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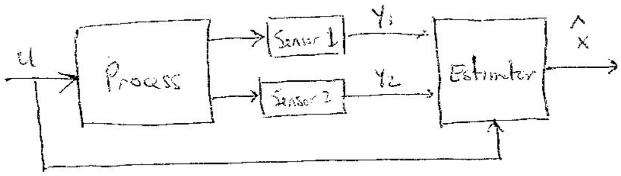
# Lecture 4-1: Sensor Fusion




$$\dot{x} = Ax + Bu + Fv$$
$$y = Cx + w$$

Disturbances: $R_v(t)$
Noise: $R_w(t)$

Problem statement

- How do we combine multiple sensors to get best estimate?
- If different sensors are available at different times, how do we incorporate data?
- How do uncertainties in individual sensor affect the way their data is included?

Start in continuous time:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$$
$$L = PC^T R_w^{-1}$$
$$\dot{P} = AP + PA^T - PC^T R_w^{-1} CP + FR_vF^T$$

Multiple sensors:

$$C = \begin{bmatrix} C_1 \\ \vdots \\ C_q \end{bmatrix} \qquad R_w = \begin{bmatrix} R_{11} & & 0 \\ & \ddots & \\ 0 & & R_{qq} \end{bmatrix} \leftarrow$$ Assumes indep measurements noise on each channel

$$L = PC^T R_w \qquad \cancel{P_1 C^T R_w^{-1} C P} = P\begin{bmatrix} C_1^T \cdots C_q^T \end{bmatrix} \begin{bmatrix} R_{11}^{-1} & & \\ & \ddots & \\ & & R_{qq}^{-1} \end{bmatrix}$$

If $P$ small $\Rightarrow$ current state has good estimate $\Rightarrow$ lower weight of measurement

Weigh individual sensors according to inverse of covariance $\Rightarrow$ accurate sensors have ~~low~~ larger gain

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(2)

Sensor fusion in discrete time (predictor/corrector)

$$\hat{x}[k|k-1] = A\hat{x}[k-1|k-1] + Bu[k]$$
$$P[k|k-1] = P[k-1|k-1] + FR_v[k]F^T$$

Prediction

$$\hat{x}[k|k] = \hat{x}[k|k-1] + L[k](y[k] - C\hat{x}[k|k-1])$$
$$P[k|k] = P[k|k-1] + P[k|k-1]C^T R_\varepsilon^{-1}[k] C P[k|k-1]$$

Correction

$$L[k] = A^T P[k|k] C^T R_\varepsilon^{-1}[k] \qquad R_\varepsilon[k] = CP[k|k]C^T + R_W$$

Again, if $R_W = \mathrm{diag}(R_1, \dots, R_q)$, get inverse covariance weighting, but complicated due to $R_\varepsilon$

## Information Filter

Idea: use inverse covariance instead of covariance

$$\mathcal{I}[k|k] = P^{-1}[k|k] \qquad \text{Information matrix}$$
$$\hat{Z}[k|k] = P^{-1}[k|k]\hat{X}[k|k] \qquad \text{weighted output}$$

$$\Omega_i[k] = C^T R_{W_i}^{-1}[k] C$$
$$\Psi_i[k] = C_i^T R_{W_i}^{-1}[k] C_i \hat{X}[k|k]$$

Claim: correction becomes

$$\left.\begin{aligned} \mathcal{I}[k|k] &= \mathcal{I}[k|k-1] + \sum_{i=1}^{q} \Omega_i[k] \\ \hat{Z}[k|k] &= \hat{Z}[k|k-1] + \sum \Psi_i[k] \end{aligned}\right\} \text{Simple form!}$$

Remarks

1. Information form allows simple addition for correction step. Intuition: add information through additional data

2. Sensor fusion: ~~covariance~~ information content = inverse covariance (for each sensor)

3. Variable rate: incorporate new information whenever it arrives. No data $\Rightarrow$ no ~~#~~ information $\Rightarrow$ prediction update only

Derivation of Information Filter

HW

## Variations on Kalman Filters

1. Converting from continuous time to discrete time

$$\dot{x} = Ax + Bu + Fv$$

$$x(t+h) \approx x(t) + h\dot{x}(t)$$
$$= x(t) + hAx(t) + hBu(t) + hFv(t)$$
$$= (I+hA)\,x(t) + (hB)\,u(t) + (hF)\,v(t)$$
$$\rightarrow x[k+1] = \underbrace{(I+hA)}_{\hat{A}}\,x[k] + \underbrace{(hB)}_{\tilde{B}}\,u[k] + \underbrace{(hF)}_{\tilde{F}}\,v[k]$$

2. Non-white noise sources

$$V \longrightarrow \boxed{\text{Process}} \longrightarrow Y$$

Q. What if V is not white?
A: Spectral factorization

$$\Lambda \longrightarrow \boxed{\text{Filter}} \xrightarrow{V} \boxed{\text{Process}} \longrightarrow Y$$

Estimate state of process <u>and</u> filter

Intuition. noise V is correlated $\Rightarrow$ need an internal model to capture correlation

Constructing model for random process V
- collect samples $V[1], V[2], \ldots, V[N]$ (assume zero mean)
- compute $R_V(\ell) = E\{V[i]\,V[i+\ell]\} = \frac{1}{N-\ell}\sum_{j=1}^{N-\ell} V[j]\,V[j+\ell]$

3. Example: MATLAB

## Derivation of Information Filter

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