CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110

Problem Set #6

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Issued: 13 Feb 06
Due: 22 Feb 06 (Wed)

Note: In the upper left hand corner of the first page of your homework set, please put the number of hours that you spent on this homework set (including reading).

Unless otherwise specified, you may use MATLAB or Mathematica as long as you include a copy of the code used to generate your answer.

1. Show the the following are norms on the linear space of piecewise continuous signals on $(-\infty, \infty)$:

   (a) $\|u\|_2 = \left(\int_{-\infty}^{\infty} |u(t)|^2\, dt\right)^{1/2}$

   (b) $\|u\|_\infty = \sup_t |u(t)|$

   (Note: if you have had Ma 108, correct the above statement so that it is rigorously true.)

2. Consider a second order system with transfer function

$$\widehat{G}(s) = \frac{1}{s^2 + 2\omega_n \zeta s + \omega_n^2}$$

   (ω_n is the natural frequency of the system and ζ is the damping ratio). Setting $\omega_n = 1$, write a short MATLAB program to generate a plot of the ∞-norm and the 2-norm as a function of the damping ratio $\zeta > 0$. (Hint: use DFT, 2.6 for computing the 2-norm and the `max` function in MATLAB for the ∞-norm.)

3. Consider the system given by

$$P(s) = \frac{1}{2s+5},\ C(s) = \frac{k}{s+1},\ F(s) = 1.$$

   Find the least positive k such that the following are true

   (a) The feedback system is internally stable.

   (b) The output y has less than 5% error at steady state with respect to a step input.

   (c) $\|y\|_\infty \leq 0.2$ for all $d(t)$ such that $\|d\|_2 \leq 1$ when $r = n = 0$.

4. Consider the cart pendulum system with the pendulum hanging *down*. The dynamics describing how the position of the cart depends on the applied force is given by the transfer function

$$P(s) = \frac{ls^2 + g}{Mls^4 + bls^3 + (M+m)gs^2 + bgs}$$

   $M = 0.5$ kg, $m = 0.2$ kg, $l = 0.3$ m, $b = 0.1$ N/m/sec, $g = 9.8$ m/sec^2

   We wish to design a control law that satisfies the following specification

- 0.1% steady state error
- Position (x) tracking within 10% up to 0.05 Hz
- Disturbance rejection of 10X for all disturbances (taken to be at the reference input) above 10 Hz

(a) Determine a performance weight $W_1(s)$ such that the first two elements of the specification can be written as $\|W_1 S\|_\infty \leq 1$.

(b) Design a control law that satisfies the specification and plot the resulting weighted sensitivity.

(c) Compute the gain margin for your compensator.