$, corresponding to the 24 hours in one day.

We also assume that there is an uncertainty in the heat demand H_{D}, i.e., $H_{\mathrm{D}}(k) = H_{\mathrm{D,pred}}(k) + e(k)$ where $H_{\mathrm{D,pred}}(k)$ is the predicted heat demand for the greenhouses at time step k. We gather the uncertainty for time steps $k,\dots,k+N_{\mathrm{p}}-1$ in the vector $\tilde{e}(k) = [e^T(k),\dots,e^T(k+N_{\mathrm{p}}-1)]^T \in \mathscr{E}$ where $\mathscr{E} = \{\tilde{e}(k) : \tilde{S}\tilde{e}(k) \leq \tilde{q}\}$ is a bounded polyhedral set. Accordingly, we can define the worst-case MPC optimization problem as

$$\min_{\tilde{u}(k)} \max_{\tilde{e}(k)\in\mathscr{E}} J(\tilde{u}(k), \tilde{e}(k)) \tag{25}$$

$$\text{s.t. } P(k)\tilde{u}(k) + Q(k)\tilde{e}(k) + q(k) \leq 0 \tag{26}$$

where J is the cost function, $\tilde{u}(k)$ is the vector of decision variables containing both continuous and binary variables as well as the continuous and binary auxiliary variables obtained from the MLD model (defined similarly to $\tilde{e}(k)$), $P(k), Q(k)$ are inequality constraint matrices and $q(k)$ is the inequality constraint constant vector, all defined according to the constraints (6)-(9) and (19)-(24). Since both the cost function J and the constraints are piecewise affine in $\tilde{u}(k)$, we can solve the optimization problem (25)-(26) as an MILP problem.

To this end, we solve the inner optimization problem first. For a given $\tilde{u}(k)$, the optimization problem

$$\max_{\tilde{e}(k)} J(\tilde{u}(k), \tilde{e}(k)) \tag{27}$$

$$\text{s.t. } \tilde{S}\tilde{e}(k) \leq \tilde{q} \tag{28}$$

$$P(k)\tilde{u}(k) + Q(k)\tilde{e}(k) + q(k) \leq 0$$

can be solved as a multi-parametric MILP (mp-MILP) problem, in which $\tilde{u}(k)$ is the parameter, using the algorithm in [8].

Let $\tilde{e}^*(\tilde{u}(k)) = \arg\max_{\tilde{e}(k)} J(\tilde{u}(k), \tilde{e}(k))$ denote the solution of the mp-MILP problem (27)-(28), which is a piecewise-affine function in $\tilde{u}(k)$ (see [5], [16]). Hence, the outer optimization problem, i.e.,

$$\min_{\tilde{u}(k)} J(\tilde{u}(k), \tilde{e}^*(\tilde{u}(k))) \tag{29}$$

$$\text{s.t. } P(k)\tilde{u}(k) + Q(k)\tilde{e}(k) + q(k) \leq 0 \tag{30}$$

can be solved as an MILP optimization problem using the available MILP solvers that are based on e.g. branch-and-bound or cutting plane algorithms [3], [19].

Note that the available mp-MILP algorithms are not very efficient when the size of the vector of parameters and the prediction horizon N_p increases. Therefore, we now discuss alternative approaches to mp-MILP. One approach is to use Monte Carlo simulation to eliminate the inner optimization problem as follows. Let $\tilde{e}^{(1)}(k), \ldots, \tilde{e}^{(M)(k)}$ denote M different noise realizations[3] belonging to the polyhedral set $\mathscr{E}$ and let $t(k) = \max_{\tilde{e}^{(1)}(k), \ldots, \tilde{e}^{(M)(k)}} (J(\tilde{u}(k), \tilde{e}^{(1)}(k)), \cdots, J(\tilde{u}(k), \tilde{e}^{(M)}(k)))$. The optimization problem (25)-(26) can be then rewritten as

$$\min_{\tilde{u}(k), t(k)} t(k) \tag{31}$$

$$\begin{aligned} \text{s.t. } & t(k) \geq J(\tilde{u}(k), \tilde{e}^{(1)}(k)) \\ & \cdots \\ & t(k) \geq J(\tilde{u}(k), \tilde{e}^{(M)}(k)) \\ & P(k)\tilde{u}(k) + Q(k)\tilde{e}^{(1)}(k) + q(k) \leq 0 \\ & \cdots \\ & P(k)\tilde{u}(k) + Q(k)\tilde{e}^{(M)}(k) + q(k) \leq 0 \end{aligned} \tag{32}$$

which can be solved as an MILP optimization problem.

Another approach is bilevel optimization. In this approach, we treat the inner optimization problem as the objective function of the outer optimization problem, i.e.,

$$\min_{\tilde{u}(k)} F(\tilde{u}(k)) \tag{33}$$

where $F(\tilde{u}(k))$ corresponds to the inner optimization problem (27)-(28). Note that for a given $\tilde{u}(k)$, the inner optimization problem can be solved as an MILP optimization problem and then the outer optimization problem (33), which is a binary/integer optimization problem, can be solved using e.g. genetic algorithms [12].

V. EXAMPLE

In this section, we solve the worst-case MPC optimization problem to obtain a day-ahead prediction plan for the heat production for a small network of greenhouses. In this model, we consider two green houses and an external producer. Each of the greenhouses has a CHP unit, a boiler, and a buffer. The aim is to minimize the heat production cost of the network while satisfying the network constraints and supply-demand balance.

We consider the cost function (16) and we assume that the heat demand is uncertain, i.e., $H_{\mathrm{D}j}(k) = H_{\mathrm{D,pred}j}(k) + e_j(k)$ where $H_{\mathrm{D,pred}j}(k)$ is the predicted heat demand for greenhouse $j \in \{1,2\}$ at time step k and $e_j(k)$ denotes the demand uncertainty such that $|e_j(k)| \leq 1$. The heat demand

[3] M can be defined based on the desired level of accuracy and computational efficiency [1].

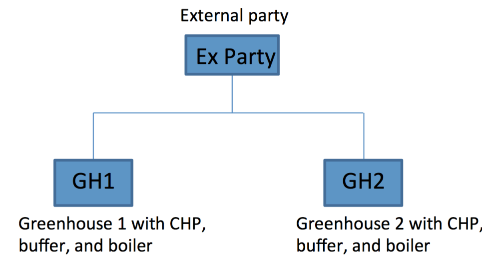

Fig. 2. Physical topology of the thermal network of the case study

is given for a random warm day of the year and the gas and electricity prices are take from the Dutch gas and electricity market.

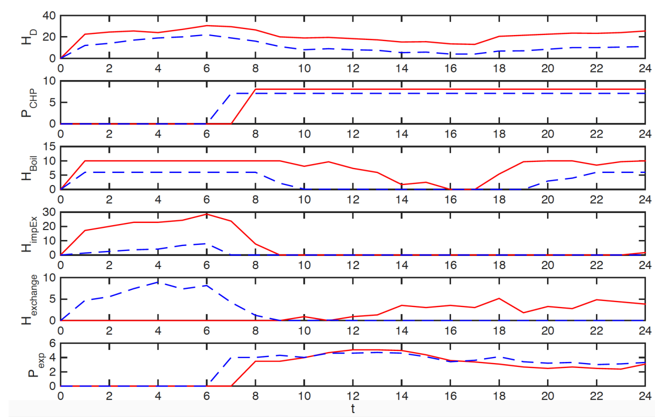

Fig. 3. Heat and electricity production and exchange plan of the thermal network; the solid line corresponds to greenhouse 1 and the dashed line corresponds to greenhouse 2.

The cost function is minimized subject to the constraints (6)-(9) and (19)-(24). At each time step we have 456 control variables (240 binary variables including the auxiliary variables from MLD model), an uncertainty vector of size 48, and 960 inequality constraints. Solving the mp-MILP optimization problem using the MPT toolbox seems very inefficient for a problem of this size. Alternatively, we use the Monte Carlo approach explained in Section IV to obtain the day-ahead heat production plan, which is solved in Matlab R2014b on a 2.6 GHz Intel Core i5 processor. To solve the optimization problem (31)-(32), we chose $M = 500$ different uncertainty vectors e and we use the MILP solver from IBM CPLEX. The computation time was 63.0458 min for a 0.95% confidence level with accuracy error of 1%. Of course there is a trade-off between the computation time and the accuracy. In practical cases, the computation time of 1 hour for a day ahead prediction under uncertainty is acceptable.

The optimization results are shown in Figures 3. The first plot shows the heat demand of each greenhouse for one day. The second and third plots shows the amount of electricity and heat that needs to be generated by the CHP and boiler units at each greenhouse. The forth plot shows the amount of imported heat from the external parties. The fifth plot illustrates the amount of heat that is imported by each greenhouse from the other one. The last plot shows the amount of electricity that each greenhouse has exported to the electricity market. Here, we assume that the production capacity of the CHP and boiler units are similar to each other for the sake of illustration; in reality, the boilers capacities are much higher and since they impose less production costs, they will be used more than the CHPs. Hence, in practice, CHPs are mainly used when the electricity price is high so the greenhouses can benefit from selling the extra generated electricity to the electricity market. Following this production plan, the total heat production cost of the network during 24 hours is 22731€. Note that the total production costs of this network for the case that the greenhouses do not exchange heat among each other and only buy from the external heat producers is 30194€. This shows that the current model is more cost-efficient comparing to the model with a single (main) producer and the greenhouses acting as a costumer rather than being a prosumer.

VI. CONCLUSION

This paper has considered control of smart thermal grids under uncertainties in demand and/or response. Thermal grids refer to energy networks whose main objective is to provide and distribute heat among their users. In this paper, we have considered a typical thermal grid, namely a network of greenhouses. Our aim was to provide a day-ahead prediction plan for heat generation in the network assuming there is uncertainty in the heat demand. To obtain an economical plan, we minimized the total heat production cost of the network using model predictive control. We assumed the uncertainty to be bounded and hence, a worst-case MPC optimization problem was solved. Since both the cost function and the constraints are linear, the optimization problem was formulated as a mixed-integer linear programming (MILP) problem. We have discussed three approaches to solve the obtained optimization problem and in a case study we obtained a day-ahead production plan for a sample network of greenhouses.

An alternative scenario to the centralized control architecture is that each greenhouse tries to maximize its own benefit and hence, they will sell heat to the other greenhouses in the network. The efficient control approach in this case is a distributed model predictive control, which is the topic of our future research.

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