CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110b

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Unless otherwise specified, you may use MATLAB or Mathematica as long as you include a copy of the code used to generate your answer.

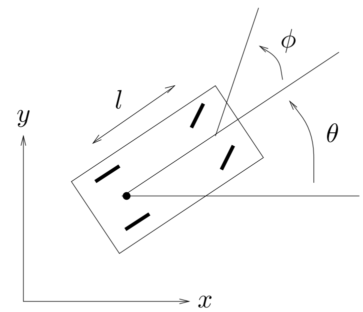
1. (Friedland 11.1, using class notation) A compensator based on a Kalman filter is to be designed for the instrument servo problem (Friedland 9.6). Only the position error e is measured, so that
$$Y = E + W$$
where W is white noise with spectral density R_W. The only excitation noise present occurs at the control input, so that the angular velocity is
$$\dot{\omega} = -\alpha\omega + \beta u + V$$
where V is white noise of spectral density R_V.
   (a) Design a state space controller that places the eigenvalues of the closed loop system (assuming full state feedback is available) at $\lambda = -0.85 \pm 0.5j$.
   (b) Find and plot the Kalman filter gains and corresponding closed-loop poles of the estimator as a function of the "signal-to-noise ratio" R_V/R_W.
   (c) Compute the gain and phase margin for the closed loop system for the following sets of desired eigenvalues of the full-state feedback system and the signal-to-noise ratio (SNR):
      - $\lambda = -0.85 \pm 0.5j$, $\lambda = -2 \pm -2$, $\lambda = \{-2, -5\}$
      - SNR = 10, 1, 0.1

   You should present your results in a 3×3 table that shows the gain and phase margin for each pair of state feedback and Kalman filter parameters.

2. (Friedland 11.2) Consider the dynamics for the inverted pendulum on a motor-driven cart, for which you are to build a full order Kalman filter as an observer.
   (a) Assume that the only excitation noise present is coincident with the control and has spectral density R_v, and that the only observation is cart displacement which is measured through white noise of spectral density R_w. Plot the Kalman filter gains and poles as a function of the ratio R_v/R_w for $1 \leq R_v/R_w \leq 10^6$ (use at least 6 points in your plot).
   (b) Following Friedland 9.10, design an LQR controller with weights of the form
$$Q = \mathrm{diag}(q_1^2, 0, q_3^2, 0) \qquad R = r^2 \qquad N = 0$$
   Using the parameters $q_1^2 = 100$, $q_3^2 = 3000$ and $r^2 = 0.01$, determine the compensator transfer function $D(s)$ for the range of R_v/R_w in part (a).
   (Note: a description of the LQR problem is given in Åström and Murray section 6.6. We will derive the theory for this later in CDS 110b; for now you can just use the MATLAB 'lqr' command to solve for the K matrix corresponding to the given cost function.)

(c) For $R_v/R_w = 10^{-3}$ and the nominal control gains in part (b), determine the range of additional gains K for which the closed loop system with loop transfer function $KD(s)P(s)$ is stable, where $P(s)$ is the transfer function for the process.

3. (Alice) Consider the problem of estimating the position of an autonomous mobile vehicle using a GPS receiver and an IMU (inertial measurement unit). The dynamics of the vehicle are given by



$$\dot{x} = \cos\theta\, v$$
$$\dot{y} = \sin\theta\, v$$
$$\dot{\theta} = \frac{1}{\ell}\tan\phi\, v,$$

We assume that the vehicle is disturbance free, but that we have noisy measurements from the GPS receiver and IMU and an initial condition error.

In this problem we will utilize the full form of the Kalman filter (including the $\dot{P}$ equation).

(a) Suppose first that we only have the GPS measurements for the xy position of the vehicle. These measurements give the position of the vehicle with approximately 1 meter accuracy. Model the GPS error as Gaussian white noise with $\sigma = 1.2$ meter in each direction and design an optimal estimator for the system. Plot the estimated states and the covariances for each state starting with an initial condition of 5 degree heading error at 10 meters/sec forward speed (i.e., choose $x(0) = (0, 0, 5\pi/180)$ and $\hat{x} = (0, 0, 0)$).

(b) An IMU can be used to measure angular rates and linear acceleration. Assume that we use a Northrop Grumman LN200 to measure the angular rate $\dot{\theta}$. Use the datasheet on the course web page to determine a model for the noise process and design a Kalman filter that fuses the GPS and IMU to determine the position of the vehicle. Plot the estimated states and the covariances for each state starting with an initial condition of 5 degree heading error at 10 meters/sec forward speed.

Note: be careful with units on this problem!