CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

ACM/CDS 202
Problem Set #3

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Reading: Abraham, Marsden, and Ratiu (MTA), sections 2.5 and 3.5

Problems:

1. MTA 2.5-3 (i), (ii) and (iv): exponential maps. You can assume (iii), which is a bit tricky to prove.
2. MTA 2.5-12: Roots of polynomials are smooth functions of polynomial coefficients.
3. MTA 3.5-1 (i)–(ii): matrix manifolds.
4. [Guillemin and Pollack, page 18, #2]
   Suppose that P is an l-dimensional submanifold of M where the differentiable structure on P is inherited from M. That is, if (ϕ, U) is a coordinate chart on M then $(\phi|_P, P \cap U)$ is a coordinate chart on P. Let $z \in P$. Show that there exists a local coordinate system $\{x_1, \ldots, x_k\}$ defined in a neighborhood U of z in M such that $P \cap U$ is defined by the equations $x_{l+1} = 0, \ldots, x_k = 0$.
5. [Guillemin and Pollack, page 18, #6; MTA 3.5-5]
   (a) If f and g are submersions/immersions, show that $f \times g$ is.
   (b) If f and g are submersions/immersions, show that $g \circ f$ is.
   (c) If f is an immersion, show that its restriction to any submanifold of its domain is an immersion.
   (d) When $\dim M = \dim N$, show that submersions/immersions $f : M \to N$ are the same as local diffeomorphisms.