CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110b

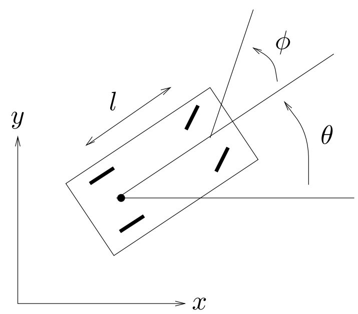
R. M. Murray
Winter 2008

Problem Set #6

Issued: 20 Feb 08
Due: 27 Feb 08

Note: Please put the number of hours that you spent on this homework set (including reading) on the back of the first page of your homework.

1. Consider the problem of estimating the position of an autonomous mobile vehicle using a GPS receiver and an IMU (inertial measurement unit). The dynamics of the vehicle are given by



$$\dot{x} = \cos\theta\, v$$
$$\dot{y} = \sin\theta\, v$$
$$\dot{\theta} = \frac{1}{\ell}\tan\phi\, v,$$

We assume that the vehicle is disturbance free, but that we have noisy measurements from the GPS receiver and IMU and an initial condition error.

In this problem we will utilize the full form of the Kalman filter (including the $\dot{P}$ equation).

(a) Suppose first that we only have the GPS measurements for the xy position of the vehicle. These measurements give the position of the vehicle with approximately 1 meter accuracy. Model the GPS error as Gaussian white noise with $\sigma = 1.2$ meter in each direction and design an optimal estimator for the system. Plot the estimated states and the covariances for each state starting with an initial condition of 5 degree heading error at 10 meters/sec forward speed (i.e., choose $x(0) = (0, 0, 5\pi/180)$ and $\hat{x} = (0, 0, 0)$).

(b) An IMU can be used to measure angular rates and linear acceleration. Assume that we use a Northrop Grumman LN200 to measure the angular rate $\dot{\theta}$. Use the datasheet on the course web page to determine a model for the noise process and design a Kalman filter that fuses the GPS and IMU to determine the position of the vehicle. Plot the estimated states and the covariances for each state starting with an initial condition of 5 degree heading error at 10 meters/sec forward speed.

Note: be careful with units on this problem!

Students who are not doing the course project should complete the following additional problems:

2. (Friedland 11.1) A compensator based on a Kalman filter is to be designed for an instrument servo whose dynamics are given by

$$\dot{\theta} = \omega, \qquad \dot{\omega} = -\alpha\omega + \beta u + V,$$

where V is white noise of spectral density R_V. Only the position θ is measured, so that

$$Y = \theta + W,$$

where W is white noise with spectral density R_W. Use parameters $\alpha = 1$ and $\beta = 3$.

(a) Design a state space controller that places the eigenvalues of the closed loop system (assuming full state feedback is available) at $\lambda = -0.85 \pm 0.5j$.

(b) Find and plot the Kalman filter gains and corresponding closed-loop poles of the estimator as a function of the "signal-to-noise ratio" R_V/R_W.

(c) Compute the gain and phase margin for the closed loop system for the following sets of desired eigenvalues of the full-state feedback system and the signal-to-noise ratio (SNR):

- $\lambda = -0.85 \pm 0.5i$, $\lambda = -2 \pm -2i$, $\lambda = \{-2, -5\}$
- SNR $= 10$, 1, 0.1

You should present your results in a 3×3 table that shows the gain and phase margin for each pair of state feedback and Kalman filter parameters.

3. (Friedland 11.2) Consider the dynamics for the inverted pendulum on a motor-driven cart for which you are to build a full order Kalman filter as an observer. Use the dynamics and parameters from Homework #3, Exercise 4.

(a) Assume that the only excitation noise present is coincident with the control and has spectral density R_v, and that the only observation is cart displacement which is measured through white noise of spectral density R_w. Plot the Kalman filter gains and poles as a function of the ratio R_v/R_w for $1 \leq R_v/R_w \leq 10^6$ (use at least 6 points in your plot).

(b) Following Friedland 9.10, design an LQR controller with weights of the form

$$Q_x = \mathrm{diag}(q_1^2, 0, q_3^2, 0) \qquad Q_u = r^2 \qquad N = 0$$

Using the parameters $q_1^2 = 100$, $q_3^2 = 3000$ and $r^2 = 0.01$, determine the compensator transfer function $D(s)$ for the range of R_v/R_w in part (a).

(c) For $R_v/R_w = 10^{-3}$ and the nominal control gains in part (b), determine the range of additional gains K for which the closed loop system with loop transfer function $KD(s)P(s)$ is stable, where $P(s)$ is the transfer function for the process. (Hint: Look at the abstract for the paper "Guaranteed margins for LQG regulators" by John Doyle, *IEEE Transactions on Automatic Control*, 23(4):756–757, 1978.)