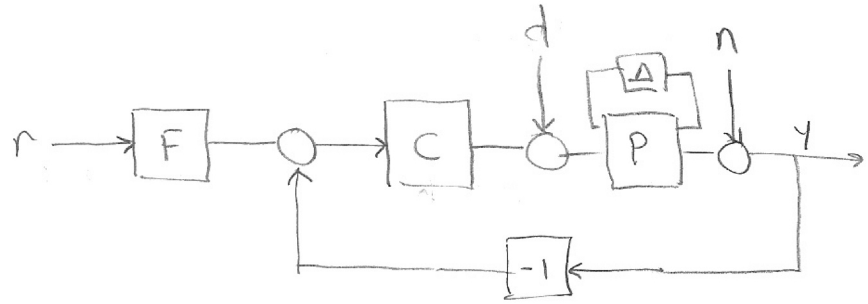
Lecture 9-2: Robust Performance



Simplified setting
(no explicit estimator,
or feedfwd input)

Suppose we have performance specs on S, T, CS, PS (gang of 4)
When do we satisfy these specs for $\tilde{P} = P + \Delta$ (or other types of uncertainty)?

Disturbance attenuation: G_{yd}

$$G_{yd} = \frac{P}{1+PC} = PS \qquad G_{yd}(P+\Delta) \approx G_{yd}(P) + \frac{dG_{yd}}{dP}\Delta$$

$$\frac{dG_{yd}}{dP} = \frac{1}{1+PC} - \frac{P}{(1+PC)^2}C = \frac{1+PC-PC}{(1+PC)^2} = \frac{1}{(1+PC)^2} = \frac{G_{yd}}{P}S$$

$$\frac{dG_{yd}}{G_{yd}} = S\frac{dP}{P} \quad \leftarrow \text{ S determines "sensitivity" to perturbations}$$

Noise attenuation: $\frac{dG_{un}}{G_{un}} = T\frac{dP}{P}$

Remarks

1. As in CDS 110a, would like to keep both T & S small, but $S+T=1$
2. Can also compute $dG_{yr}/dG_{yr} = S\, dP/P$, and other Gang of 6 sensitivities

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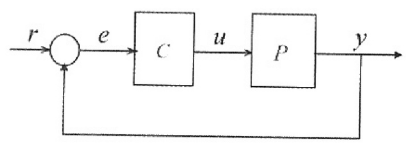
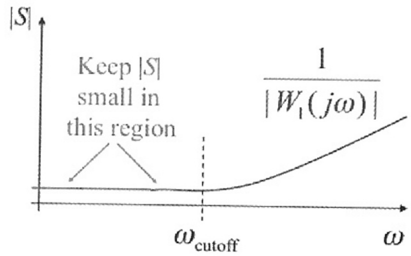
Performance Specification via Weighted Sensitivity



$$H_{er} = \frac{1}{1+PC} =: S$$

Use frequency weighting to specify performance over given range

- Doesn't make sense to ask for small error over *all* frequencies
- Plant can't track reference at extremely high frequencies



$$|S(j\omega)| \le \frac{1}{|W_1(j\omega)|}$$

$$\Updownarrow$$

$$\boxed{|W_1(j\omega)S(j\omega)| \le 1}$$

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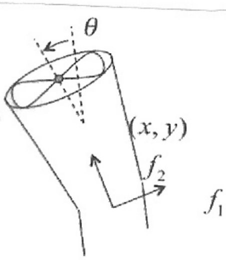
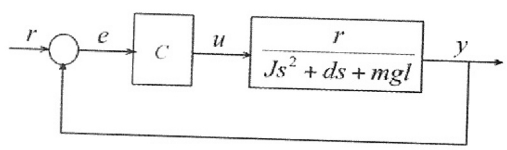
Example: Ducted fan





Performance specification on H_{er}

- Less than 5% tracking error up to 2 Hz
- Less than 10% tracking error up to 5 Hz

Parameter values

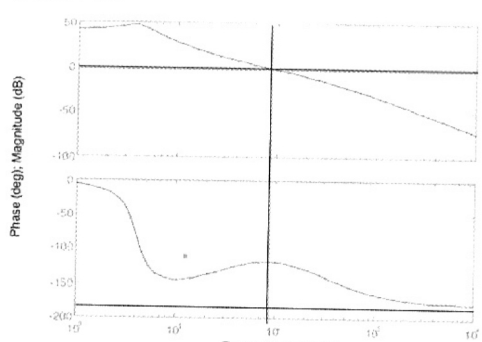
J	0.0475	inertia around pitch axis, kg m^2
m	1.5	mass of fan, kg
r	0.25	distance to flaps, m
g	10	gravitational constant, m/sec^2
d	0.1	damping factor (estimated)
l	0.05	offset of center of mass, m

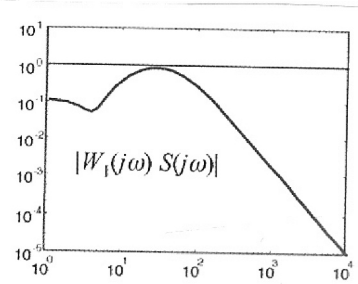
Specification/design

$$C(s) = 20\frac{(s+25)}{(s+300)}$$

$$W_1 = \frac{20}{(s/12+1)^2}$$

Bode Plot





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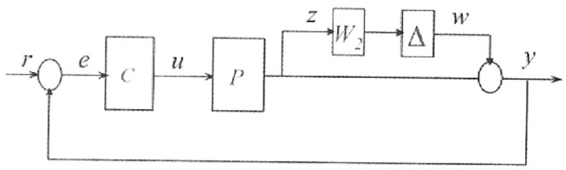
Preview: Robust *Performance*



$$|S(j\omega)| \le \frac{1}{|W_1(j\omega)|}$$

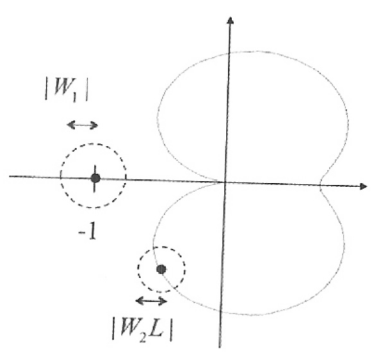
Thm C provides *robust performance* to multiplicative uncertainty if and only if

$$\max_\omega(|W_1 S| + |W_2 T|) < 1$$

Remarks

- Gives conditions for *guaranteed* robust performance
- Given W_1 and W_2, still need to find C that works

Nyquist Interpretation of Robust Performance



Stability and performance conditions:

1. Nyquist plot outside –1
   ⇒ nominal stability
2. Nyquist plot outside W_1 ball
   ⇒ nominal performance
   (generalizes gain/phase margin)
3. Nyquist plot outside W_2 ball
   ⇒ robust stability
4. Combined non-intersection
   ⇒ robust performance

Notes

- Size of balls varies as frequency changes
- Condition is equivalent to

$$\max_\omega(|W_1 S| + |W_2 T|) < 1$$

(Optional)

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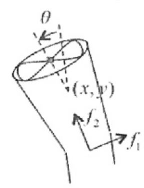
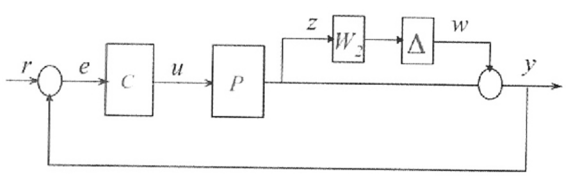
Example: Caltech Ducted Fan Design Problem (1/2)



$$m\ddot{x} = f_1 \cos\theta - f_2 \sin\theta$$
$$m\ddot{y} = f_1 \sin\theta + f_2 \cos\theta - mg$$
$$J\ddot{\theta} = rf_1$$

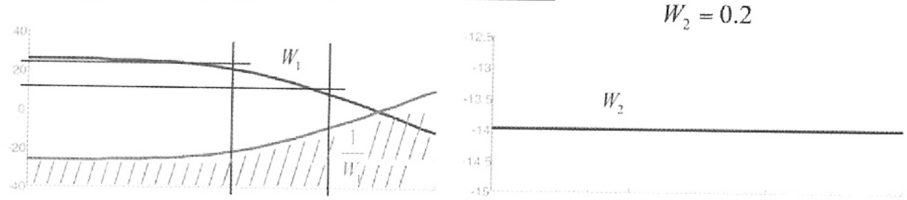
$$\frac{\theta(s)}{u(s)} = \frac{r}{Js^2 + ds + mgl}$$



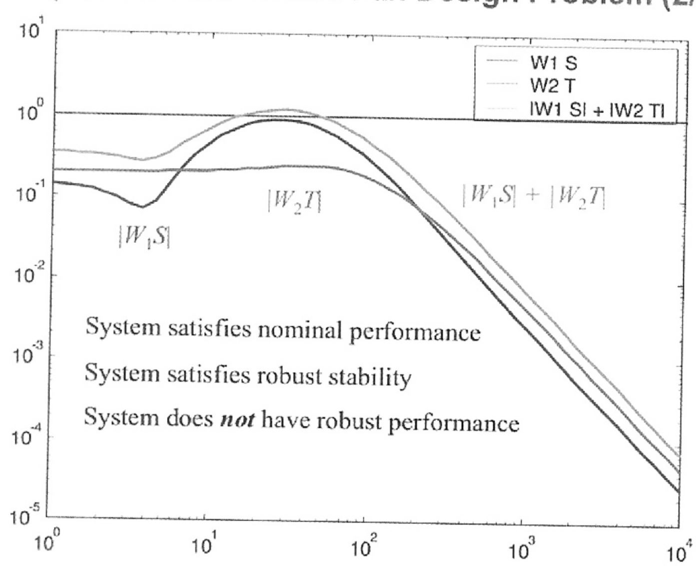
$$|S(j\omega)| \le \frac{1}{|W_1(j\omega)|}$$

$$W_1 = \frac{20}{(s/12+1)^2}$$

$$W_2 = 0.2$$



Example: Caltech Ducted Fan Design Problem (2/2)



System satisfies nominal performance

System satisfies robust stability

System does ***not*** have robust performance