CALIFORNIA INSTITUTE OF TECHNOLOGY

ME/CS 132b, Spring 2011

Problem Set #1 Due: Apr 7th, 2011

Readings: LaValle, Chapter 1 & 3 (all), Chapter 13 (Sections 13.1, 13.2, and 13.3 only).

1. [$3+3+3+5+5+6 = 25$ points] LaValle, Problems 1–5 and 7 of Chapter 13 (6 problems in total).

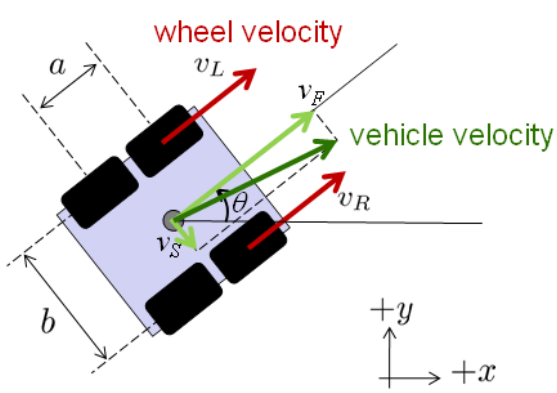
   Note: In equation (13.19) of the book, the equation for $\dot{\theta}_1$ should be

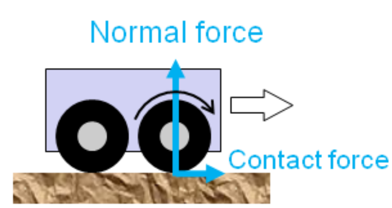
$$\dot{\theta}_1 = \frac{s}{d_1}\sin(\theta_0 - \theta_1).$$

2. [25 points] Derive the state-space model of a 4-wheel skid-steered vehicle. Use the following notations:

   - a: wheel track
   - b: wheel base
   - v_L: circumferential velocity of left wheels (input)
   - v_R: circumferential velocity of right wheels (input)
   - v_F: forward speed of the vehicle
   - v_S: lateral speed of the vehicle
   - m: vehicle mass
   - J: moment of inertia around C.G. (assume C.G. is at the vehicle center)
   - x, y: vehicle position
   - θ: vehicle heading
   - C: tire stiffness
   - μ_0: friction coefficient

   Use a simple friction model: $P = Cv_{\text{slip}}$ when $Cv_{\text{slip}} \leq \mu_0 W$ and $P = \mu_0 W$ otherwise, where v_{slip} is the slip velocity, and W is the weight acting on the ground contact point of the tire.





3. [20 points] Implement a dynamics model of a skid-steered vehicle in MATLAB (or any other programming languages of your choice). The function should look like:

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[dxdt] = dynamics_skid_steer(x, u)
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4. [15 points] Implement an Euler integrator and integrate this with the model developed in Problem #3 in MATLAB (if you have trouble solving #3, take any model covered in the lecture). Use the following parameters:

Parameter	Value	Unit
a	0.4	m
b	0.5	m
m	8	kg
J	2	$\text{kg} \cdot \text{m}^2$
C	5000	N/(m/s)
μ_0	0.4	(no unit)
g	9.8	m/s^2

Run the simulation with four different integration step sizes dt: 0.01 sec, 0.1 sec, 1.0 sec, and 2.0 sec, and plot the vehicle's states (x, y, θ etc.) as a function of time, in a separate 2D plot. Start from the origin (all states are zeros), and use the following control input:

$$v_L = \begin{cases} 0.1t & 0 \leq t \leq 20, \\ 0 & t > 20, \end{cases}$$

$$v_R = \begin{cases} 0.2t & 0 \leq t \leq 10, \\ 2 & 10 < t \leq 20, \\ 0 & t > 20. \end{cases}$$

5. [15 points] Implement the 4th-order Runge-Kutta integration scheme. Run the simulation with the same setup used in Problem #4 and compare the results.