CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CS 142

R. Murray, M. Chandy | **Homework Set #2** | Issued: 9 Oct 2019
Fall 2019 | | Due: 16 Oct 2019

Note: Please keep track of the number of hours that you spent on this homework set (including reading) and enter this when you turn in your homework on Moodle.

1. Show that the following Hoare triple holds

$$\{x \geq 2\} \quad x := x - y + 3 \quad \{x + y \geq 0\}$$

2. For each of the statements below, indicate whether the statement is true or false. If true, give a proof. If false, give a counterexample.

   Constants:

   (a) $\{\textbf{false}\}$ **next** $\{Q\}$

   (b) $\{P\}$ **next** $\{\textbf{true}\}$

   (c) **true next false**

   Junctivity:

   (d) $(P_1 \textbf{ next } Q_1) \wedge (P_2 \textbf{ next } Q_2) \quad \Longrightarrow \quad (P_1 \wedge P_2) \textbf{ next } (Q_1 \wedge Q_2)$

   (e) $(P_1 \textbf{ next } Q_1) \wedge (P_2 \textbf{ next } Q_2) \quad \Longrightarrow \quad (P_1 \vee P_2) \textbf{ next } (Q_1 \vee Q_2)$

   Weakening:

   (f) $(P \textbf{ next } Q) \wedge [Q \implies Q'] \quad \Longrightarrow \quad (P \textbf{ next } Q')$

3. For each of the statements below, indicate whether the statement is true or false. If true, give a proof. If false, give a counterexample.

   (a) $\textbf{stable}(P) \wedge \textbf{stable}(Q) \implies \textbf{stable}(P \wedge Q)$

   (b) $\textbf{stable}(P) \wedge \textbf{stable}(Q) \implies \textbf{stable}(P \vee Q)$

   (c) $\textbf{stable}(P) \wedge [P \implies P'] \implies \textbf{stable}(P')$

4. Give a proof for the following parts of the earliest meeting time algorithm in Sivilotti, Section 4.4 (similar to the proof provided for FindMax in Section 4.2).

   (a) Show that $r = f(r) = g(r) = h(r)$ is a fixed point for the program.

   (b) Show that $\textbf{invariant}(r \leq M)$.

5. Consider the "average consensus" problem: we are given N agents indexed $0, \ldots, N-1$, where $N > 2$. Each agent j has a real number x_j. Let A be the average of the x_j values and let V be the variance. We wish to have the agents compute and reach consensus on the average value of their initial measurements.

   Consider the following program that is designed to compute the average by picking two i and j non-deterministically (with weak fairness) and performing a local averaging operation:

Program	$AverageConsensus$
constant	N $\{$*number of agents*$\}$
	$\mathcal{G}$ $\{$*interconnection graph*$\}$
	$0 < \alpha < 1$ $\{$*averaging factor*$\}$
var	x : array of N numbers
assign	

   $$\big(\,[\!]\, i, j : (i,j) \in \mathcal{G} : x_i := \alpha x_i + (1-\alpha)x_j \parallel x_j := \alpha x_j + (1-\alpha)x_i\big)$$

   For simplicity, you can take $\alpha = 0.5$.

   (a) Prove that the variance never increases. More formally, show that:

   $$\forall K : \textbf{stable}(V \le K)$$

   (b) Prove the following Hoare triple.

   $$\{x_i \neq x_j \wedge V = K\} \qquad x_i, x_j := (x_i + x_j)/2, (x_i + x_j)/2 \qquad \{V < K\}$$

   This implies that performing the the assignment action corresponding to any x_j, x_k that are not equal decreases the variance V.