

LPE Reading Group  
25 July 2001  
Consensus continued  
(Lynch ch 25)

## Last time:

- 1) Define simplified agreement problem
- 2) Failure detector algorithm  
(PSynchFD)
- 3) algorithm to solve agreement  
in Synch network if  $f \leq 1$  rounds  
Floodset (6.2)
- 4) cannot solve agreement in Synch network  
in fewer than  $f$  rounds  
"chain argument" (6.2)

Transform synch algorithm A to  
partially synch algorithm A'

A' algorithm

- 1.)  $P_i = Q_i \times R_i$        $R_i$ : main process  
                                  $Q_i$ : PSynchFD
- 2) simulate each round  $r$  of A
- ①  $R_i$  use msgs. from A to determine  $\text{Send}(m)_{i,j}$  &  $t_i$
  - ② for each  $i \in I$   $R_i$  waits until either  
it receives message from  $R_j$   
or  
 $R_i$  times out
  - ③  $R_i$  determines new state using  $\text{trans}_i$  from A

$$\text{let } L = \frac{L_1}{f}$$

Upper bound  
 time from 1st init until all non-faulty processes  
 decide is at most  $f(Ld + d) + d + O(fLd_2)$

$$\text{let } \alpha = \alpha \in \text{exec}(A')$$

$$S = Ld + d + O(fLd_2)$$

$S$  is max time from failure <sub>$i$</sub>  to timeout <sub>$i$</sub>

def a series of times  $T(0), T(1), T(2), \dots$  s.t.

$T(0)$  is time that last init occurs in  $\alpha$

$$T(1) = \begin{cases} T(0) + d_2 + S & \text{if stop by } T(0) + d_2 \\ T(0) + d_1 + d & \text{otherwise} \end{cases}$$

$$T(r) = \begin{cases} T(r-1) + d_2 + S & \text{if stop in } [T(r-1) + d_2, T(r-1) + d_1] \\ T(r-1) + d_1 + d & \text{otherwise} \end{cases}$$

### Claim 25.5

$\forall r \geq 0$ ,  $T(r)$  is upper bound on time for all processes to complete round  $r$ .

$T(f+1)$  is upper bound to complete  $f+1$  rounds  
there at most  $f$  failures

$$T(f+1) \leq T(0) + f(\frac{1}{2} + S) + (\frac{1}{2} + d)$$

$\uparrow$  time of last init       $\uparrow$   $f$  rounds with failure       $\uparrow$  round with no failure

## Lower bound

No algorithm guarantees agreement  
before time  $(f+1)\Delta$

pf sketch

- assume algorithm exists, show contradiction
- restrict model (strongly timed model) is cor. 802

$\forall r \in \mathbb{N}$  all messages sent  $[rd, (r+1)\Delta)$   
are delivered at  $(r+1)\Delta$

- last message received at  $F\Delta$
- decision must occur  $[f\Delta, (f+1)\Delta)$
- but this corresponds to deciding  
after  $f$  rounds in sync algorithm.







