CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 131

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Homework Set #7

Issued: 13 Nov 2019
Due: 20 Nov 2019

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. [DFT 4.5] Consider the unity feedback system with

$$P(s) = \frac{1}{(s+1)(s+\alpha)} \qquad C(s) = \frac{1}{s}.$$

   For what range of α is the feedback system internally stable? Find the upper and lower gain margins as functions of α.

2. [DFT 4.6] Consider the unity feedback system with $C(s) = 10$ and plant

$$P(s) = \frac{1}{s-a},$$

   where a is *real*.

   (a) Find the range of a for the system to be internally stable.

   (b) For $a = 0$ the plant is $P(s) = 1/s$. Regarding a as a perturbation, we can write the plant as

$$\widetilde{P} = \frac{P}{1 + \Delta W_2 P}$$

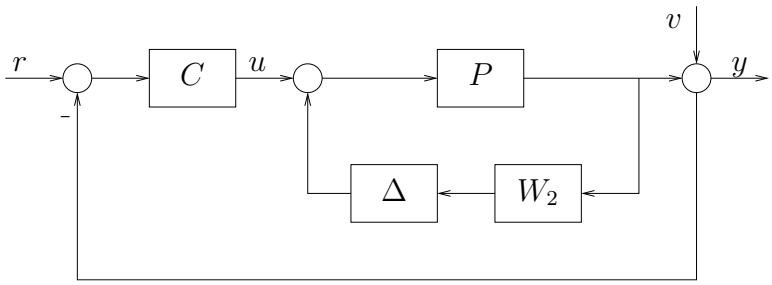
   with $W_2(s) = -a$. Then $\widetilde{P}$ equals the true plant when $\Delta(s) = 1$. Apply robust stability theory to see when the feedback system $\widetilde{P}$ is internally stable for all $\|\Delta\|_\infty \leq 1$. Compare this to your result for part (a).

3. [DFT 4.9] Consider the class of perturbed plants of the form

$$\frac{P}{1 + \Delta W_2 P},$$

   where W_2 is a fixed stable weighting function with W_2P strictly proper and Δ is a variable, stable transfer function with $\|\Delta\|_\infty \leq 1$. Assume that C is a controller achieving internal stability for the nominal plant P. Prove that C provides internal stability for the perturbed plant if $\|W_2PS\|_\infty < 1$.

4. Consider the system shown below. The performance objective is $\|W_1 H_{uv}\|_\infty < 1$ for all $\|\Delta\|_\infty < 1$, where H_{uv} is the transfer function from v to u.



(a) Derive a set of necessary and sufficient conditions for robust stability of the system.

(b) Derive a set of necessary and sufficient conditions for robust performance. These conditions may be written in terms of W_1, W_2, L and P, but should not contain C or Δ.

5. This problem concerns robust stability of the unity-feedback system. Suppose that P and C are nominal transfer functions for which the feedback system is internally stable. Instead of allowing perturbations in just P, this problem allows perturbations in C too. Suppose that P may be perturbed to

$$P(1 + \Delta_1 W_1)$$

and C may be perturbed to

$$C/(1 + \Delta_2 W_2)$$

The transfer functions W_1 and W_2 are fixed, while Δ_1 and Δ_2 are variable transfer functions having ∞-norms no greater than 1. Making appropriate additional assumptions, find a sufficient condition, depending only on the four functions P, C, W_1, W_2, for robust stability. Prove sufficiency. (A weak sufficient condition is the goal; for example, the condition $W_1 = W_2 = 0$ would be too strong.)