CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110
Problem Set #16

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Due: 12 March 03

Note: In the upper left hand corner of the first page of your homework set, please put the number of hours that you spent on this homework set (including reading).

Unless otherwise specified, you may use MATLAB or Mathematica as long as you include a copy of the code used to generate your answer.

1. (DFT 7.2) Assume that we have a plant with relative degree 1 and multiplicative uncertainty with weighting function
$$W_2(s) = \frac{10s+1}{20(0.01s+1)}.$$
For the performance weight, we choose
$$|W_1(j\omega)| = \begin{cases} a & \text{if } 0 \leq \omega \leq 1 \\ 0 & \text{otherwise,} \end{cases}$$
where a is unspecified. Following the approach given in Example 1 of Section 7.3 of DFT, determine what performance level a you can achieve.

2. (FPE 8.5) The following transfer function is a lead network designed to add about 60 degrees of phase at $\omega_1 = 3$ rad/sec:
$$H(s) = \frac{s+1}{0.1s+1}$$
   (a) Assume a sampling period of $T = 0.25$ sec and compute the pole locations of the digital implementation of H using (1) the forward difference approximation and (2) Tustin's method. For each case, compute the amount of phase lead provided by the network at $z_1 = e^{j\omega_1 T}$.

   (b) Using a log-log scale for the frequency range $\omega = 0.1$ to $\omega = 100$ rad/sec, plot the magnitude Bode plots for each of the equivalent digitial sytsems you found in part (a) and compare with $H(s)$. (Hint: magnitude bode plots are given by $|H(z)| = |H(e^{j\omega T})|$.

   Note: you may need to do a bit of reading to sort out how to do this problem since I covered it very quickly in class. In addition to the handout from class (available outside 109 Steele if you weren't there), you are encouraged to look at the textbooks on reserve in the Sherman Fairchild Library.

3. (FPE 8.11) Consider the following discrete-time system in state-space form:
$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u(k).$$
Use state feedback to relocate all of the sytems poles to 0.5.

Note: You can find the solutions for all DFT problems on the web if you look hard enough. You are not allowed to use the posted solutions to solve the problems on this homework set.