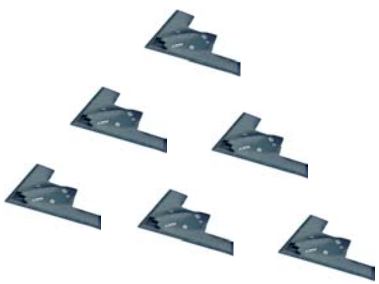
Model Predictive Control :

Extension to coordinated multi-vehicle formations & Real-time implementation



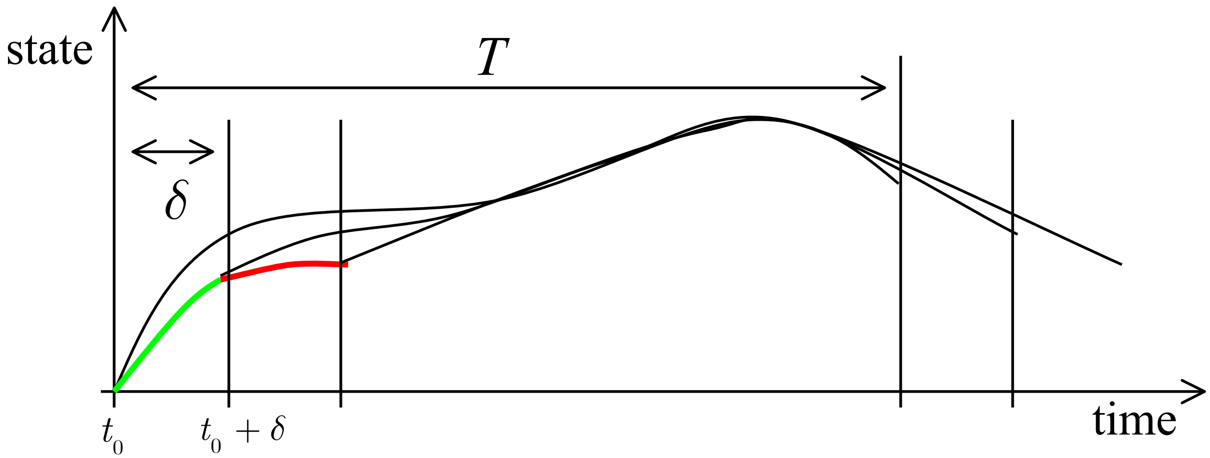
UCAV Swarm

William Dunbar
Control and Dynamical Systems
Candidacy Defense Exam- December 18, 2001

Outline

I. Overview of MPC
II. Multi-vehicle Formations and Extension of MPC
III. Multi-vehicle MPC Simulations
IV. Experimental Results (Caltech dfan) – Trajectory Generation & MPC
V. Conclusions and Future Directions

I. Overview: Model Predictive Control



$$u^*_{[t_0, t_0+\delta]} = \arg\min \int_{t_0}^{t_0+T} q(x(\tau), u(\tau))\, d\tau + V(x(t_0+T))$$

$$\text{s.t.} \quad \dot{x} = f(x,u), \quad x_0 = x(t_0)$$

$$(x,u) \in X \times U, \quad x(t_0+T) \in \mathrm{X}_f$$

System:	$f(x,u)$
Constraints:	X, U, X_f
Mission:	$q(x,u)$
Final Penalty:	$V(x)$

- Solve optimization from $x(t_0)$, implement u^* until new state update $x(t_0+\delta)$ is received
- Repetitive trajectory optimization: gives state feedback control law
- Good performance, where on-line computation is *feasible* ……Stability →

I. Overview: Stability of MPC

Given: $$J_T^0(x_0) = \min_{u(\cdot)} \int_0^T q(x(\tau), u(\tau))d\tau + V(x(T))$$

with terminal constraint set X_f and stabilizing controller $\kappa_f(\cdot)$

Stability Approach: Employ $J_T^0(\cdot)$ as a *Lyapunov function*

with requirements on $V(\cdot),\ X_f,\ \kappa_f(\cdot)$ such that

$$\left[\dot{J}_T^0 + L\right](x_0, \kappa_T(x_0)) \leq 0, \qquad \forall x_0 \in X_T$$

Results to Date:

- terminal constraint set only (dual-mode control)
- CLF as terminal cost (absence of constraints)

Open Issues:

- robustness, disturbance rejection
- receding horizon trajectory tracking
- proofs for *SE(2,3)* with constraints

II. A Multi-vehicle Formation (MVF) Problem

Consider k vehicles on $X = T\left(SE(2)\right)$ or $\subseteq \mathbb{R}^4$

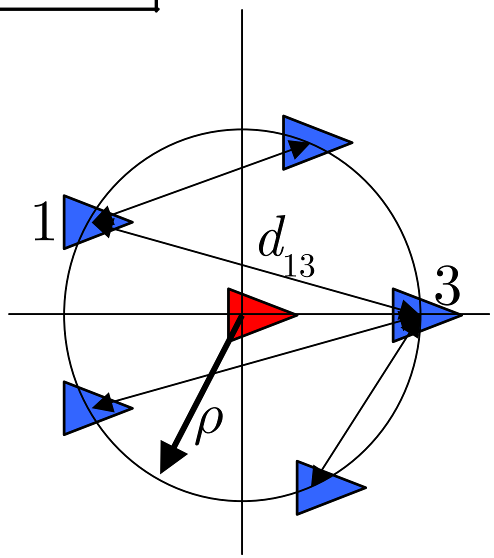
$$\dot{x}_i = f_i(x_i, u_i), \;\; x_i(t_0) = x_{i0}, \;\; t \in [t_0, \infty),$$
$$f_i : X \times U \to F, \;\; \forall i = 1, \ldots, k$$

<u>Given:</u> an invariant set M and a formation reference x_{ref}, a k-vehicle formation is defined as

$$F(M, k, x_{ref}) = \left\{ \vec{x} \in X^k \mid (\vec{x} - \vec{x}_{ref}) \in M \right\}$$

An example invariant set M is

$$M = \Big\{ \vec{x} \in X^k \mid \|z_i\| = \rho_i, \;\; \|z_i - z_p\| = d_{ip}, \\ z_i \neq z_j, \;\; \dot{z}_i = 0, \; \forall i, j = 1, \ldots, k, \\ i \neq j \text{ and some } p \in \{1, \ldots, k\} \Big\}.$$



II. Extension of MPC to MVF

Theorem based on definition of stability of invariant sets and Lasalle's theorem provide extension of MPC stability result to MVF's where we want stability to a set *M*.

Idea: Lyapunov type function *V* that is (locally) positive everywhere except on an invariant set AND the desired formation is this (smallest) invariant set where d/dt $V = 0$.

(collision problem not addressed)

Here come the movies…

III. Multi-Vehicle Optimization-Based Control

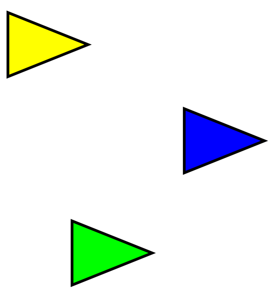
Assume we have real-time, finite horizon optimal control as a *primitive*

$$\left.\begin{array}{c} u_{[t,t+\Delta T]} = \arg\min \int_t^{t+T} L(x,u)dt + V(x(t+T)) \\ x_0 = x(t) \quad x_f = x_d(t+T) \\ \dot{x} = f(x,u) \quad g(x,u) \le 0 \end{array}\right\}$$

Choose f, g, L to represent the *coupling* between the various subsystems

Cooperation depends on how we model "rest of the world"

$$\ddot{q}_1 = 0 \quad |u_1| \le L$$

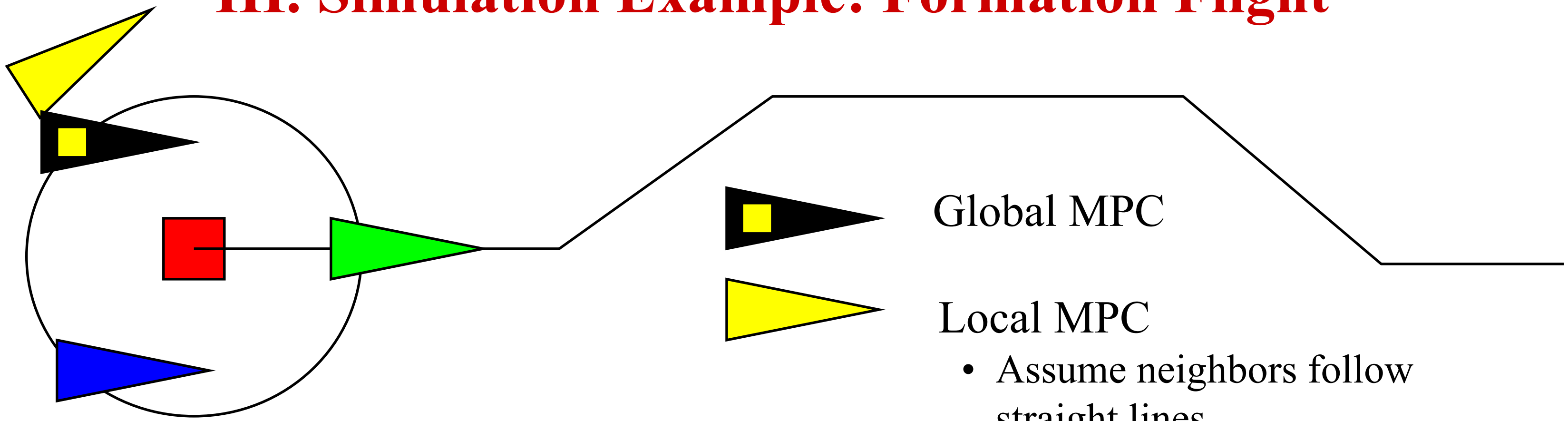


$$u_{2,[t,t+\Delta T]} = \arg\min \int_t^{t+T} L(q_{1,}x_2, q_3, u_2)dt + V$$
$$\dot{x}_2 = f(x_2,u_2) \quad g(x_2,u_2) \le 0$$

$$\ddot{q}_3 = 0 \quad |u_3| \le L$$

Reconfigurable based on condition, mission, environment

III. Simulation Example: Formation Flight

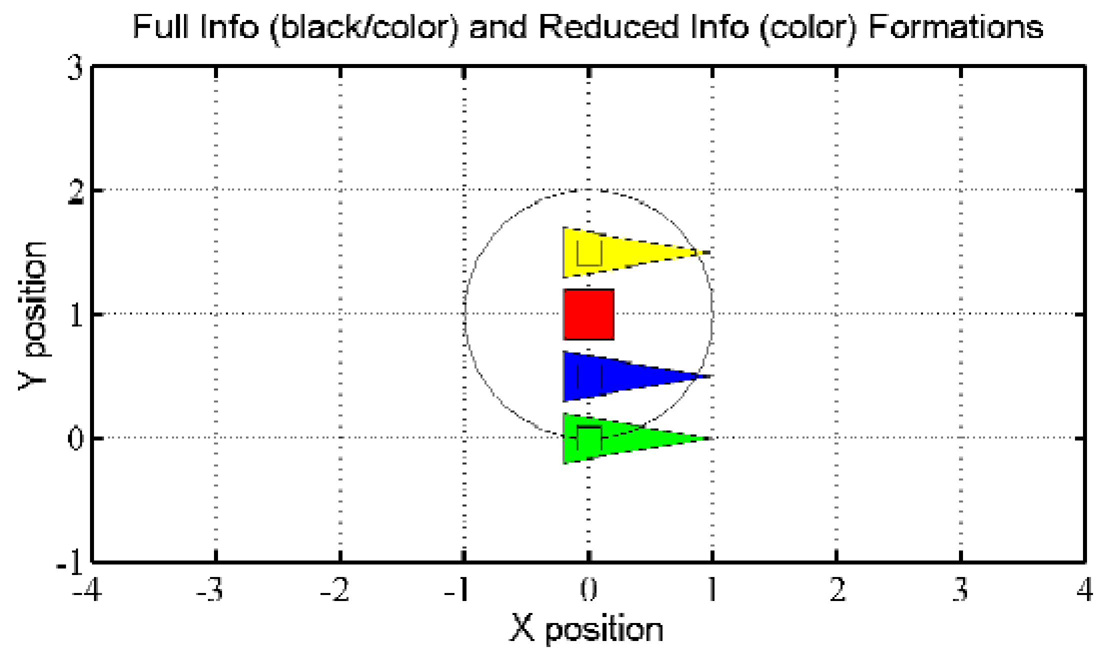


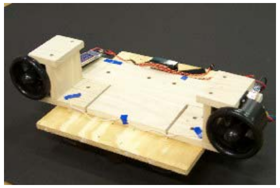
Task:

- Maintain equal spacing of vehicles around circle
- Follow desired trajectory for center of mass

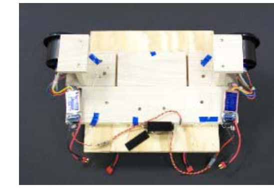
Parameters:

- Horizon: 2 sec
- Update: 0.5 sec
- **High damping**



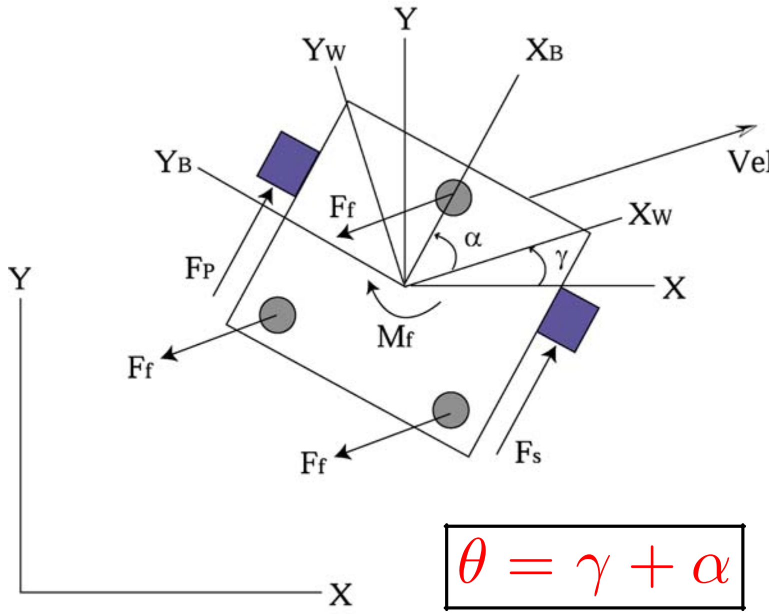


III. Wilson Vehicle on *SE(2)*



Equations of motion, input constraints and formation set:

$$m\ddot{x} = -\eta\dot{x} + \left(F_S + F_P\right)\cos\theta$$
$$m\ddot{y} = -\eta\dot{y} + \left(F_S + F_P\right)\sin\theta$$
$$J\ddot{\theta} = -\psi\dot{\theta} + \left(F_S - F_P\right)r$$



$$\theta = \gamma + \alpha$$

$$0 \le F_S, F_P \le 6N$$

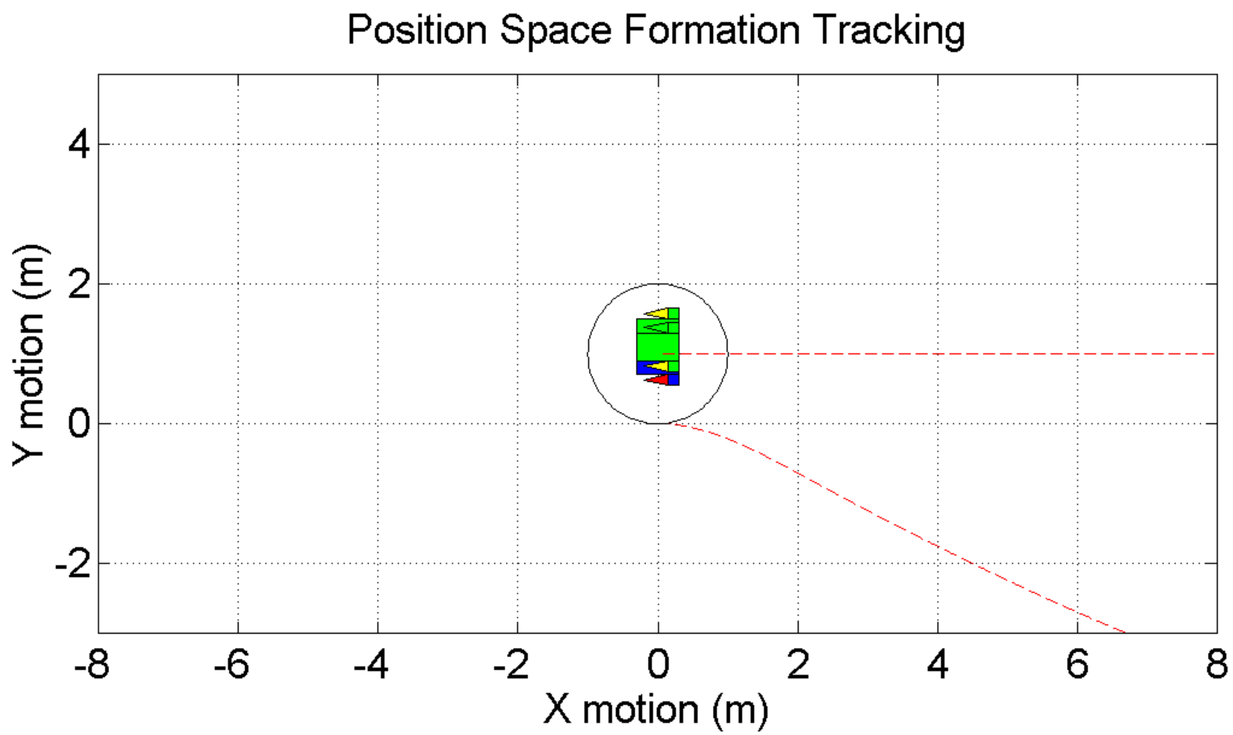
$$M = \left\{ \hat{x} = (\vec{x} - \vec{x}_{ref}) \in X^3 \mid \left\|(\hat{x}_i, \hat{y}_i)\right\| = 1,\ \left\|(\hat{x}_i, \hat{y}_i) - (\hat{x}_j, \hat{y}_j)\right\| = \sqrt{3}/2,\ (\hat{x}_i, \hat{y}_i) \ne (\hat{x}_j, \hat{y}_j),\ \dot{\hat{x}}_i = \dot{\hat{y}}_i = 0,\ i, j = 1,2,3,\ i \ne j \right\}.$$

III. One Wilson Vehicle Formation Simulations

- The Blue moves according to a *nontrivial* formation reference

- The Green is tracking a circle with center that moves according to Blue reference by MPC: $(T,\delta) = (8.0,1.0)$

- Initial coefficients are set to 1.0 for first optimization (warm-start afterwards)

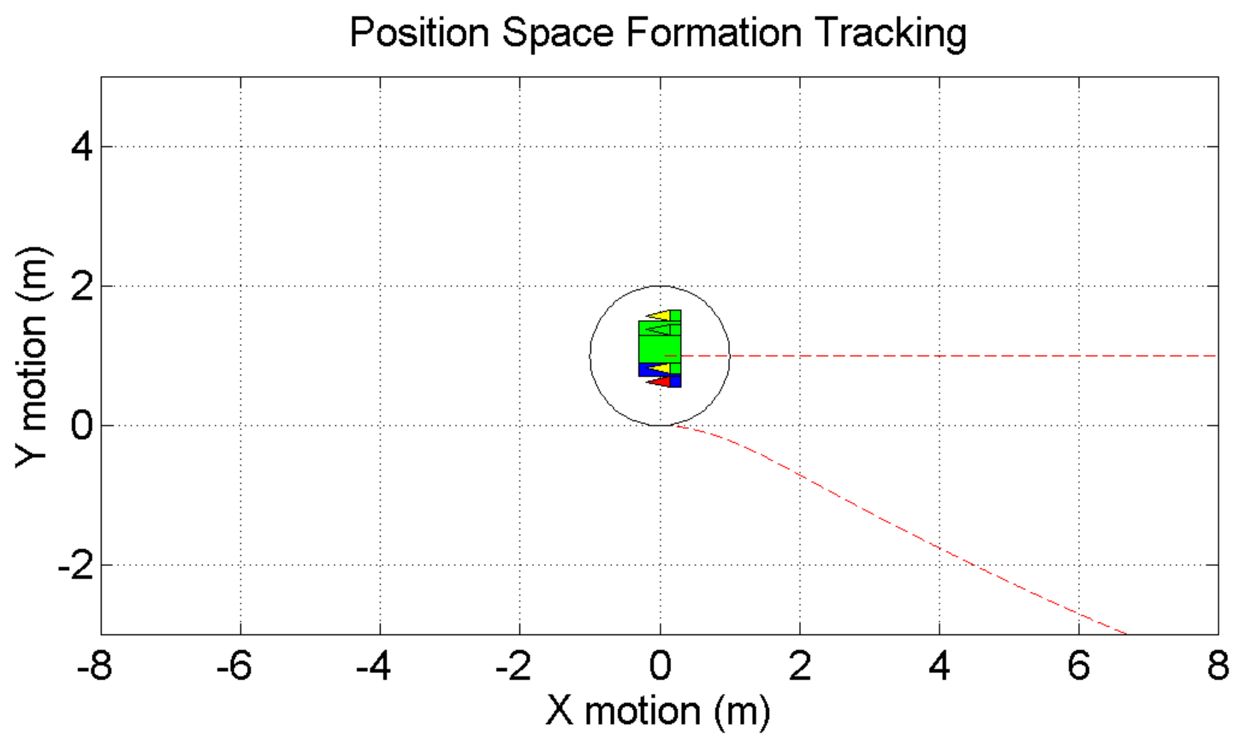


III. One Wilson Vehicle Formation, cont'd

- The Blue moves according to the same formation reference

- The Green initial coefficients are evenly spaced over (-0.5,1.0).

•<u>Punch line:</u>
With only integrated costs, MPC stability not insured!

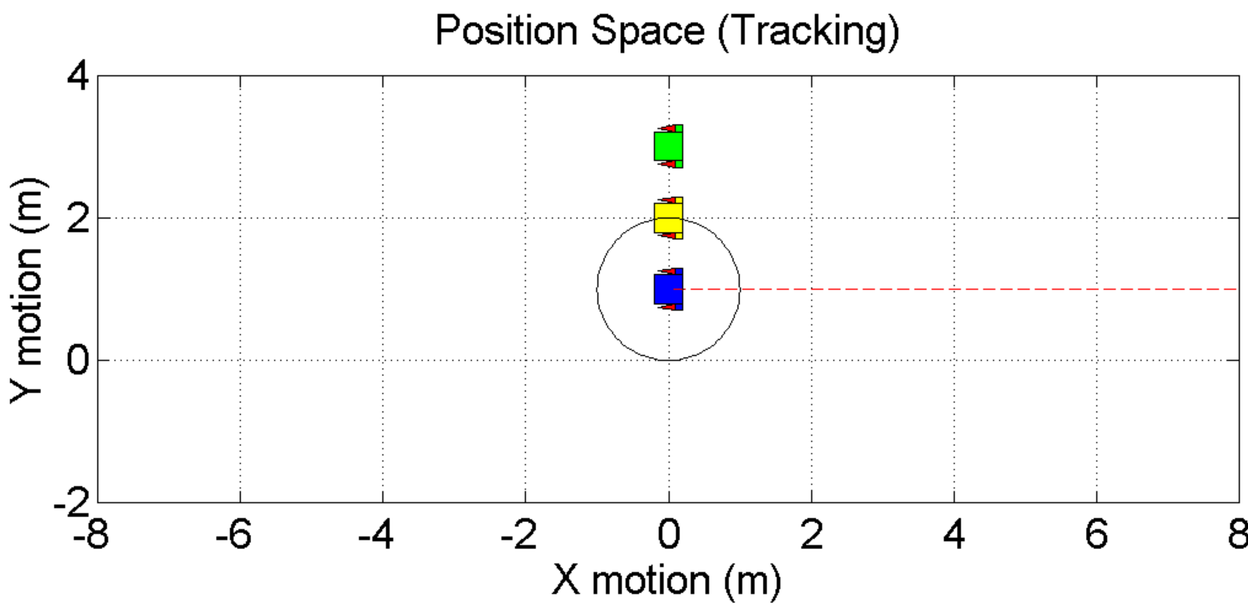


III. Three Wilson Formation Simulations

- Formation *M,* Black moves as form reference on straight line with constant velocity

- Global MPC with (T,δ) = (5.0,1.0)

- **Only integrated cost** → there is no guarantee of stability



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- Same formation *M* - black moves as reference on straight line with constant velocity

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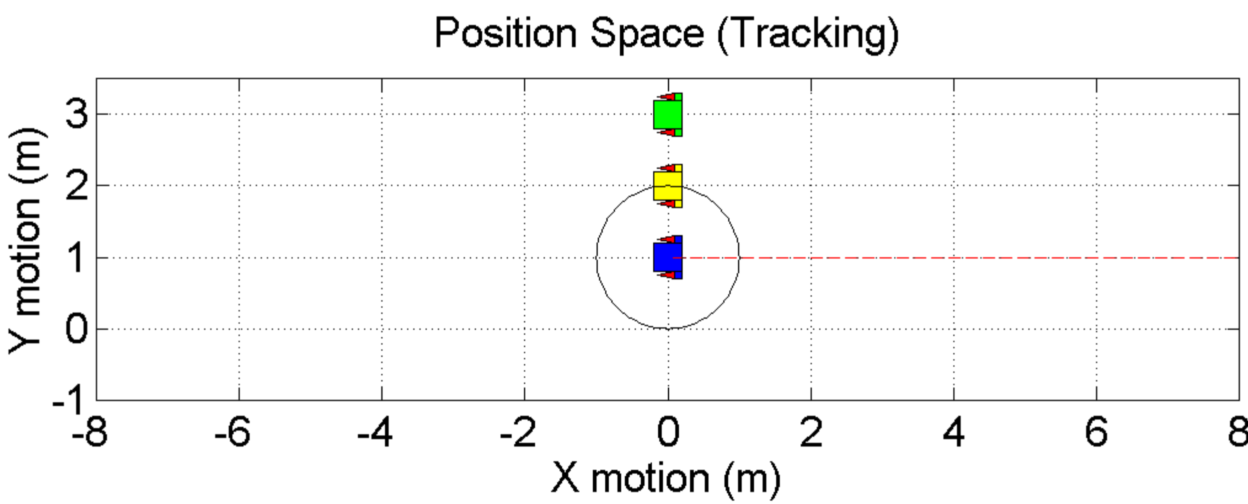
- **Add terminal cost** (scheduled quadratic TC for corresponding LQR linearization)



Stability Recovered!

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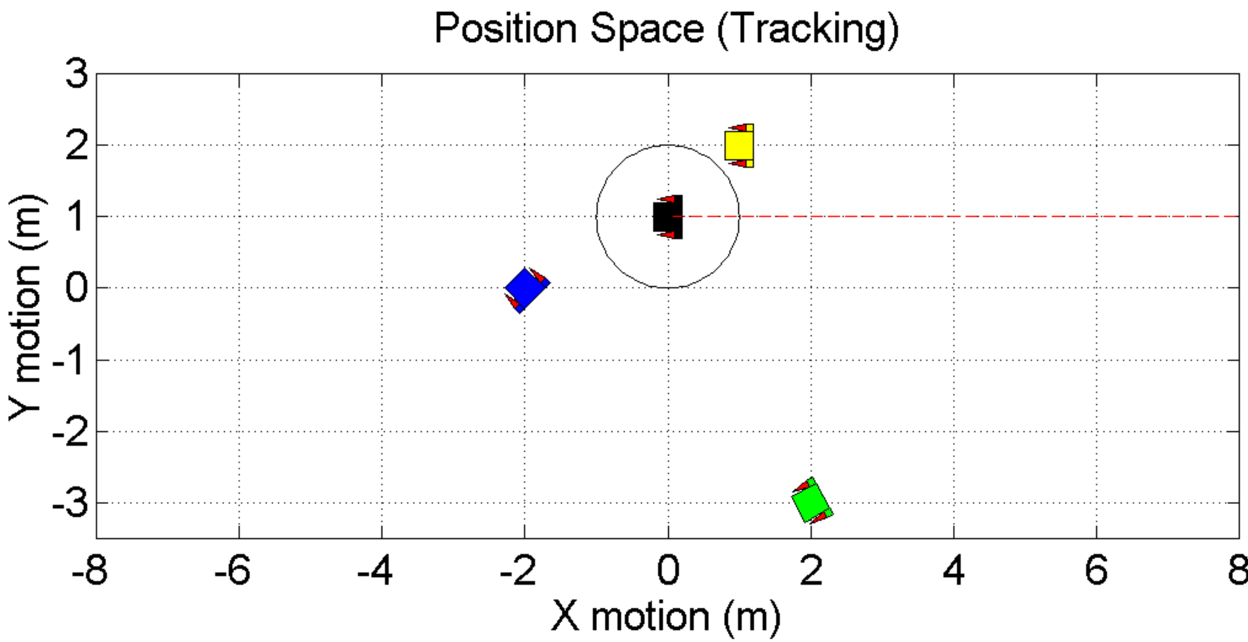

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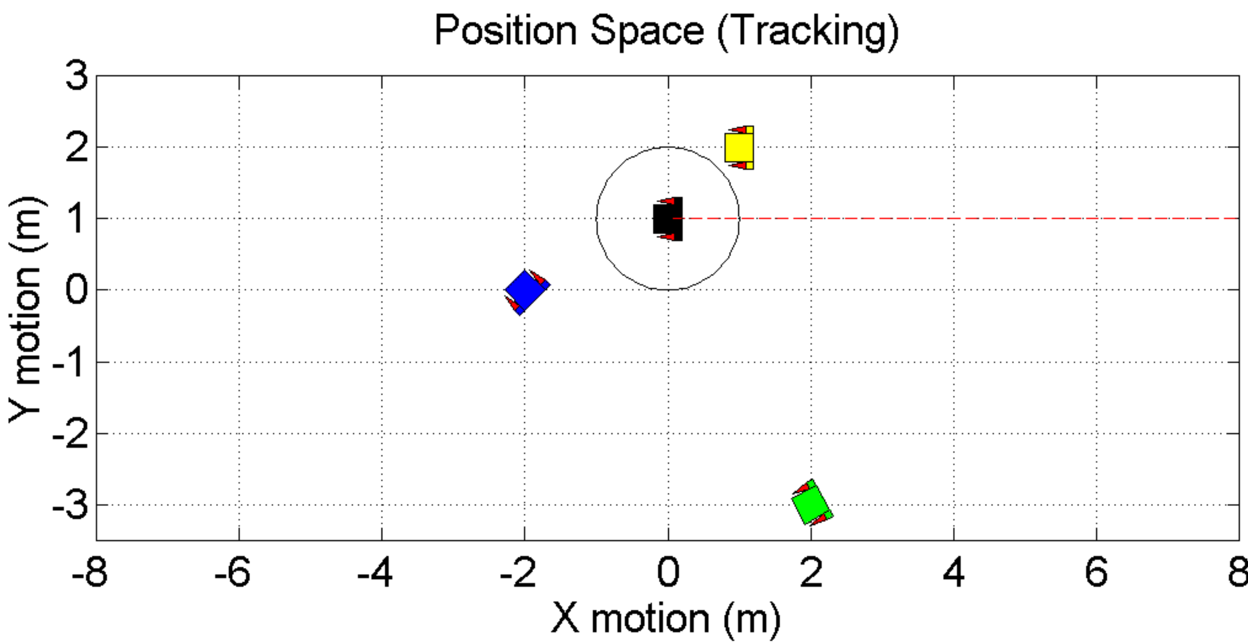
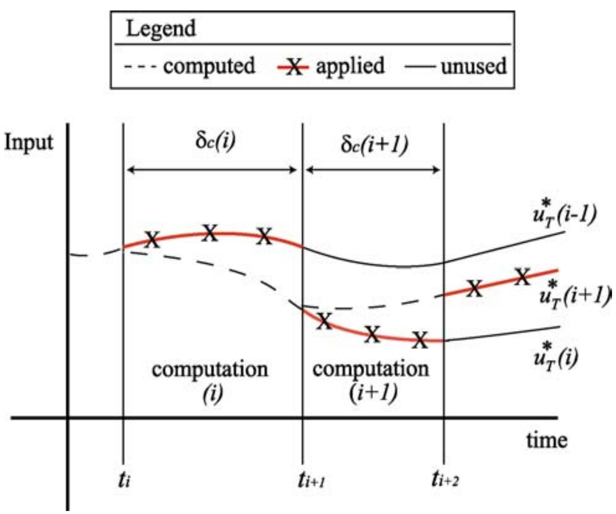


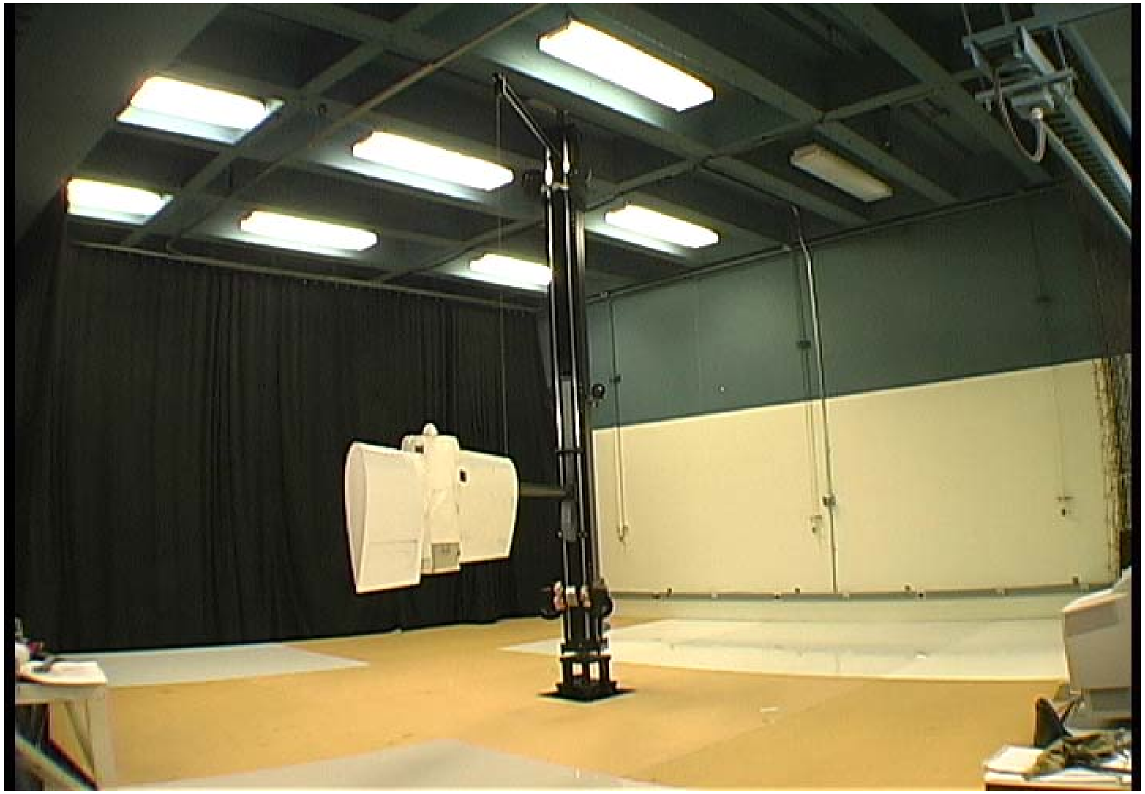
Stability Recovered!

IV. Real-time MPC of ducted fan

MPC problem
- LQR of planar model around hover with quadratic (Riccati) terminal cost
- Ramp input of 16 meters in horizontal, Step input of 1m in altitude

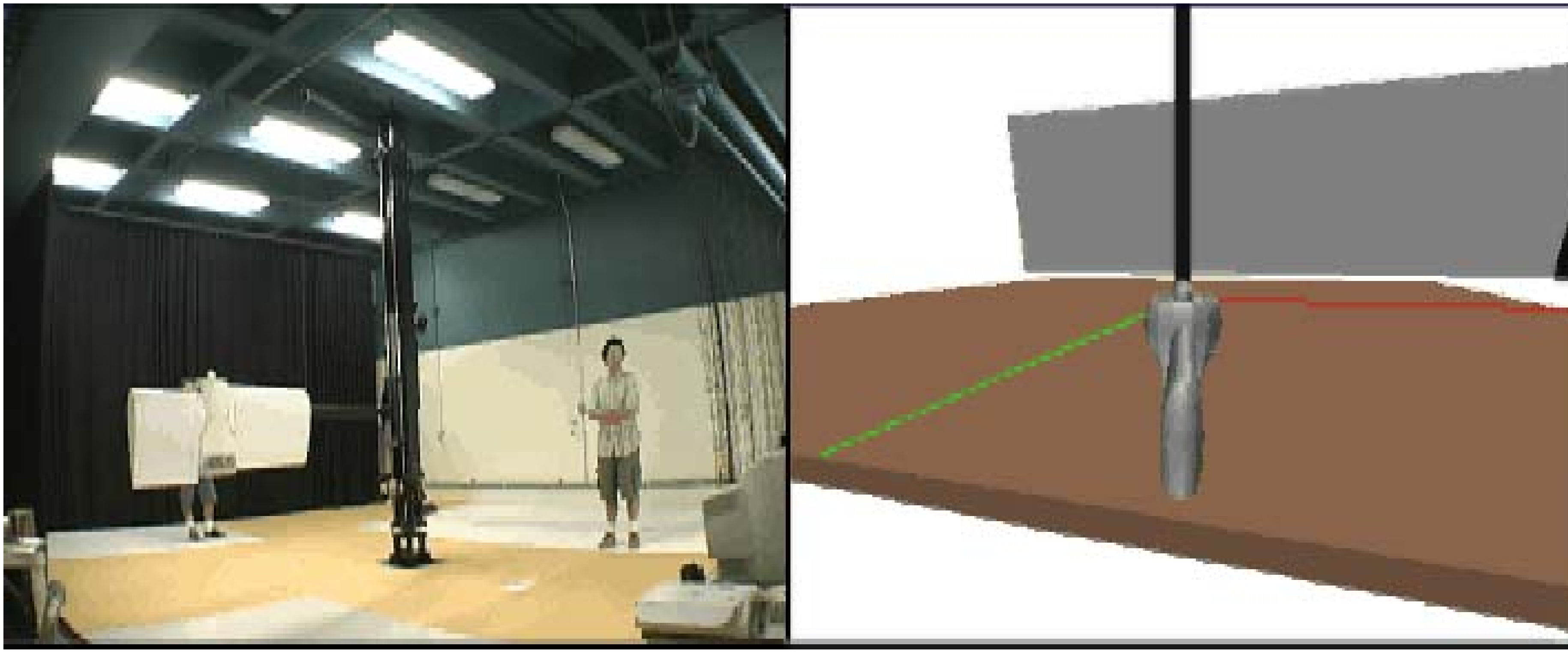
Timing set-up – update *asap*



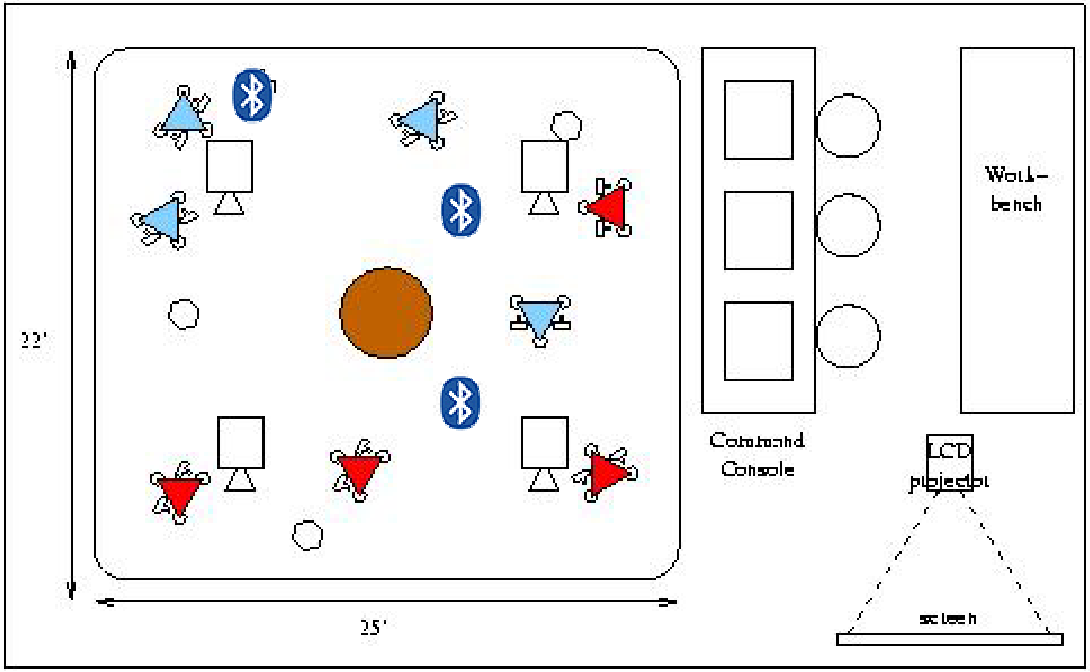


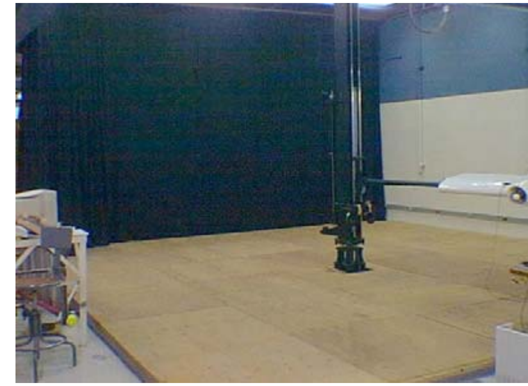
IV. Trajectory Generation for ducted fan

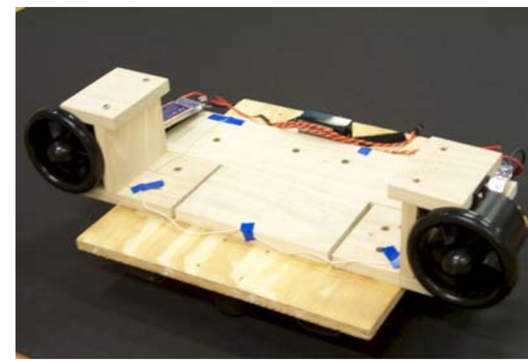
Adaptation to new constraints – terrain avoidance

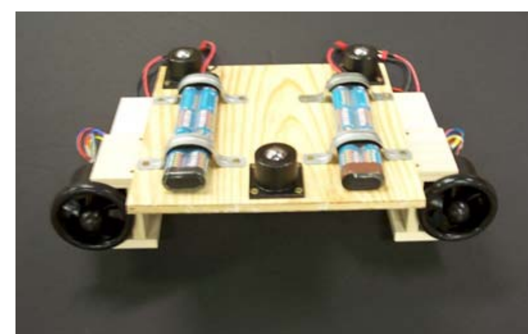


Multi-Vehicle Wireless Testbed for Integrated Control, Communications and Computation









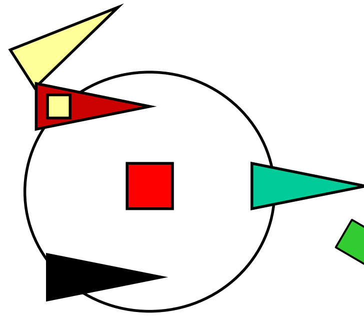
Testbed features

- Distributed computation on individual vehicles + command and control console
- Point to point, ad-hoc networking (bluetooth) + local area networking (802.11)
- Cooperative control in dynamic, uncertain, and adversarial environments

IV. Experimental Extensions

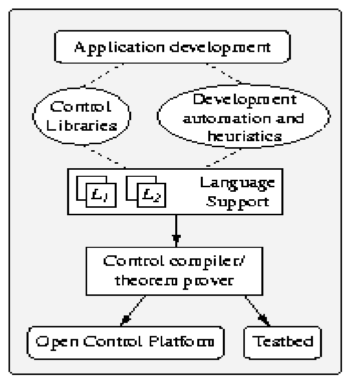
Optimization-Based Control

- Real-time model predictive control for online control customization: theory and software
- Online implementation on Caltech Ducted Fan



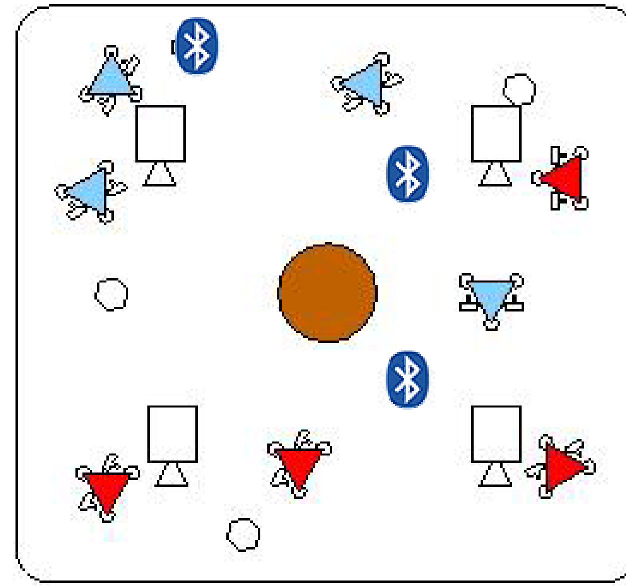
Cooperative Control

- Linked cost functions



Software Environments

- Logical programming environments for embedded control systems design



Multi-Vehicle Testbed

- Implementation on multi-vehicle, wireless testbed using Open Control Platform
- Bluetooth-based point to point communications with ad-hoc networking