CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110

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Problem Set #7

Issued: 22 Feb 06
Due: 1 Mar 06

Note: In the upper left hand corner of the first page of your homework set, please put the number of hours that you spent on this homework set (including reading).

Unless otherwise specified, you may use MATLAB or Mathematica as long as you include a copy of the code used to generate your answer.

1. Let $G(s) = \frac{Ke^{-\tau s}}{Ts+1}$ and $K \in [10, 12]$, $\tau \in [0, 0.5]$, $T = 1$. Find a nominal model $G_0(s)$ that is a real, rational stable transfer function, and a real, rational stable weighting function $W(s)$ such that

$$G(s) \in \{G_0(s)(1 + W(s)\Delta(s)) : \Delta \text{ a real, rational stable matrix, } \|\Delta\| \leq 1\}$$

2. This problem shows that the stability margin is critically dependent on the type of perturbation. The setup is a unity-feedback loop with controller $K(s) = 1$ and plant $P_{nom}(s) + \Delta(s)$, where

$$P_{nom}(s) = \frac{10}{s^2 + 0.2s + 1}$$

   (a) Assume $\Delta(s)$ is a stable transfer function. Compute the largest β such that the feedback system is internally stable for all $\|\Delta\|_\infty < \beta$.

   (b) Repeat but with $\Delta \in \mathbb{R}$.

3. (DFT 4.3) For the unity-feedback system with $P(s) = k/s$, does there exist a proper controller $C(s)$ such that the feedback system is internally stable for both $k = +1$ and $k = -1$?

4. (DFT 4.9) Consider the class of perturbed plants of the form

$$\frac{P}{1 + \Delta W_2 P},$$

   where W_2 is a fixed stable weighting function with W_2 strictly proper and Δ is a variable stable transfer function with $\|\Delta\|_\infty \leq 1$. Assume that C is a controller achieving internal stability for P. Prove that C provides internal stability for the perturbed plant if $\|W_2 PS\|_\infty < 1$.

Note: You can find the solutions for all DFT problems on the web if you look hard enough. You are not allowed to use the posted solutions to solve the problems on this homework set.