CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110b

R. M. Murray | **Problem Set #5** | Issued: 6 Feb 06
Winter 2006 | | Due: 13 Feb 06

Note: In the upper hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

Unless otherwise specified, you may use MATLAB or Mathematica as long as you include a copy of the code used to generate your answer.

1. (Friedland 11.1) A compensator based on a Kalman filter is to be designed for the instrument servo problem of HW #2, problem 2 (Friedland 9.6). Only the position error e is measured, so that

$$y = e + w$$

where w is white noise with spectral density W. The only excitation noise present occurs at the control input, so that the angular velocity is

$$\dot{\omega} = -\alpha\omega + \beta u + v$$

where v is white noise of spectral density V.

   (a) Find and plot the Kalman filter gains and corresponding closed-loop poles as a function of the signal-to-noise ratio V/W.

   (b) Using the optimized gains determined in HW #2, problem 2(a) (Friedland 9.6a), find the gain margin for several values of q_1^2 and several values of V/W. Tabulate the results as functions of q_1^2 and V/W

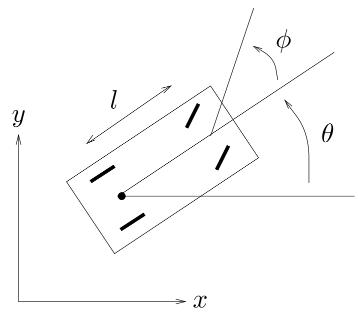
2. (Friedland 11.2) Consider the dynamics for the inverted pendulum on a motor-driven cart from HW #2 and HW #4, for which you are to build a full order Kalman filter as an observer.

   (a) Assume that the only excitation noise present is coincident with the control and has spectral density v^2, and that the only observation is cart displacement which is measured through white noise of spectral density w^2. Plot the Kalman filter gains and poles as a function of the ratio v^2/w^2 for $1 \leq v^2/w^2 \leq 10^6$ (use at least 6 points in your plot).

   (b) Using the gains from HW #2, problem 3(a) (Friedland 9.10a) with $r^2 = 0.01$, determine the compensator transfer functoin $D(s)$ for the range of v^2/w^2 in part (a).

   (c) For $r^2 = 0.01$ and $v^2/w^2 = 10^{-3}$ in part (b), determine the range of gains K for which the closed loop system with loop transfer function $KD(s)P(s)$ is stable, where $P(s)$ is the transfer function for the plant.

3. (Alice) Consider the problem of estimating the position of an autonomous mobile vehicle using a GPS receiver and an IMU (inertial measurement unit). The dynamics of the vehicle are given by



$$\dot{x} = \cos\theta\, v$$
$$\dot{y} = \sin\theta\, v$$
$$\dot{\theta} = \frac{1}{\ell}\tan\phi\, v,$$

We assume that the vehicle is disturbance free, but that we have noisy measurements from the GPS receiver and IMU and an initial condition error.

In this problem we will utilize the full form of the Kalman filter (including the $\dot{P}$ equation).

(a) Suppose first that we only have the GPS measurements for the xy position of the vehicle. These measurements give the position of the vehicle with approximately 1 meter accuracy. Model the GPS error as Gaussian white noise with $\sigma = 1.2$ meter in each direction and design an optimal estimator for the system. Plot the estimated states and the covariances for each state starting with an initial condition of 5 degree heading error at 10 meters/sec forward speed (i.e., choose $x(0) = (0, 0, 5\pi/180)$ and $\hat{x} = (0, 0, 0)$).

(b) An IMU can be used to measure angular rates and linear acceleration. Assume that we use a Northrop Grumman LN200 to measure the angular rate $\dot{\theta}$. Use the datasheet on the course web pgae to determine a model for the noise process and design a Kalman filter that fuses the GPS and IMU to determine the position of the vehicle. Plot the estimated states and the covariances for each state starting with an initial condition of 5 degree heading error at 10 meters/sec forward speed.

Note: be careful with units on this problem!