CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101

Problem Set #7

R. M. Murray — Fall 2015

Issued: 16 Nov 2015
Due: 25 Nov 2015, 2 pm

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

Reminder: You get two grace periods of no more than 2 days each for late homework. After that, late homework will not be accepted without a note from the dean or the health center.

1. Consider a second-order process of the form

$$P(s) = \frac{k}{s^2 + 2\zeta\omega_0 s + \omega_0^2} \qquad k, \zeta, \omega_0 > 0.$$

In this problem we will explore various methods for designing a PID controller for the system.

(a) (Eigenvalue assignment) Suppose that we want the closed loop dynamics of the system to have a characteristic polynomial given by

$$p(s) = s^3 + a_1 s^2 + a_2 s + a_3.$$

Compute a formula for the controller parameters of a PID controller (k_p, k_i and k_d) that gives the desired closed loop response.

(b) (Eigenvalue assignment) Let the process parameters be given by $k = 1$, $\zeta = 0.5$ and $\omega_0 = 2$. Using the formulas from part (a), compute a feedback control law that places the closed loop poles of the system at $\lambda = \{-1, -2 \pm j\}$. Plot the step response and frequency response for the closed loop systems, and compute the gain and phase margins for your design.

(c) (Ziegler–Nichols step response) Using the same process parameters as above, plot the step response for the corresponding system and use one of the Ziegler–Nichols rules to design a PID controller. Plot the closed loop step response and frequency response for your design, and compute the gain and phase margins.

2. For the control systems below, design a P, PI, PD or PID control law that stabilizes the system, gives less than 1% error at zero frequency and gives at least 30° phase margin. You may use any method (loop shaping, Ziegler–Nichols, eigenvalue assignment, etc) and you only need to design one type of controller (as long as it meets the specification), but be sure to explain why you chose your controller, and include appropriate plots or calculations showing that all specifications are met. For the closed loop system, determine the steady-state error in response to a step input and the maximum frequency for which the closed loop system can track with less than 25% error.

(a) Drug administration/compartment model (Section 4.6):

$$P(s) = \frac{1.5s + 0.75}{s^2 + 0.7s + 0.05}$$

(b) Disk drive read head positioning system:

$$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10}$$

CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 110a

R. M. Murray	**Problem Set #7**	Issued: 16 Nov 2015
Fall 2015		Due: 25 Nov 2015, 2 pm

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

Reminder: You get two grace periods of no more than 2 days each for late homework. After that, late homework will not be accepted without a note from the dean or the health center.

1. For the control systems below, design a P, PI, PD or PID control law that stabilizes the system, gives less than 1% error at zero frequency and gives at least 30° phase margin. You may use any method (loop shaping, Ziegler–Nichols, eigenvalue assignment, etc) and you only need to design one type of controller (as long as it meets the specification), but be sure to explain why you chose your controller, and include appropriate plots or calculations showing that all specifications are met. For the closed loop system, determine the steady-state error in response to a step input and the maximum frequency for which the closed loop system can track with less than 25% error.

   (a) Drug administration/compartment model (Section 4.6):

   $$P(s) = \frac{1.5s + 0.75}{s^2 + 0.7s + 0.05}$$

   (b) Disk drive read head positioning system:

   $$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10}$$

2. Consider a first-order system with a PI controller given by

   $$P(s) = \frac{b}{s+a} \qquad C(s) = k_p\Big(1 + \frac{1}{T_i s}\Big).$$

   In this problem we will explore how varying the gains k_p and T_i affect the closed loop dynamics.

   (a) Suppose we want the closed loop system to have the characteristic polynomial

   $$s^2 + 2\zeta\omega_0 s + \omega_0^2.$$

   Derive a formula for k_p and T_i in terms of the parameters a, b, ζ and ω_0.

   (b) Suppose that we choose $a = 1$, $b = 1$ and choose ζ and ω_0 such that the closed loop poles of the system are at $\lambda = \{-20 \pm 10j\}$. Compute the resulting controller parameters k_p and T_i and plot the step and frequency responses for the system.

(c) Using the process parameters from part (b) and holding T_i fixed, let k_p vary from 0 to ∞ (or something very large). Plot the location of the closed loop poles of the system as the gain varies. You should plot your results in two different ways:

- A pair of plots showing the real and imaginary parts of the poles as a function of the gain k_p, similar to Figure 4.18a in the text.
- A parametric plot, showing the location of the eigenvalues on the complex plane, as k_p varies. Label the gains at which any interesting features in this plot occur. (This type of plot is called a *root locus* diagram.)

You may find it convenient to use the `subplot` command in MATLAB so that you can present all of your results in a single figure.

3. In this problem we will design a PID compensator for a vectored thrust aircraft (see Example 2.9 in the text for a description). Use the following transfer function to represent the dynamics from the lateral input to the roll angle of the aircraft:

$$P(s) = \frac{r}{Js^2 + cs + mgl} \qquad \begin{array}{lll} g = 9.8 \text{ m/s}^2 & m = 1.5 \text{ kg} & c = 0.05 \text{ kg/s} \\ l = 0.05 \text{ m} & J = 0.0475 \text{ kg m}^2 & r = 0.25 \text{ m} \end{array}$$

(these parameters correspond to a laboratory-scale experiment that we have a Caltech). Design a feedback controller that tracks a given reference input with the following specifications:

- Steady-state error of less than 1%
- Tracking error of less than 5% from 0 to 1 Hz (remember to convert this to rad/s).
- Phase margin of at least 30°.

(a) Plot the open loop Bode plot for the system and mark on the plot the various frequency domain constraints in the above specification, as we did in class.

(b) Design a compensator for the system that satisfies the specification. You should include appropriate plots or calculations showing that all specifications are met.

(c) Plot the step and frequency response of the resulting closed loop control. For the step response, compute the steady-state error, rise time, overshoot and settling time of your controller.

(Hint: you may not need all of the terms in a PID controller.)

4. Consider a PI controller of the form $C(s) = 1 + 1/s$ for a process with input that saturates when $|u| > 1$, and whose linear dynamics are given by the transfer function $P(s) = 1/s$. Simulate the response of the system to step changes in the reference signal of magnitude 1, 1.5 and 3. Repeat the simulation when the windup protection scheme in Figure 11.11 is used.

.

Note: Use $k_t = 1$ and steps of magnitude 1, 1.5 and 3 to demonstrate the effect of the anti-windup compensation.