1. Show that for a unity feedback system it suffices to check only two transfer functions to determine internal stability.

2. Let
\[ \hat{P}(s) = \frac{1}{10s + 1}, \quad \hat{C}(s) = k, \quad \hat{F}(s) = 1. \]
Find the least positive gain \( k \) such that the following are all true:
(a) The feedback system is internally stable
(b) \( |e(\infty)| \leq 0.1 \) when \( r(t) \) is the unit step and \( n = d = 0 \).
(c) \( \|y\|_\infty \leq 0.1 \) for all \( d(t) \) such that \( \|d\|_2 \leq 1 \) when \( r = n = 0 \).

3. Consider a linear input/output system \( \Sigma \) with a minimal realization given by \( (A, B, C, D) \) and let the associated transfer function be \( H(s) = C(sI - A)^{-1}B + D \). For simplicity, you may also assume that the system is SISO.
(a) Show that if the linear system \( \dot{x} = Ax \) is asymptotically stable then the induced input/output norm of the system \( \Sigma \) is bounded.
(b) Show that if a linear input/output system \( \Sigma \) has bounded induced input/output norm, then the linear system \( \dot{x} = Ax \) is asymptotically stable.
(c) Show that if a linear system is input/output stable then \( \|H\|_\infty \) is bounded.
(d) Show via example that \( \|H\|_\infty \) being bounded is not a sufficient condition for stability of the underlying system.

4. Consider the linear system (7.20). Let \( u = -Kx \) be a state feedback control law obtained by solving the linear quadratic regulator problem. Prove the inequality
\[ (I + L(-i\omega))^{T}Q_u(I + L(i\omega)) \geq Q_u, \]
where
\[ K = Q_u^{-1}B^TS, \quad L(s) = K(sI - A)^{-1}B. \]
(Hint: Use the Riccati equation (7.33), add and subtract the terms \( sS \), multiply with \( B^T(sI + A)^{-T} \) from the left and \( (sI - A)^{-1}B \) from the right.)

For single-input single-output systems this result implies that the Nyquist plot of the loop transfer function has the property \( |1 + L(i\omega)| \geq 1 \), from which it follows that the phase margin for a linear quadratic regulator is always greater than 60°.