MPC-Lectures → SEC Chapter
On-line Control Customization via Optimization Based Control.

2 weeks

1) §1–4 (Intro., Preliml. MPC theory, NTG)

2) §5 & Multi-vehicle MPC examples/simulation (ppl)

§1 Intro

A − T

e.g. traj

Problem: UAV, auton. vehicles, robotic applic.

- Real-time trajectory generation
- Stabilize to traj. ~~[illegible]~~

} in presence of (a) noise, uncertainty

(b) terrain avoidance

(c) reconfiguration

Alt. Approaches

- storage of trajectories (can't handle dyn. changes in constraints (Real-time))
- trad. Optim. Control Real-time
  - shooting (RIOTS) • problems: (numerical instability)
  - Collaction • (too slow)

Motivation for Optim. Based Approach On-line

$\min_u L(u)$ vs. $\min L(u)$ s.t. $f(u) \le 0$

Allow for customization of controller based on changes in

- mission (costfunction, constraints, ...)
- condition (model, faults)
- environment (terrain avoidance)

Contribution: Comp: NTG software pckge. (uses collocation & differential flatness)

Theory: MPC w/ CLF terminal cost (guarantee stability, improve comp. feasibility compared to terminal constraint schemes)

Applic: RT experim. implem.

§7. Prelimin. §7.1 Use of differential flatness for trajectory generation

Def'n system $\dot{x} = f(x,u)$ (eq (1)) is diff. flat if $\exists$ a function

$z(x, u, \dot{u}, \ldots, u^{(p)})$ such that all feasible solution of (1) can be written as

$$x = \alpha(z, \dot{z}, \ldots, z^{(r)}) \qquad z \in \mathbb{R}^m \qquad m < n+s$$
$$u = \beta(z, \dot{z}, \ldots, z^{(q)}) \qquad u \in \mathbb{R}^s, \; x \in \mathbb{R}^n$$

E.g. pdfan say want $\int_0^T (x^T Q x + u^T R u)\,dt + x^T P x\Big|_{t=T}$

$x^T = [x, y, \theta, \dot{x}, \dot{y}, \dot{\theta}]$ s.t. $\dot{x} = f(x,u)$ (eq. 3)

$u^T = [f_1, f_2]$ $-5 \le f_1 \le 5, \quad 0 \le f_2 \le 10$

$(4), (5) \longrightarrow \quad \theta = \tan^{-1}\left(\frac{-\ddot{z}_1}{\ddot{z}_2 + g}\right) = g_0(\underset{\sim}{z})$

$\underset{\sim}{z} = [z_1, z_2, \dot{z}_1, \dot{z}_2, \ldots]$

$(4) \longrightarrow \quad x = z_1 + \frac{J}{mr}\sin(g_0(\underset{\sim}{z})) \triangleq g_1(\underset{\sim}{z}), \quad y = z_2 - \frac{J}{mr}\cos(g_0(\underset{\sim}{z})) \triangleq g_2(\underset{\sim}{z})$

$(2) \longrightarrow \quad f_1 = \frac{J\ddot{\theta}}{r} = h_1(\underset{\sim}{z}), \quad f_2 = h_2(\underset{\sim}{z})$

$\underset{\sim}{g}(\underset{\sim}{z})^T \triangleq [g_0(\underset{\sim}{z}) \ldots \dot{g}_2(\underset{\sim}{z})]$

$\underset{\sim}{h}(\underset{\sim}{z})^T \triangleq [h_1(\underset{\sim}{z}) \; h_2(\underset{\sim}{z})]$

<u>FHOCP:</u> $\displaystyle\int_0^T \left(\underset{\sim}{g}(\underset{\sim}{z})^T Q\, \underset{\sim}{g}(\underset{\sim}{z}) + \underset{\sim}{h}(\underset{\sim}{z})^T R\, \underset{\sim}{h}(\underset{\sim}{z})\right) dt + \underset{\sim}{g}(\underset{\sim}{z})^T P\, \underset{\sim}{g}(\underset{\sim}{z})\Big|_{t=T}$

s.t. $-5 \le h_1(\underset{\sim}{z}) \le 5$

$0 \le h_2(\underset{\sim}{z}) \le 10$

$z_1^T Q z_1$

$\star$ Can ~~solve~~ use <u>Algebraic</u> <u>methods</u> b/c problem no longer dynamic!!

$\star$ Parameterize flat outputs

$z = \sum a_i \phi_i(t) = \underset{\sim}{a}^T \underset{\sim}{\phi}(t)$

coefficients

Basis functions

§3. $\underbrace{\text{MPC}}$ + CLF terminal cost (Yo)

solve FHOCP $[0,T]$ sec., apply for $0 < \delta \ll T$, resolve.

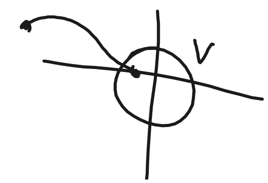
Pf. for nominal stability (assume no model uncertainty or noise).

Seen that $J_\infty^*(x) > 0$

$(\dot{J}_\infty^* + q)(x,u) \le 0$

$\Gamma_\infty := \{x \in \mathbb{R}^n \mid J_\infty^*(x) < \infty\}$

Find tc s.t. $x(T) \rightsquigarrow$ origin



$J_T^*(x(\delta)) \le J_T^*(x(0)) - \int_0^\delta q\,d\tau$

props. $V(x) > 0$, $\min_u [\dot{V} + q](x,u) \le 0$

$x(T) \in \Omega_{r_v}$

§4 RT traj. gen. & Diff flatness (NTG)

min $J(x,u)$ s.t. $\begin{cases} \dot{x} = f(x) + g(x)u & (12) \\ lb \le c(x,u) \le ub \end{cases}$

$\mathbb{R}$

§4.1 Collocation

NLP problem

- discrete time interval $[t_1, t_2, \dots, t_N]$
- x, u approx. by piecewise polynomials
- collocation ensures satisfaction of (12) at interval midpoints.

solve (16) – (Finite Dim. Approx) for $\underline{x}$, $\underline{u}$

Better (Seywald) solve for u in terms of states

- elliminate u (via $u = g(x, \dot{x}, \dots)$) and solve only for x (states).
- use Finite diff for derivations

⟹ get improvement (substantial) over $\underbrace{\text{trad. collocation}}$

§4.2 use flatness (better still) → NTG
& collocation

idea: Keep going with algebraic reduction

Ideally, system (12) is differentially flat ⇒ eliminate all $\overbrace{\text{(dynamic) equality constraints}}^{\text{"differential constraints"}}$

Forget finite difference → Use B-spline basis functions
- (ease of enforcing continuity at collocation points)
- (ease of computing derivatives)

Get (19). User Supply:
a) Spline representation quantities for each "output" (fig. 3)
b) State problem in terms of outputs (costs, constraints)
c) provide derivatives of