CALIFORNIA INSTITUTE OF TECHNOLOGY
Computing and Mathematical Sciences

CDS 131

R. Murray — **Homework Set #1** — Issued: 3 Oct 2018
Fall 2018 — Due: 10 Oct 2018

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. [DFT 2.1] Suppose that $u(t)$ is a continuous signal whose derivative $\dot{u}(t)$ is also continuous. Which of the following quantities qualifies as a norm for u:

   (a) $\sup_t |\dot{u}(t)|$
   (b) $|u(0)| + \sup_t |\dot{u}(t)|$
   (c) $\max\{\sup_t |u(t)|, \sup_t |\dot{u}(t)|\}$
   (d) $\sup_t |u(t)| + \sup_t |\dot{u}(t)|$

   Make sure to give a thorough answer (not just yes or no).

2. [DFT 2.4] Let D be a pure time delay of τ seconds with transfer function

   $$\widehat{D}(s) = e^{-s\tau}.$$

   A norm $\|\cdot\|$ on transfer functions is *time-delay invariant* if for every bounded transfer function $\widehat{G}$ and every $\tau > 0$ we have

   $$\|\widehat{D}\widehat{G}\| = \|\widehat{G}\|$$

   Determine if the 2-norm and ∞-norm are time-delay invariant.

3. [DFT 2.5] Compute the 1-norm of the impulse response corresponding to the transfer function

   $$\frac{1}{\tau s + 1} \qquad \tau > 0.$$

4. [DFT 2.7] Derive the ∞-norm to ∞-norm system gain for a stable, proper plant $\widehat{G}$. (Hint: write $\widehat{G} = c + \widehat{G}_1$ where c is a constant and $\widehat{G}_1$ is strictly proper.)

5. [DFT 2.8] Let $\widehat{G}$ be the transfer function for a stable, proper plant (but not necessarily strictly proper).

   (a) Show that the ∞-norm of the output y given an input $u(t) = \sin(\omega t)$ is $|\widehat{G}(jw)|$.
   (b) Show that the 2-norm to 2-norm system gain for $\widehat{G}$ is $\|\widehat{G}\|_\infty$ (just as in the strictly proper case).

6. [DFT 2.11] Consider a system with transfer function

   $$\widehat{G}(s) = \frac{s+2}{4s+1}$$

and input u and output y. Compute

$$\|G\|_1 = \sup_{\|u\|_\infty = 1} \|y\|_\infty$$

and find an input that achieves the supremum.

7. [DFT 2.12] For a linear system with input u and output y, prove that

$$\sup_{\|u\| \leq 1} \|y\| = \sup_{\|u\| = 1} \|y\|$$

where $\| \cdot \|$ is any norm on signals.

8. Consider a second order mechanical system with transfer function

$$\widehat{G}(s) = \frac{1}{s^2 + 2\omega_n \zeta s + \omega_n^2}$$

(ω_n is the natural frequence of the system and ζ is the damping ratio). Setting $\omega_n = 1$, plot the ∞-norm as a function of the damping ratio $\zeta > 0$. (You may use a computer to to this, but if you do then make sure to turn in a copy of your code with your solutions.)