CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101

Problem Set #8

R. M. Murray
Fall 2015

Issued: 23 Nov 2015
Due: 4 Dec 2015, 2 pm

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

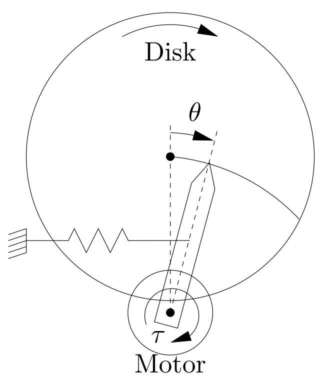
Reminder: You get two grace periods of no more than 2 days each for late homework. After that, late homework will not be accepted without a note from the dean or the health center.

1. Consider the system in Example 12.1, where the process and controller transfer fucctions are given by

$$P(s) = 1/(s-a), \qquad C(s) = k(s-a)/s.$$

Choose the parameters $a = -1$ and compute the time (step) and frequency responses for all the transfer functions in the Gang of Four for controllers with $k = 0.2$ and $k = 5$.

2. The figure below shows a simple mechanism for positioning a disk drive read head and the associated equations of motion:



$$J\ddot{\theta} = -b\dot{\theta} - kr\sin\theta + \tau_m$$
$$\dot{\tau}_m = -a(\tau_m - u)$$

The system consists of a spring-loaded arm that is driven by a small motor. The motor applies a force against the spring and pulls the head across the platter. The input to the system is the desired motor torque, u. In the diagram above, the force exerted by the spring is a nonlinear function of the head position due to the way it is attached. All constants are positive.

We wish to design a controller that holds the drive head at a given location θ_d.

(a) Show that the transfer function of the process can be written as

$$P(s) = \frac{a}{a+s} \cdot \frac{k}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

(b) Assume that the system parameters are such that $K = 0.001$, $\zeta = 0.5$, $\omega_n = 0.1$ and $a = 1$. Design a compensator that provides tracking with less than 10% error up to 1 rad/s and has a phase margin of 60°.

(c) Plot the Nyquist plot for the (open loop) system corresponding to your control design and compute the gain margin, phase margin and stability margin.

(d) Compute and plot the Gang of Four for your system. Comment on any of the transfer functions that might lead to large errors or control signals and indicate the conditions under which this might occur.

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1. Consider a control system with process and controller dynamics given by

$$P(s) = \frac{1}{s(s+c)} \qquad C(s) = k.$$

where $k > 0$.

(a) Show that the closed loop (tracking) response of the system can be written as

$$G(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

and give formulas for ζ and ω_0 in terms of c and k.

(b) Show that the phase margin for the system is given by

$$\varphi_m = \tan^{-1}\Big(\frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4} - 2\zeta^2}}\Big)$$

(Hint: compute the frequency at which $|L(i\omega)| = 1$ and then find the phase at that frequency.)

(c) Plot ζ as a function of the phase margin (φ_m on the x-axis, ζ on the y axis). Use this plot to determine the phase margin at which a second order system would be critically damped (defined in Chapter 7 of the text).

(d) Combine the result of part 1b with the formula in Table 7.1 to plot the overshoot for a second order system as a function of the phase margin. Your plot should look similar to the plot shown in lecture.

2. Consider the problem of stabilizing the orientation of a flying insect, modeled as a rigid body with moment of inertia $J = 0.41$ and damping constant $D = 1$.[1] We assume there is a small delay $\tau = 0.01$ sgiven by the neural circuitry that implements the control system. The resulting transfer function for the system is taken to be

$$P(s) = \frac{1}{Js^2 + Ds}e^{-\tau s}.$$

[1] Based loosely on "Biologically Inspired Feedback Design for Drosophila Flight", M. Epstein, S. Waydo, S. B. Fuller, W. Dickson, A. Straw, M. H. Dickinson and R. M. Murray, 2007 American Control Conference.

(a) Suppose that we can measure the orientation of the insect relative to its environment and we wish to design a control law that gives zero steady state error, less than 10% tracking error from 0 to 0.5 Hz and has an overshoot of no more than 10%. Convert these specifications to appropriate bounds on the loop transfer function and sketch the resulting constraints on a Bode plot. (Hint: Try using problem 1 to convert the overshoot requirement to a phase margin requirement.)

(b) Using a lead compensator, design a controller that meets the specifications in part (a). Provide whatever plots are required to verify that the specification is met. You may use a Padé approximation for the time delay, but make sure that it is a good approximation over a frequency range that includes your gain crossover frequency.

(c) Plot or sketch the Nyquist plot corresponding to your controller and the process. You can again use a Padé approximation for the time delay. Show the gain and phase margin on your plot.

(d) Plot the "gang of 4" for the system. If any of the magnitudes of the closed loop transfer functions are substantially greater than one in some frequency range, explain the consequences of this in terms of one of the input/output responses of your system. (You are not required to fix these problems.)

(e) Extra credit: genetically modify a fly to implement your controller, using the fly visual system as your input.

3. Consider the dynamics of the magnetic levitation system from lecture. The transfer function from the electromagnet input voltage to the IR sensor output voltage is given by

$$P(s) = \frac{k}{s^2 - r^2}$$

with $k = 4000$ and $r = 25$ (these parameters are slightly different than those used in the MATLAB files distributed with the lecture).

(a) Design a stabilizing compensator for the process, assuming unity feedback. Compute the poles and zeros for the loop transfer function and for the closed loop transfer function between the reference input and measured output.

(b) Plot the Nyquist plot corresponding to your compensator and the process dynamics, and verify that the Nyquist criterion is satisfied.

(c) Plot the log of the magnitude of the sensitivity function, $\log |S(j\omega)|$, versus ω on a *linear* scale and numerically verify that the Bode integral formula is (approximately) satisfied. (Hint: you can do the integration numerically in MATLAB, using the `trapz` function. Make sure to choose your frequency range sufficiently large.)