Appendix B

Decentralised Data Fusion

B.1 The Information Filter

A key tool in decentralised data fusion systems is the information filter. The information filter allows standard continuous estimation and control functions to be decentralised. The information filter is summarised in this section.

Consider a system described in standard linear form

$$\mathbf{x}(k) = \mathbf{F}(k)\mathbf{x}(k-1) + \mathbf{B}(k)\mathbf{u}(k) + \mathbf{G}(k)\mathbf{w}(k), \tag{B.1}$$

where $\mathbf{x}(j)$ is the state of interest at time j, $\mathbf{F}(k)$ is the state transition matrix from time $k-1$ to k, $\mathbf{B}(k)$ and $\mathbf{G}(k)$ the control input and noise input transition matrices, and where $\mathbf{u}(k)$ and $\mathbf{w}(k)$ are the associated control input and process noise input modeled as an uncorrelated white sequence with $\mathrm{E}\{\mathbf{w}(i)\mathbf{w}^T(j)\} = \delta_{ij}\mathbf{Q}(i)$. The system is observed by a sensor according to the linear equation

$$\mathbf{z}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{v}(k) \tag{B.2}$$

where $\mathbf{z}(k)$ is the vector of observations made at time k, $\mathbf{H}(k)$ the observation matrix or model, and where $\mathbf{v}(k)$ is the associated observation noise modeled as an uncorrelated white sequence with $\mathrm{E}\{\mathbf{v}(i)\mathbf{v}^T(j)\} = \delta_{ij}\mathbf{R}(i)$.

The conventional Kalman filter algorithm generates estimates for the state $\hat{\mathbf{x}}(k \mid k)$ at a time k given all observations up to time k, together with a corresponding estimate covariance $\mathbf{P}(k \mid k)$ as:

$$\hat{\mathbf{x}}(k \mid k) = \hat{\mathbf{x}}(k \mid k-1) + \mathbf{W}(k)\left[\mathbf{z}(k) + \mathbf{H}(k)\hat{\mathbf{x}}(k \mid k-1)\right] \tag{B.3}$$

$$\mathbf{P}(k \mid k) = \mathbf{P}(k \mid k-1) - \mathbf{W}(k)\mathbf{S}(k)\mathbf{W}^T(k) \tag{B.4}$$

where $\mathbf{W}(k)$ is the gain matrix, $\mathbf{S}(k)$ the innovation covariance. The information form of the Kalman filter is obtained by re-writing the state estimate and covariance in terms of two new variables

$$\hat{\mathbf{y}}(i \mid j) \triangleq \mathbf{P}^{-1}(i \mid j)\hat{\mathbf{x}}(i \mid j), \qquad \mathbf{Y}(i \mid j) \triangleq \mathbf{P}^{-1}(i \mid j), \tag{B.5}$$

and also the information associated with an observation in the form

$$\mathbf{i}(k) \triangleq \mathbf{H}^T(k)\mathbf{R}^{-1}(k)\mathbf{z}(k), \qquad \mathbf{I}(k) \triangleq \mathbf{H}^T(k)\mathbf{R}^{-1}(k)\mathbf{H}(k) \tag{B.6}$$

With these definitions, the information filter can be summarised

Prediction:

$$\begin{aligned}\hat{\mathbf{y}}(k \mid k-1) &= \left[\mathbf{1} - \mathbf{\Omega}(k)\mathbf{G}^T(k)\right]\mathbf{F}^{-T}(k)\hat{\mathbf{y}}(k-1 \mid k-1) + \\ &\quad + \mathbf{Y}(k \mid k-1)\mathbf{B}(k)\mathbf{u}(k)\end{aligned} \tag{B.7}$$

$$\mathbf{Y}(k \mid k-1) = \mathbf{M}(k) - \mathbf{\Omega}(k)\mathbf{\Sigma}(k)\mathbf{\Omega}^T(k) \tag{B.8}$$

where

$$\mathbf{M}(k) = \mathbf{F}^{-T}(k)\mathbf{P}^{-1}(k-1 \mid k-1)\mathbf{F}^{-1}(k), \tag{B.9}$$

$$\mathbf{\Omega}(k) = \mathbf{M}(k)\mathbf{G}(k)\mathbf{\Sigma}^{-1}(k), \tag{B.10}$$

and

$$\mathbf{\Sigma}(k) = \left[\mathbf{G}^{T}(k)\mathbf{M}(k)\mathbf{G}(k) + \mathbf{Q}^{-1}(k)\right]. \tag{B.11}$$

Estimate:

$$\hat{\mathbf{y}}(k \mid k) = \hat{\mathbf{y}}(k \mid k-1) + \mathbf{i}(k) \tag{B.12}$$

$$\mathbf{Y}(k \mid k) = \mathbf{Y}(k \mid k-1) + \mathbf{I}(k). \tag{B.13}$$

The information-filter form has the advantage that the update Equations B.12 and B.13 for the estimator are computationally simpler than the equations for the Kalman Filter, at the cost of increased complexity in prediction. The value of this in decentralized sensing is that estimation occurs locally at each node, requiring partition of the estimation equations which are simpler in their information form. Prediction, which is more complex in this form, relies on a propagation coefficient which is independent of the observations made and so is again simpler to decouple and decentralize amongst a network of sensor nodes.

The information form of the Kalman filter, while widely known, is not commonly used because the update terms are of dimension the state, whereas in the distributed Kalman filter updates are of dimension the observation. For single sensor estimation problems, this argues for the use of the Kalman filter over the information filter. However, in multiple sensor problems, the opposite is true. The reason is that with multiple sensor observations

$$\mathbf{z}_i(k) = \mathbf{H}_i(k)\mathbf{x}(k) + \mathbf{v}_i(k), \qquad i = 1, \cdots, N$$

the estimate can not be constructed from a simple linear combination of contributions from individual sensors

$$\hat{\mathbf{x}}(k \mid k) \neq \hat{\mathbf{x}}(k \mid k-1) + \sum_{i=1}^{N} \mathbf{W}_i(k) \left[\mathbf{z}_i(k) - \mathbf{H}_i(k)\hat{\mathbf{x}}(k \mid k-1)\right],$$

as the innovation $\mathbf{z}_i(k) - \mathbf{H}_i(k)\hat{\mathbf{x}}(k \mid k-1)$ generated from each sensor is correlated because they share common information through the prediction $\hat{\mathbf{x}}(k \mid k-1)$. However, in information form, estimates can be constructed from linear combinations of observation information

$$\hat{\mathbf{y}}(k \mid k) = \hat{\mathbf{y}}(k \mid k-1) + \sum_{i=1}^{N} \mathbf{i}_i(k),$$

as the information terms $\mathbf{i}_i(k)$ from each sensor are uncorrelated. Once the update equations have been written in this simple additive form, it is straight-forward to distribute the data fusion problem (unlike for a Kalman filter); each sensor node simply generates the information terms $\mathbf{i}_i(k)$, and these are summed at the fusion center to produce a global information estimate.

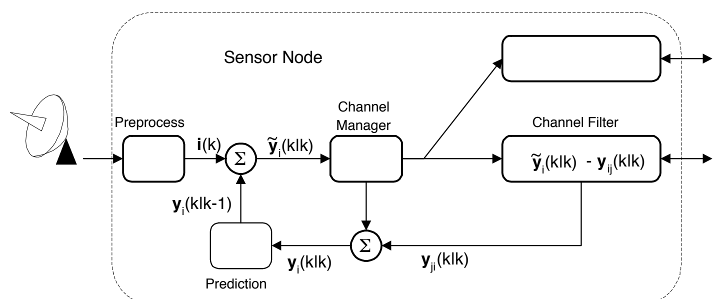

Figure B.1: Algorithmic structure of a decentralised sensing node.

To decentralise the information filter all that is necessary is to replicate the central fusion algorithm (summation) at each sensor node and simplify the result. This yields a surprisingly simple nodal fusion algorithm. The algorithm is described graphically in Figure B.1. Essentially, local estimates are first generated at each node by fusing (adding) locally available observation information $\mathbf{i}_i(k)$ with locally available prior information $\hat{\mathbf{y}}_i(k \mid k-1)$. This yields a local information estimate $\tilde{\mathbf{y}}_i(k \mid k)$. The difference between this local estimate and prediction (corresponding to new information gained) is then transmitted to other nodes in the network. In a fully connected or broadcast network, this results in every sensing node getting all new information. Communicated information is then assimilated simply by summing with the local information. An important point to note is that, after this, the locally available estimates are *exactly* the same as if the data fusion problem had been solved on a single central processor using a monolithic formulation of the conventional Kalman filter.

It is also worth noting the manner in which the control input enters the prediction stage of the information form; through the term $\mathbf{Y}(k \mid k-1)\mathbf{B}(k)\mathbf{u}(k)$. In general $\mathbf{H}_i(k)$ is a function of state which is dependent on control action. Thus, the control input also influences the observed information update.