# A Mathematical Introduction to Robotic Manipulation R. M. Murray, Z. Li, and S. S. Sastry 

## Errata

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Changes from first printing to August 28, 2005

Conventions: each line of text and each line of a displayed equation are counted as a line. Negative line numbers mean number of lines from the bottom of the page. First line on page $=1$, last line on page $=-1$. Entries in boldface indicate errors which might change the interpretation of the text.

## Changes to first and second printing (Oct 1994)

The following changes apply to the first and second printings of the text. Changes which apply only to the first printing of the book are listed in the next section.

- Page 30, line 16. (C. A. Vanelli, 10/4/96) The expression for $\omega$ in equation (2.18) is singular when $\theta$ is $\pi$. To fix this, use the diagonal terms to determine the axis of rotation.
- Page 40, figure 2.5. (W. Wray, $1 / 12 / 95)$ The expression $p(t$ should be $p(t)$ in the figure (occurs twice).
- Page 75, line -2. (W. Wray, 1/12/95) $S O(3)$ should be $S O(2)$.
- Page 114, line -6. (R. Mason, 4/16/95) The phrase "choosing fewer eight" should be "choosing fewer than eight".
- Page 173, line 2. (P. Stellman, $8 / 2 / 05$ ) In the expression for $J_{3}$, the $(1,1)$ entry of the matrix should read $-l_{1} c_{2}-r_{2} c_{23}$ (the $l_{2}$ term should have been $l_{1}$ ).
- Page 176, line -5. (J.-H. Lee, 8/27/95) The displayed equation should read

$$
\begin{align*}
\frac{\partial M_{i j}}{\partial \theta_{k}}=\sum_{l=\max (i, j)}^{n}\left(\left[A_{k i} \xi_{i}, \xi_{k}\right]^{T} A_{l k}^{T} \mathcal{M}_{\uparrow}^{\prime} \mathcal{A}_{\downarrow \mid} \xi_{\mid}\right. & \\
& \left.+\xi_{i}^{T} A_{l i}^{T} \mathcal{M}_{\uparrow}^{\prime} \mathcal{A}_{\llbracket \|}\left[\mathcal{A}_{\| \mid} \xi_{\mid}, \xi_{\| \|}\right]\right) \tag{1}
\end{align*}
$$

- Page 177, line 11-13. (J.-H. Lee, 8/27/95) The computation for the partial derivative has multiple errors for the indices in lines $2-4$. The correct expression is

$$
\begin{aligned}
\frac{\partial}{\partial \theta_{k}}\left(A_{l j} \xi_{j}\right) & =\left(\frac{\partial}{\partial \theta_{k}}\left(g_{l j} \widehat{\xi}_{j} g_{l j}^{-1}\right)\right)^{\vee}=\left(\frac{\partial g_{l j}}{\partial \theta_{k}} \widehat{\xi}_{j} g_{l j}^{-1}+g_{l j} \widehat{\xi}_{j} \frac{\partial g_{l j}^{-1}}{\partial \theta_{k}}\right)^{\vee} \\
& =\left(-g_{l, k} \widehat{\xi}_{k} g_{k j} \widehat{\xi}_{j} g_{l j}^{-1}+g_{l j} \widehat{\xi}_{j} g_{k j}^{-1} \widehat{\xi}_{k} g_{l k}^{-1}\right)^{\vee} \\
& =\operatorname{Ad}_{g_{l k}}\left(-\widehat{\xi}_{k} g_{k j} \widehat{\xi}_{j} g_{k j}^{-1}+g_{k j} \widehat{\xi}_{j} g_{k j}^{-1} \widehat{\xi}_{k}\right)^{\vee} \\
& =A_{l k}\left[A_{k j} \xi_{j}, \xi_{k}\right] .
\end{aligned}
$$

- Page 193, line -1. (S. Laubach, $5 / 9 / 94)$ The $N(\theta, \dot{\theta})$ term (gravity and friction) is missing from equation (4.52). Note that the gravity portion of this term must be cancelled (or absent) to achieve asymptotic stability. The statement of the theorem will probably change in the next edition of the book to indicate this explicitly.
- Page 195, line 15. (J. Wendlandt, 10/18/94) The term $\frac{1}{2} \dot{M}$ in the last line of the multiline equation should be $\dot{M}-C$.
- Page 198, line 12. (S. Misra, 4/30/05) The terms $C(\theta, \dot{\theta})$ and $N(\theta, \dot{\theta})$ should be $\tilde{C}(\theta, \dot{\theta})$ and $\tilde{N}(\theta, \dot{\theta})$.
- Page 207, line -15 . (W. Wray, 1/12/95) The phrase 'of the' is repeated.
- Page 231, line -11. (R. Mason, 3/10/95) "the these bounds" should be "these bounds".
- Page 237, line 16. (R. Mason, 3/13/95) The word "restatment" should be "restatement".
- Page 258, line 8. (RMM/SSS, 2/27/94) The following paper should be cited as one of the references on kinematics of rolling contact:
A. Cole, J. Hauser, S. S. Sastry. Kinematics and Control of Multifingered Hands with Rolling Contact, IEEE Transactions on Automatic Control, 34(4):398-404, 1989.
- Page 269, line -1. (J. C. K. Chan, 3/27/97) The term $A M^{-1} A$ should be $A M^{-1} A^{T}$.
- Page 296, line -6. (W. K. Fung, 12/29/94) The coupling matrix has an extra factor of 2 in the $(1,1)$ entry and the second row is incorrect. The correct coupling matrix is:

$$
P(\theta)={\frac{\partial h^{T}}{\partial \theta}}^{T}=\left[\begin{array}{cccc}
-\sqrt{a^{2}+b^{2}} \sin \left(\tan ^{-1}\left(\frac{a}{b}\right)+\frac{\theta_{1}}{2}\right) & -R_{1} & R_{1} & R_{1} \\
-R_{2} & 0 & 0 & R_{2}
\end{array}\right]
$$

- Page 327, line 11. (W. Wray, 1/16/95) $T_{q}^{*} \mathbb{R}$ should be $T_{q}^{*} \mathbb{R}^{n}$.
- Page 345, line -7. (W. Wray, 1/16/95) Extra bracket in the $\left[g_{2}, g_{3}\right]$ term in displayed equation.
- Page 382, line 4. (W. Goodwine, 8/21/96) Change "paper of Lafferriere and Sussmann in [62]" to "paper of Lafferriere and Sussmann [55]".
- Page 401, line -7. (R. Mason, 3/13/95) The word "oragami" should be "origami".
- Page 406, line 4. (R. Mason, 10/28/94) $T M$ should be $T^{*} M$.
- Page 432, line -14. (W. Wray, 1/12/95) The word 'seqment' should be 'segment'.
- Page 442, line 14 (R. Mason, 6 Nov 95). "Managment" should be "Management".
- Page 443, line -18. The order of the authors for reference [43] should be Venkataraman and Iberall.


## Changes to first printing (Mar 1994)

The following corrections apply to the first printing only; all of these changes have been implemented in the second printing of the book (Dec 1994).

- Front cover. (RMM/SSS, 2/94) The figure on the front cover is courtesy of John Hauser at the University of Colorado, Boulder.
- Title page. (RMM, 3/18/94) In some versions of the book the middle initial of the first author is incorrect on the title page. The correct middle initial for R. Murray is ' M ' (as in Martin). The initial appears correctly on the cover and the verso.
- Page xviii, line 24. (M. Spong, 4/13/94) "greatful" should be "grateful".
- Page 44, figure. (S. Laubach, 5/11/94) The orientation of the $B$ frame in Figure 2.6 is misleading. The axes of the coordinate frame should be aligned with those of $A$ when $\alpha=0$.
- Page 45 , line -11. (S. Laubach, 5/9/94) "screws motions" shoudl be "screw motions".
- Page 81, line -3. (S. Laubach, 5/9/94) "joint" is repeated.
- Page 128, line -4. (S. Laubach, 5/9/94) "producted" should be "produced".
- Page 137, line 8. (M. Milam, 2/4/94) The displayed equation has a number of typos. The correct equation should be

$$
\begin{aligned}
-r-l_{1} \sin \theta_{11}+r \cos \left(\theta_{11}+\theta_{12}\right) & =x=r-l_{2} \sin \theta_{21}-r \cos \left(\theta_{21}+\theta_{22}\right) \\
l_{1} \cos \theta_{11}+r \sin \left(\theta_{11}+\theta_{12}\right) & =y=h+l_{2} \cos \theta_{21}-r \sin \left(\theta_{21}+\theta_{22}\right) \\
\theta_{11}+\theta_{12} & =\phi=\theta_{21}+\theta_{22}
\end{aligned}
$$

- Page 137, line -10. (RMM, 2/6/94) The $(2,2)$ term in the matrix on the right hand side of the equals sign should be $-r+l_{2} \sin \theta_{21}$.
- Page 159, line 2. (L. Crawford, 7/26/94) The angle $\psi$ should be $\phi$ in the caption.
- Page 160, line -10. (H. Choi, 4/9/94) There are errors in the $\sin ^{2} \theta$ terms in $\frac{d}{d t} \frac{\partial L}{\partial \dot{\phi}}$. The correct expression should read:

$$
\frac{d}{d t} \frac{\partial L}{\partial \dot{\phi}}=\frac{d}{d t}\left(m l^{2} \sin ^{2} \theta \dot{\phi}\right)=m l^{2} \sin ^{2} \theta \ddot{\phi}+2 m l^{2} \sin \theta \cos \theta \dot{\theta} \dot{\phi}
$$

- Page 160, line -7. (H. Choi, 4/9/94) The $\sin \theta$ term in the inertia matrix should be $\sin ^{2} \theta$ (this is a propogation of an error from the previous displayed equation).
- Page 165, line 6. (R. Behnken, $4 / 27 / 94) \beta$ should be $m_{2} l_{1} r_{2}$.
- Page 165, line 10. (R. Behnken, 4/27/94) Equation (4.11) should read

$$
\left[\begin{array}{cc}
\alpha+2 \beta c_{2} & \delta+\beta c_{2} \\
\delta+\beta c_{2} & \delta
\end{array}\right]\left[\begin{array}{l}
\ddot{\theta}_{1} \\
\ddot{\theta}_{2}
\end{array}\right]+\left[\begin{array}{cc}
-\beta s_{2} \dot{\theta}_{2} & -\beta s_{2}\left(\dot{\theta}_{1}+\dot{\theta}_{2}\right) \\
\beta s_{2} \dot{\theta}_{1} & 0
\end{array}\right]\left[\begin{array}{l}
\dot{\theta}_{1} \\
\dot{\theta}_{2}
\end{array}\right]=\left[\begin{array}{l}
\tau_{1} \\
\tau_{2}
\end{array}\right] .
$$

( $\beta$ should be replaced with $2 \beta$ ).

- Page 171, line -3. (S. Laubach, 5/9/94)"amount" should be "among"
- Page 175, line 12. (S. Laubach, 5/9/94) $S O(3)$ should be $s o(3)$.
- Page 175, line -1. (S. Laubach, 5/9/94) $S E(3)$ should be $s e(3)$.
- Page 187, line -8. (R. Behnken, 4/27/94) In the vector form of $\dot{V}$, a transpose is missing from the leftmost vector and the bottom right entry of the matrix should be $B-\epsilon M$.
- Page 194, line 17. (S. Laubach, 5/9/94) "invarient" should be "invariant".
- Page 195, line 16. (F. Bullo, 10/13/94) "negative semidefinite" should be "negative definite".
- Page 219, line 1. (R. Fearing, 3/28/94) The table is missing the pictures. The corrected table should look like Table 5.2, reproduced at the end of this document.
- Page 222, line -8. (S. Sur, 4/15/94) In the expressions for $p_{c_{1}}$ and $p_{c_{2}}$, the constant $a$ should be $r$ (the radius of the box, as shown in Figure 5.6).
- Page 222, line -1. (S. Sur, 4/15/94) In the expression for $G$, the constant $a$ should be $r$.
- Page 236, line -1. (S. Sur, 4/15/94) The adjoint in the bottom right hand entry of the hand Jacobian matrix should be $\mathrm{Ad}_{s_{k} c_{k}}^{-1}$.
- Page 240, line -1. (M. Milam, 2/20/94) The displayed equation is not correct. The correct equation is

$$
J_{s_{i} f_{i}}^{s}=\left[\begin{array}{cccc}
0 & l_{1} \cos \theta_{1} & l_{1} \cos \theta_{1}+l_{2} \cos \left(\theta_{1}+\theta_{2}\right) & 0 \\
0 & l_{1} \sin \theta_{1} & l_{1} \sin \theta_{1}+l_{2} \sin \left(\theta_{1}+\theta_{2}\right) & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0
\end{array}\right] .
$$

- Page 272, line -9. (S. Laubach, 5/13/94) In equation (6.9), the second $\dot{q}_{1}$ should be $\dot{q}_{2}$.
- Page 300, line -2. (S. Laubach, 5/9/95) "input" is repeated.
- Page 310, line 7. (S. Laubach, $5 / 9 / 95) G^{T} \dot{x}_{f}$ should be $G^{+T} \dot{x}_{f}$ where $G^{+T}$ is the pseudoinverse of $G^{T}$. Not that although $G^{T}$ is not necessarily surjective, $G^{+T} \dot{x}_{f}$ is still the right quantity to use for $\dot{x}_{b}$ because the $\dot{x}_{f}=J \dot{\theta}$ satisfies the grasp constraint and hence lies in the range of $G^{T}$.
- Page 324, line -2. (L. Crawford, 7/26/94) The displayed equation should read $[f, g](q)=$ $(B A-A B) q$.
- Page 334, figure. (L. Crawford, $7 / 26 / 94$ ) The labeling in the figure is incorrect. The angles $\phi_{1}$ and $\phi_{2}$ should be $\psi_{1}$ and $\psi_{2}$ to match the text.
- Page 334-335, example. (L. Crawford, 7/26/94) The symbol $r$ is used both for the radius of the central body and the as the position of the center of mass. The position of the center of mass should be labelled $p$ to avoid confusion.
- Page 335, line 18. (L. Crawford, $7 / 26 / 94$ ) The minus sign in front of the $\frac{\partial L}{\partial \theta}$ term is incorrect.
- Page 383, line -1. (SSS, 6/7/94) $\dot{q}_{4}$ should be $\dot{q}_{5}$.
- Page 408, line 18. (N. Getz, $5 / 11 / 94$ ) In the displayed equation, $Y_{p}$ should be $X_{p}$.
- Page 412, line 6. (S. Sur, 2/3/94) "both sides equal" should be "both sides are equal".
- Page 414, line 9. (F. Bullo, 10/13/94) $A^{-1} v$ should be $A^{-1} p$ is displayed equation.

Table 5.2: Common contact types.

| Contact type | Picture | Wrench basis | FC |
| :--- | :--- | :--- | :--- |
| Frictionless <br> point contact |  |  |  |

