CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110

Problem Set #8

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Note: In the upper left hand corner of the first page of your homework set, please put the number of hours that you spent on this homework set (including reading).

Unless otherwise specified, you may use MATLAB or Mathematica as long as you include a copy of the code used to generate your answer.

1. (DFT 6.5) Let $\epsilon := \|W_1 S\|_\infty$ and $\delta := \|CS\|_\infty$, so that ϵ is a measure of tracking performance while δ measures control effort. In a practical design, we would like $\epsilon < 1$ and δ not too large. Derive the following inquality, showing that ϵ and δ cannot both be very small in general: for every point s_0 with $\text{Re} s_0 \geq 0$,

$$|W_1(s_0)| \leq \epsilon + |W_1(s_0)P(s_0)|\delta.$$

2. (DFT 6.7) Let P be a system with transfer function

$$P(s) = \frac{1}{s^2 - s + 4}.$$

We want to find a controller C that achieves the following

$$\begin{aligned}&\text{internal stability}\\ &|S(j\omega)| \leq \epsilon \text{ for } 0 \leq \omega < 0.1\\ &|S(j\omega)| \leq 2 \text{ for } 0.1 \leq \omega < 5\\ &|S(j\omega)| \leq 1 \text{ for } 5 \leq \omega < \infty.\end{aligned}$$

Find a (positive) lower bound on the achievable ϵ.

3. (20 pts) Consider the following model for the pitch dynamics of the Caltech ducted fan:

$$P(s) = \frac{r}{Js^2 + bs + mgl} \qquad \begin{array}{lll} g = 9.8 \text{ m/sec}^2 & m = 1.5 \text{ kg} & b = 0.05 \text{ kg/sec} \\ l = 0.05 \text{ m} & J = 0.0475 \text{ kg m}^2 & r = 0.25 \text{ m} \end{array}$$

(note that these parameters are slightly different than what was used in lecture). We wish to design a controller that satisfied the following: performance specification:

- Steady state error of less than 1%
- Tracking error of less than 5% from 0 to 1 Hz (remember to convert this to rad/sec).
- High frequency disturbance rejection from reference to input (H_{ur}) of at least 10X above 100 Hz.

(a) Write the above specification as one or more weighted sensitivity specifications. Give explicit formulas for the frequency weight(s).

(b) Consider a plant perturbation of 20% variation in the value of r around the nominal value. Design a controller that provides robust performance with respect to this perturbation.

(c) Suppose that the actuator for the system has an unknown time delay of τ seconds where $\tau \leq 0.01$, so that

$$\tilde{P} = Pe^{-\tau s}.$$

Using the bound described on page 48 of DFT (suitably modified), design a compensator the provides robust performance with respect to this perturbation. If you cannot find such a compensator, back off on the performance specification as needed.

(d) Suppose that both the variation in r and the unknown time delay are present at the same time. Derive a single multiplicative uncertainty that covers both of these variations at the same time and design a controller that provides robust performance with respect to this perturbation.

Note: You can find the solutions for all DFT problems on the web if you look hard enough. You are not allowed to use the posted solutions to solve the problems on this homework set.