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CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110
Problem Set #10

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Due: 21 Jan 03

Note: In the upper left hand corner of the first page of your homework set, please put the number of hours that you spent on this homework set (including reading).

Unless otherwise specified, you may use MATLAB or Mathematica as long as you include a copy of the code used to generate your answer.

1. The output $c(t)$ in a position-control system is governed by

$$J\ddot{c} = u,$$

where $u(t)$ is applied force.

   (a) Write down a state space realization (find A and B).

   (b) Use the matrix Riccati equation to find the feedback control law minimizing

$$\int_0^\infty (c^2 + qu^2)dt.$$

   (c) Show that the optimal control system has damping ratio $\frac{1}{\sqrt{2}}$.

   (d) What is the corresponding optimal value of natural frequency?

2. (Friedland 9.6) Consider the dynamics of a DC motor driving an inertial load (see Friedland, page 231 for a picture):

$$\dot{\theta} = \omega$$
$$\dot{\omega} = -\alpha\omega + \beta u$$

where θ is the angular position of the load, ω is the angular velocity, u is the applied voltage, and α and β are constants that depend on the physical parameters of the motor and load. For this problem, let $\alpha = 1$ and $\beta = 3$.

   (a) Let $e = \theta - \theta_d$. For the performance criterion

$$V = \int_t^\infty (q_1^2 e^2 + u^2)\, d\tau$$

   find and tabulate the control gains and corresponding closed-loop poles for $q_1 = 0.1, 1, 10$.

   (b) Plot the transient response (e as a function of t) for the inital error of unity for the values of q_1 in part (a). (Note: you should use the MATLAB `initial` function to get the transient response to an initial error. Set the initial condition for θ appropriately.)

   (c) In addition to weighting the position error it is also desired to limit the velocity by using a performance criterion

$$V = \int_t^\infty (q_1^2 e^2 + q_2^2 \dot{e}^2 + u^2)\, d\tau.$$

   For the values of q_1^2 used in part (a) and $q_2^2 = 0.1q_1^2, q_1^2, 10q_1^2$ find the control gains and corresponding closed loop poles.

(d) Plot the transient response as in part (b) for a range of q_1^2 and q_2^2 (you need not include all 9 plots; just the "interesting" ones). Compare the results with those of part (b). Are the results as expected?

3. (Friedland 2.1, 3.6, 7.2, 9.10) Consider the motor-driven inverted pedulum on a cart from problem 3 of homework set #9. We wish to optimize the gains using a preformance criterion of the form

$$V = \int_t^\infty \left(q_1^2 x_1^2 + q_3^2 x_3^2 + r^2 u^2 \right) \, dt$$

A pendulum angle much greater than 1 degree = 0.017 rad would be precarious. Thus a heavy weighting error on $\theta = x_3$ is indicated: $q_3^2 = 1/(0.017)^2 \approx 3000$. For the physical dimensions of the system, a position error of the order of 10 cm = 0.1 m is not unreasonable. Hence $q_1^2 = 1/(0.1)^2 = 100$.

(a) Using these values of q_1^2 and q_3^2, determine and plot the gain matrices and corresponding closed loop poles as a functoin of the control weighting parameter r^2 for $0.001 < r^2 < 50$.

(b) Repeat part (a) for a heavier weighting: $q_1^2 = 10^4$ on the cart displacement.

4. (FPE 7.55) Consider a system with state matrices

$$A = \begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 3 \end{bmatrix}$$

We wish to track a reference signal $r(t)$ using state feedback plus feedforward compensation.

(a) Using a feedback of the form $u(t) = -Kx(t) + Nr(t)$ where N is a nonzero scalar, move the poles to $-3 \pm 3j$.

(b) Choose N so that if r is a constant, the system has zero steady-state error; that is $y(\infty) = r$.

(c) Show that if A changes to $A + \delta A$ where δA is an arbitrary 2×2 matrix, then your choice of N in part (b) will no longer make $y(\infty) = r$. Therefore the ystem is not robust to changes to the system parameters in A.

(d) The system steady-state error performance can be made robust by augmenting the system with an integrator and using unity feedback: that is by setting $x_i = r - y$ where x_i is the state of the integrator. To see this, first use state feedback of the form $u = -Kx - K_i x_i$ so that the poles of the augmented systems are at $-3, -2 \pm j\sqrt{3}$.

(e) Show that the resulting system will yield $y(\infty) = r$ no matter how the matrices A and B are changed, as long as the closed-loop system remains stable.