CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 202
Problem Set #8

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Reading: Boothby, VI.1–2 (except Reimannian mfds portion), VI.4–VI.5

1. Let ω be an n-form on a compact, orientable, smooth manifold M. Assume that we have two atlases for M, each with an associated partition of unity. Show that $\int_M \omega$ is defined, independent of choice of atlas.

2. Let $H^n \subset \mathbb{R}^n$ be an n-dimensional half space and let $\phi : W \subset \mathbb{R}^n \to \mathbb{R}^n$ be a smooth function such that $\phi|_{W \cap H^n} = 0$. Show that $D\phi(x) = 0$ for all $x \in W \cap H^n$.

3. [Guillemin and Pollack, page 185, #3]
   Use the generalized Stokes' theorem to prove the divergence theorem: let W be a compact domain in $\mathbb{R}^3$ with smooth boundary and let $\vec{F} = (f_1, f_2, f_3)$ be a smooth vector field on W. Then
   $$\int_W (\text{div } \vec{F}) dx dy dz = \int_{\partial W} (\vec{n} \cdot \vec{F}) dA.$$
   (Here $\vec{n}$ is the outward normal to ∂W. See GP for extra hints.)