CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

ACM/CDS 202
Problem Set #7

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Due: 29 May 2013

Reading: Abraham, Marsden, and Ratiu (MTA), Section 5.3; and Kelly and Murray (1994) [from web site]

Problems:

1. [Boothby, page 151, #4]
   Find the one-parameter subgroups of $GL(2, \mathbb{R})$ generated by

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

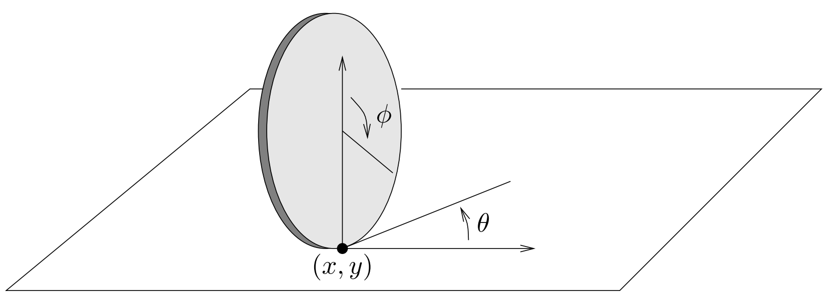
   Find the corresponding actions on $\mathbb{R}^2$ and their infinitesimal generators, starting from the natural action of $GL(2, \mathbb{R})$ on $\mathbb{R}^2$.

2. MTA 5.3-1: semidirect product groups

3. MTA 5.3-2: SE(3)

4. MTA 5.3-4: tangent bundle of a Lie group

5. Consider the locomotion system given by a disk rolling on the plane,



   where we assume we can control the angles θ and ϕ.

   (a) Let $Q = SE(2) \times S^1$ represent the configuration space for the system. Compute the Lagrangian for the sytem and show that is invariant under the action of $SE(2)$ given by translation and rotation as well as the subgroup of actions given just by translation.

   (b) Compute the kinematic connection for the system $A : TQ \to \mathfrak{g}$ corresponding to the system rolling without slipping.

   (c) Determine if the system is totally controllable and/or fiber controllable. (Note: you should do this just by computing Lie brackets and applying Chow's theorem, assuming you can control the velocities in θ and ϕ. If you try to use the conditions in Kelly and Murray (1994), you'll need material that we won't cover until next week.)

   (d) (Optional) Construct an explicit trajectory that moves the system from an arbitrary initial configuration $q = (x_0, y_0, \theta_0, \phi_0)$ to the origin.