CDS 110b

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Winter 2007   Due: 22 Feb 07
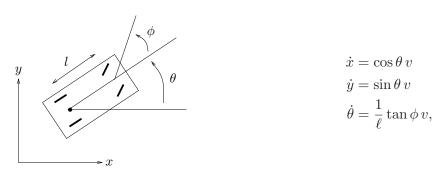
1. Consider a nonlinear control system with gain scheduled feedback

$$\dot{e} = f(e, v) \qquad v = k(\mu)e,$$

   where $\mu(t) \in \mathbb{R}$ is an externally specified parameter (eg, the desired trajectory) and $k(\mu)$ is chosen such that the linearization of the closed loop system around the origin is stable for each fixed μ.
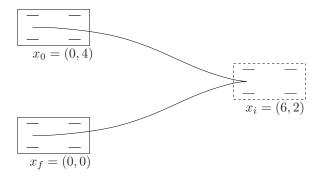
   Show that if $|\dot{\mu}|$ is sufficiently small then the equilibrium point is locally asymptotically stable for the full nonlinear, time-varying system. (Hint: find a Lyapunov function of the form $V = x^T P(\mu)x$ based on the linearization of the system dynamics for fixed μ and then show this is a Lyapunov function for the full system.)

2. Show that all reachable linear control systems are differentially flat and give a formula for the flat output. (Hint: start by assuming the system is in reachable canonical form.)

3. Consider the lateral control problem for an autonomous ground vehicle, as shown below:

$$\dot{x} = \cos\theta\, v$$
$$\dot{y} = \sin\theta\, v$$
$$\dot{\theta} = \frac{1}{\ell}\tan\phi\, v,$$

   Let (x, y, θ) represent the state and (v, ϕ) the inputs. For simplicity we will assume $\ell = 1$ meter.

   (a) Show that this system is differentially flat using $z = (x, y)$ and solve explicitly for the state and input in terms of the flat output and its derivatives. Note any special situations in which additional assumptions are required (eg, if you equations go singular at some point, you may need to stay away from these points).

   (b) Using the fact that the system is differentially flat, find an explicit trajectory that solves the following parallel parking manuever:

Your solution should consist of two segments: a curve from x_0 to x_i with $v > 0$ and a curve from x_i to x_f with $v < 0$. For the trajectory that you determine, plot the trajectory in the plane (x versus y) and also the inputs v and ϕ as a function of time.

4. A simplified model of the steering control problem is derived in Åström and Murray, Example 5.12. The model has the form

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} \alpha \\ 1 \end{bmatrix} u$$

$$\eta = x_1$$

where $x = (y/\ell, \theta)$ and α is a parameter related to the speed of the vehicle. Suppose that we wish to track a piecewise constant reference trajectory

$$\eta_r = \texttt{square}(2\pi t/20),$$

where $\texttt{square}$ is the square wave function in MATLAB. Suppose further that the speed of the vehicle varies according to the formula

$$\alpha = 2 + 2\sin(2\pi t/50).$$

Design and implement a gain-scheduled controller for this system by computing an LQR controller with cost function

$$L(x, u) = x_1^2 + x_2^2 + u^2$$

at three different parameter values: $\alpha = 0, 2, 4$. You should use linear interpolation to compute the gain for values of α between these fixed values. Compare the performance of the gain scheduled controller to a simple controller that assumes $\alpha = 2$ for the purpose of the control design (but leaving α time-varying in your simulation).

Note: a MATLAB file with the vehicle dynamics is available on the course web page. You can use this if you like to get the reference trajectory and parameter variation.