Kinematic and Dynamic Models

ME/CS 132b
Advanced Robotics: Navigation and Perception
3/29/2011

Lecture Outline

Nomenclature

Kinematic Constraints and Models

Dynamic Constraints and Models

Applications

Current and Active Research

Nomenclature

State $\mathbf{x} = [\ \mathbf{r} \quad \mathbf{o} \quad \mathbf{v} \quad \omega \quad \ldots\ ]^T$

State Space $\mathbf{X}$ Configuration Space $\mathcal{C}$

Configuration $\mathbf{q}$

Action $\mathbf{u}$

Action Space $\mathbf{U}(\mathbf{x})$ (state-space) $\mathbf{U}(\mathbf{q})$ (configuration-space)

State/Configuration Transition Equation $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$ (state-space) $\dot{\mathbf{q}} = \mathbf{f}(\mathbf{q}, \mathbf{u})$ (configuration-space)

Trajectory $\mathbf{x}(t) = [\ \mathbf{x}(t_0) \quad \mathbf{x}(t_1) \quad \mathbf{x}(t_2) \quad \ldots\ ]^T$ (state-space)

Actions

$\mathbf{U}$ is the set of all possible actions over all states

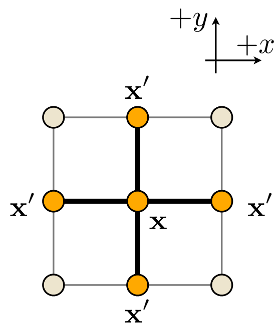
$$\mathbf{U} = \bigcup_{\mathbf{x} \epsilon \mathbf{X}} \mathbf{U}(\mathbf{x})$$

Example: Moving on a 2D grid

$$\mathbf{x} = \begin{bmatrix} x & y \end{bmatrix}^T$$

$$\mathbf{x}' = \mathbf{f}(\mathbf{x}, \mathbf{u}) = \mathbf{x} + \mathbf{u}$$

$$\mathbf{U} = \{(0,1), (0,-1), (1,0), (-1,0)\}$$



Actions (cont.)

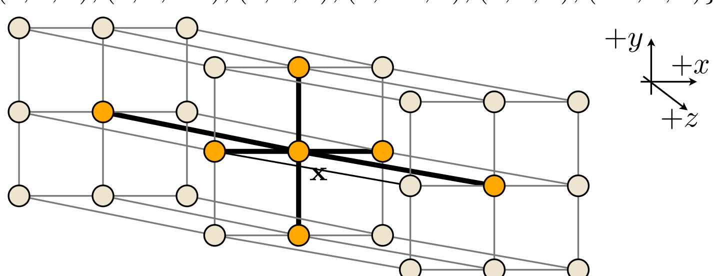
Example: Moving on a 3D grid

$$\mathbf{U} = \bigcup_{\mathbf{x} \epsilon \mathbf{X}} \mathbf{U}(\mathbf{x})$$

$$\mathbf{x} = \begin{bmatrix} x & y & z \end{bmatrix}^T$$

$$\mathbf{x}' = \mathbf{f}(\mathbf{x}, \mathbf{u}) = \mathbf{x} + \mathbf{u}$$ (same as before, just a higher dimensional state space)

$$\mathbf{U} = \{(0,0,1), (0,0,-1), (0,1,0), (0,-1,0), (1,0,0), (-1,0,0)\}$$



Actions (cont.)

In continuous spaces the next state is determined by a integrating the state transition equation

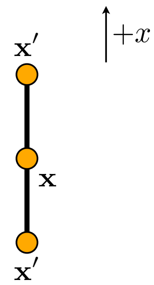
$$\mathbf{U} = \bigcup_{\mathbf{x} \epsilon \mathbf{X}} \mathbf{U}(\mathbf{x})$$

Example: One-dimensional particle

$$\mathbf{x} = [x]^T$$

$$\dot{\mathbf{x}} = v_x$$

$$\mathbf{U} = \{v_x\} = [-1, 1]$$



Kinematic Constraints

Implicit $\mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}) \bowtie 0$

Parametric $\dot{\mathbf{q}} = \mathbf{f}(\mathbf{q}, \mathbf{u})$

Pfaffian $g_1(\mathbf{q})\dot{q_1} + g_2(\mathbf{q})\dot{q_2} + \ldots + g_n(\mathbf{q})\dot{q_n} = 0$

Kinematic Constraints (cont.)

Pfaffian $g_1(\mathbf{q})\dot{q}_1 + g_2(\mathbf{q})\dot{q}_2 + \ldots + g_n(\mathbf{q})\dot{q}_n = 0$

Configuration transition equation

$$\begin{aligned} \dot{q}_1 &= u_1 \\ \dot{q}_2 &= u_2 \\ \dot{q}_{n-k} &= u_{n-k} \end{aligned} \qquad \begin{aligned} \dot{q}_{n-k+1} &= f_{n-k+1}(\mathbf{q}, \mathbf{u}) \\ \dot{q}_{n-k+2} &= f_{n-k+2}(\mathbf{q}, \mathbf{u}) \\ \dot{q}_n &= f_n(\mathbf{q}, \mathbf{u}) \end{aligned}$$

(these are determined by solving for the remaining variables)

Kinematic Constraints (cont.)

A quick example...

$\mathcal{C} = \mathbb{R}^3$ (three dimensional state space, n=3)

$7\dot{q}_1 - 3\dot{q}_2 + \dot{q}_3 = 0$ (one Pfaffian constraint, k=1)

(two control inputs (n - k = 2)

$$\begin{aligned} \dot{q}_1 &= u_1 \\ \dot{q}_2 &= u_2 \end{aligned}$$

(substituting for configuration rates)

$7u_1 - 3u_2 + \dot{q}_3 = 0$

(configuration transition equation)

$$\begin{aligned} \dot{q}_1 &= u_1 \\ \dot{q}_2 &= u_2 \\ \dot{q}_3 &= -7u_1 + 3u_2 \end{aligned}$$

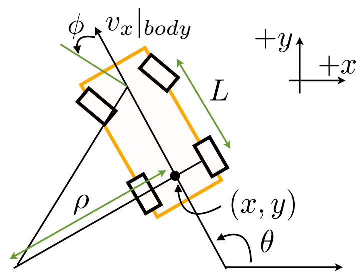
Simple Car Model

Rigid body that moves in a two-dimensional configuration space

$$\mathcal{C} = \mathbb{R}^2 \times \mathbb{S}^1$$

$$q = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$$

$$\begin{aligned} \dot{x} &= f_1(x, y, \theta, v_x|_{body}, \phi) \\ \dot{y} &= f_2(x, y, \theta, v_x|_{body}, \phi) \\ \dot{\theta} &= f_3(x, y, \theta, v_x|_{body}, \phi) \end{aligned}$$



Simple Car Model

Rigid body that moves in a two-dimensional configuration space

$$\mathcal{C} = \mathbb{R}^2 \times \mathbb{S}^1$$

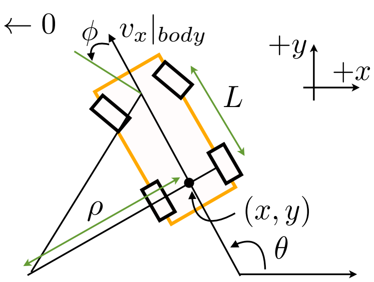
$$q = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$$

$$\frac{dy}{dx} = tan(\theta) = \frac{sin(\theta)}{cos(\theta)} = \frac{\dot{y}}{\dot{x}} \text{ as } t \leftarrow 0$$

$$-\dot{x} sin(\theta) + \dot{y} cos(\theta) = 0$$

(rewritten as a Pfaffian constraint)

$$\begin{aligned} \dot{x} &= cos(\theta) = v_x|_{body} cos(\theta) \\ \dot{y} &= sin(\theta) = v_x|_{body} sin(\theta) \end{aligned}$$



Simple Car Model

How are these constraints found?

$$\mathcal{C} = \mathbb{R}^2 \times \mathbb{S}^1$$

$$q = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$$

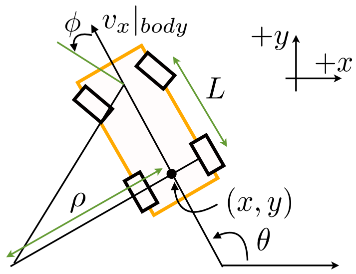
$$-\dot{x} sin(\theta) + \dot{y} cos(\theta) = 0$$

(n=2, k=1)

$$\begin{aligned} \dot{x} &= u_1 \\ \dot{y} &= u_1 tan(\theta) \end{aligned}$$

(substituting for u_1)

$$\begin{aligned} \dot{x} &= cos(\theta) \\ \dot{y} &= cos(\theta)\frac{sin(\theta)}{cos(\theta)} = sin(\theta) \end{aligned}$$



Simple Car Model

Rigid body that moves in a two-dimensional configuration space

$$\mathcal{C} = \mathbb{R}^2 \times \mathbb{S}^1$$

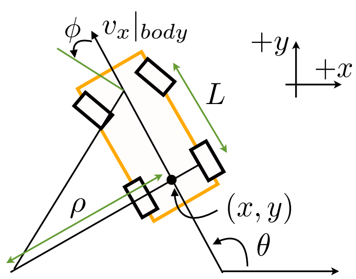
$$q = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$$

(distance traveled) $d\omega = \rho d\theta$

$$d\theta = \frac{tan(\phi)}{L} d\omega$$

(divide by dt)

$$\dot{\theta} = \frac{v_x|_{body}}{L} tan(\phi)$$



Simple Car Model

With control inputs of the form

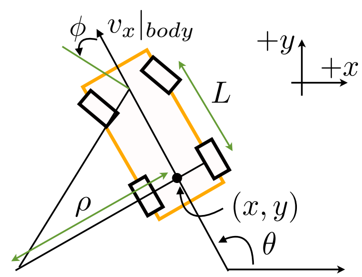
$$u = (u_{v_x|_{body}}, u_\phi)$$

the configuration transition equation is

$$\begin{aligned} \dot{x} &= u_{v_x|_{body}} cos(\theta) \\ \dot{y} &= u_{v_x|_{body}} sin(\theta) \\ \dot{\theta} &= \frac{u_{v_x|_{body}}}{L} tan(u_\phi) \end{aligned}$$

$$\mathcal{C} = \mathbb{R}^2 \times \mathbb{S}^1$$

$$q = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$$



Variations of the Simple Car Model

Simple Car

$$U = [-1, 1] \times (-\phi_{max}, \phi_{max})$$ (cannot turn in place)

Tricycle

$$U = [-1, 1] \times (-\frac{\pi}{2}, \frac{\pi}{2})$$ (can turn in place)

Reed-Shepp Car

$$U = \{-1, 0, 1\} \times (-\phi_{max}, \phi_{max})$$ (can drive forwards and backwards)

Dubins Car

$$U = \{0, 1\} \times (-\phi_{max}, \phi_{max})$$ (cannot drive backwards)

Differential Drive Model

With inputs of the form

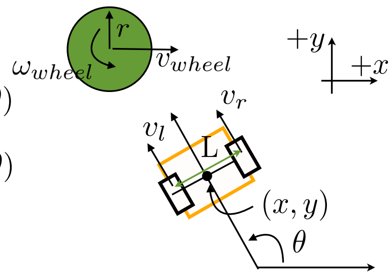
$$u = (u_{\omega_r}, u_{\omega_l})$$

$$\mathcal{C} = \mathbb{R}^2 \times \mathbb{S}^1$$

$$q = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$$

The configuration transition equation is

$$\begin{aligned} \dot{x} &= \frac{r}{2}(u_{\omega_l} + u_{\omega_r})cos(\theta) \\ \dot{y} &= \frac{r}{2}(u_{\omega_l} + u_{\omega_r})sin(\theta) \\ \dot{\theta} &= \frac{r}{L}(u_{\omega_r} - u_{\omega_l}) \end{aligned}$$

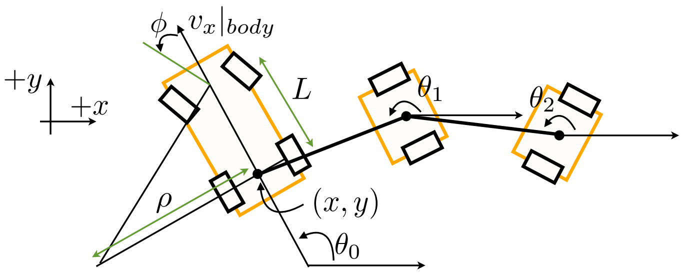


N-Trailers

A variation of the simple car model involves one that is pulling N passively controlled trailers

$$\mathcal{C} = \mathbb{R}^2 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$$

$$q = \begin{bmatrix} x & y & \theta_0 & \theta_1 & \theta_2 \end{bmatrix}^T$$



Dynamic Constraints

Implicit $\mathbf{g_i}(\ddot{\mathbf{q}}, \dot{\mathbf{q}}, \mathbf{q}) = 0$

similar to the kinematic constraints

Parametric $\ddot{\mathbf{q}} = \mathbf{f}(\dot{\mathbf{q}}, \mathbf{q}, \mathbf{u})$

Phase Space

Increase the dimension to remove higher-order derivatives

define a phase vector

$$\mathbf{x} = (x_1, x_2)$$

set the values of the phase vector

$$x_1 = y$$
$$x_2 = \dot{y}$$

creates new constraints

$$\dot{x_2} = \ddot{y}$$
$$\dot{x_1} = x_2$$

a single constraint

$$4\ddot{y} - 5\dot{y} + y = 0$$

system is converted into two first-order differential equations

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= \frac{5x_2 - x_1}{4} \end{aligned}$$

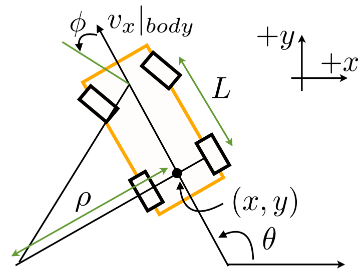
Improved Car Model

Use a double integrator to ensure a continuously varying steering angle

$$\mathbf{x} = [\ x \quad y \quad \theta \quad \phi\ ]^T$$

$$\mathbf{u} = (u_{v_x|_{body}}, u_\omega)$$

$$\begin{aligned}
\dot{x} &= v_x|_{body} cos(\theta) \\
\dot{y} &= v_x|_{body} sin(\theta) \\
\dot{\theta} &= \frac{v_x|_{body}}{L} tan(\phi) \\
\dot{\phi} &= u_\omega
\end{aligned}$$

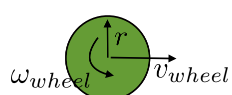


Improved Differential Drive Model

This can similarly be applied for the differential drive model

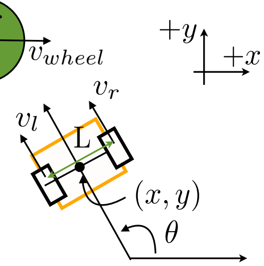
$$\mathbf{x} = [\ x \quad y \quad \theta \quad \omega_l \quad \omega_r\ ]^T$$

$$\mathbf{u} = (u_{\alpha_l}, u_{\alpha_r})$$



$$\begin{aligned}
\dot{x} &= \frac{r}{2}(\omega_l + \omega_r) cos(\theta) \\
\dot{y} &= \frac{r}{2}(\omega_l + \omega_r) sin(\theta) \\
\dot{\theta} &= \frac{r}{L}(\omega_r - \omega_l)
\end{aligned}$$

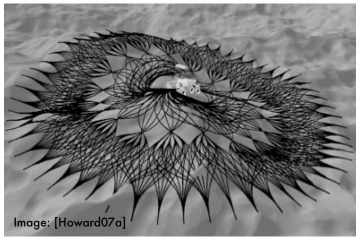
$$\dot{\omega_l} = u_{\alpha_l}$$
$$\dot{\omega_r} = u_{\alpha_r}$$

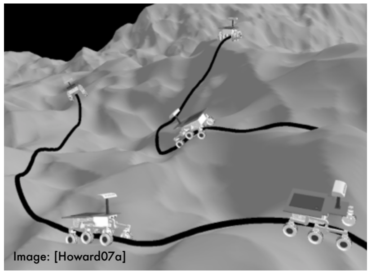


Predictive Model Applications

Predictive Terrain Compensation [Howard07a]

Exploited a motion model that simulated the effects of terrain geometry to plan paths that considered rough terrain and wheel slip models

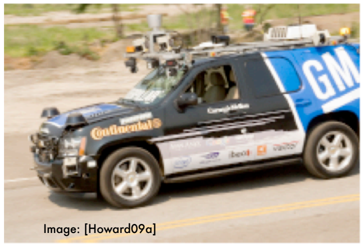


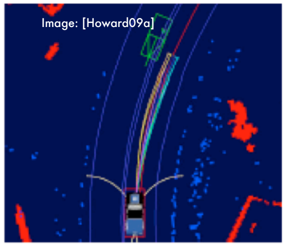


Predictive Model Applications (cont.)

Autonomous automobile navigation [Ferguson08a]

Modeled curvature, curvature rate, safety controller constraints





Current and Active Research

Online Slip Identification [Rogers-Marcovitz10a]

Extended Kalman filter to calibrate a parameterized predictive motion model in real-time

Planar slip disturbances modeled as second-order functions of velocity and curvature

Demonstrated on multiple platforms

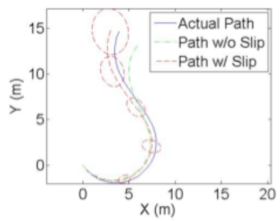


Image: [Rogers-Marcovitz10a]


Image: [Rogers-Marcovitz10a]

Next

Lecture 2: Motion simulation (3/31)

- Newton-Euler Mechanics
- Motion of Particles
- Motion Simulation (Linear, Euler, Runge-Kutta)
- Homework #1 Assigned

Document/Image References

[Ferguson08a] David Ferguson, Thomas Howard, and Maxim Likhachev, "Motion Planning in Urban Environments: Part I," Proceedings of the IEEE/RSJ 2008 International Conference on Intelligent Robots and Systems, September, 2008.

[Howard07a] Thomas Howard and Alonzo Kelly, "Optimal Rough Terrain Trajectory Generation for Wheeled Mobile Robots," International Journal of Robotics Research, Vol. 26, No. 2, March, 2007, pp. 141-166.

[Howard 09a] T.M. Howard, "Adaptive Model-Predictive Motion Planning for Navigation in Complex Environments". Ph.D. Thesis, Carnegie Mellon University, August 2009

[LaValle 06a] S. LaValle, "Planning Algorithms". Cambridge: Cambridge University Press. ISBN 0521862051.

[Rogers-Marcovitz10a] Forrest Rogers-Marcovitz and Alonzo Kelly, "On-line Mobile Robot Model Identification using Integrated Perturbative Dynamics," 12th International Symposium on Experimental Robotics, December, 2010.