CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 131

Homework Set #9

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Fall 2019

Issued: 27 Nov 2019
Due: 6 Dec 2019

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. [DFT 6.5] Define $\epsilon = \|W_1 S\|_\infty$ and $\delta = \|CS\|_\infty$, so that ϵ is a measure of tracking performance and δ measures control effort. Show that for every point s_0 with Re $s_0 \geq 0$,

$$|W_1(s_0)| \leq \epsilon + |W_1(s_0)P(s_0)|\,\delta.$$

Hence ϵ and δ cannot both be very small and so we cannot get good tracking without exerting some control effort.

2. [DFT 6.6] Let ω be a frequency such that $j\omega$ is not a pole of P. Suppose that

$$\epsilon := |S(j\omega)| < 1.$$

Derive a lower bound for $C(j\omega)$ that blows up as $\epsilon \to 0$. Hence good tracking at a particular frequency requires large controller gain at this frequency.

For the next three problems, $M_\text{s} := \|S\|_\infty$ is the maximum sensitivity, $M_\text{t} := \|T\|_\infty$ is the maximum complementary sensitivity, and $s_\text{m} = 1/M_\text{s}$ is the stability margin.

3. [FBS 14.3] Consider a closed loop system consisting of a first-order process and a proportional controller. Let the loop transfer function be

$$L(s) = P(s)C(s) = \frac{k}{s+1},$$

where parameter $k > 0$ is the controller gain. Show that the sensitivity function can be made arbitrarily small.

4. [FBS 14.7] Consider a process with the loop transfer function

$$L(s) = k\frac{z-s}{s-p},$$

with positive z and p. Show that the system is stable if $p/z < k < 1$ or $1 < k < p/z$ and that the largest stability margin is $s_m = |p-z|/(p+z)$, which is obtained for $k = 2p/(p+z)$. Determine the pole/zero ratios that give the stability margin $s_m = 2/3$.

5. [FBS 14.16] Consider a process with the transfer function

$$P(s) = \frac{e^{-s\tau}}{s-p}\bar{P}(s),$$

where $\bar{P}(s)$ has no poles and zeros in the right half-plane. Show that the sensitivity functions have the properties listed in Table 14.1:

$$M_{\mathrm{t}} \geq e^{p\tau}, \qquad M_{\mathrm{s}} \geq e^{p\tau} - 1.$$

Hint: Example 14.9 in FBS2e may be useful with a different choice of weighting function.

This bound shows that a combination of a “fast” unstable pole with a time delay places severe limitations on the achievable performance/robustness of a feedback system.