CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

ACM/CDS 202
Problem Set #5

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Reading: Abraham, Marsden, and Ratiu (MTA), Sections 5.1 and 5.2

Problems:

1. MTA 5.1-1: properties of the adjoint map on $\mathrm{GL}(n)$
2. [Warner, page 135, #16; MTA 5.1-5]
   (a) Let G be a Lie group. Show that the set of right invariant vector fields on G forms a Lie algebra under the Lie bracket operation and that it is naturally isomorphic to T_eG.
   (b) Let $\phi : G \to G$ be the diffeomorphism defined by $\phi(g) = g^{-1}$. Prove that if $X \subset TG$ is a left invariant vector field on G then $\phi_*(X)$ is a right invariant vector field whose value at e is $-X(e)$. Further show that $X \mapsto \phi_*(X)$ gives a Lie algebra isomorphism of the Lie algebra of left invariant vector fields with the Lie algebra of right invariant vector fields on G.
   (A Lie algebra isomorphism is a linear mapping $A : V \to V$ which preserves the Lie bracket: $A[\xi, \eta] = [A\xi, A\eta]$.)
3. MTA 5.2-1, parts (i)–(iii): calculations on $\mathrm{SO}(3)$
4. MTA 5.2-5, part (i): the Euclidean group
5. [Boothby, page 151, #6]
   Prove that if A is a nonsingular $n \times n$ matrix and $X \in \mathbb{R}^{n\times n}$ then $Ae^XA^{-1} = \exp(AXA^{-1})$. From this deduce that $\det e^X = e^{\mathrm{tr}\, X}$. Use this to determine those matrices A such that e^{At}, $t \in \mathbb{R}$, is a one-parameter subgroup of $\mathrm{SL}(n, \mathbb{R})$, the real special linear group.