CDS 101/110 Recitation

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Agenda

› Administrative Info

› Overview of HW8

› Useful MATLAB commands

› Designing Controllers

Administrative Info

- HW7 due Today
- HW8 already available, due next Friday (12/4)
- Office hour next week:
  - Wednesday (12/2) 3-4pm,   ANB 243
  - Thursday (12/3) 7-9pm,   ANB 106
- Final available on 12/4, hard deadline on 12/11
- Review class by Richard on 12/4

Overview of Homework

- CDS 101:
  - Problem 1: Step/frequency response, Gang of Four
  - Problem 2: Compensator design + Verification
- CDS 110:
  - Problem 1: Overshoot vs Phase Margin for second order system
  - Problem 2 & 3: Compensator design + Verification

CDS 101 Problem 1

› To compute step response from frequency response:

  – You don't have to find the analytic expression
  – Just use `step()` function from MATLAB

CDS 101 Problem 2

- (a) the location θ is generally small, so can take linearization around $\theta=0$
- (b) Hint: you may want to try a second order controller
- (c) to find the stability margin,
  - Simply plot $||L+1||$ versus s and look for the minimal value
  - Or derive the analytic formula for $||L+1||$ and minimize over s
- (d) you should explicitly state which transfer functions (if any) have high gain at which frequencies, and the possible effects

CDS 110 Problem 1

- (b) Phase of $a+bi$ is
  - $\phi=\tan^{-1}(b/a)$
- (c) Critical damping:
  - $\zeta=1$
  - Hint: try solve for ζ^2 first.
- (d) The overshoot formula is
  - $M_p=e^{-\pi\zeta/\sqrt{1-\zeta^2}}$
  - Compare to the lecture notes

CDS 110 Problem 2

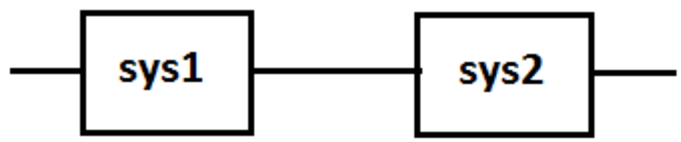
- (a) Note time delay does not change the overshoot (why?)
  - Can use problem 1 to convert overshoot requirement to phase margin requirement
- (b) Padé approximation: `[num,den]=pade(T,n)`
  - Choose the right order so that it gives good approximation around the crossover frequency
- (d) Again, you should explicitly state which transfer functions (if any) have high gain at which frequencies, and the possible effects
- (e) You are strongly encouraged to complete this part, which boosts your grade to A+ directly

CDS 110 Problem 3

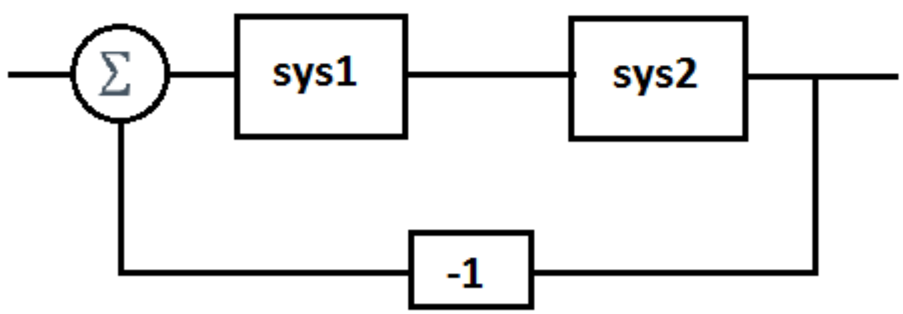
- (a) Unity feedback
  - there is no block on the feedback path
- (c) The Bode integral formula will be covered next Monday
  - Numerical integration: `trapz`

Useful MATLAB commands

- `margin(sys)`: Bode plot + gain/phase margin
- `series(sys1,sys2)`: Concatenate `sys1` and `sys2`



- `feedback(sys1,sys2)`: Concatenate `sys1` and `sys2`, and feed `sys2` back to `sys1`



Useful MATLAB commands

- `[den,num]=pade(T,n)`: n-th order Padé approximation of e^{-Ts}
- `trapz(x,y)`: Numerical integration
  - Example: to compute $\int_0^1 x^2 \, dx$
    - `x = 0:0.1:1`
    - `y = x.^2`
    - `trapz(x,y)`
  - `x` specifies the stepsize: smaller stepsize gives better approximation

Designing Controllers

- Lead Compensator:
  - Key idea: we are "shaping" the loop by adding a pole-zero pair
  - This corrects the phase, and also the gain
  - General form:

    $C(s)=k(s+a)/s+b$
  - k only affects the magnitude plot
  - Zero occurs before the pole ($|a|<|b|$)
    - Zero increases the slopes of the magnitude and the phase plots
    - Pole decreases the slopes of the magnitude and the phase plots

Designing Controllers

- Typical loop constraints
  - High gain at low frequency
    - Good tracking, disturbance rejection at low frequencies
  - Low gain at high frequency
    - Avoid amplifying noise
  - Sufficiently high bandwidth
    - Good rise/settling time
  - Shallow slope at crossover
    - Sufficient phase margin for robustness, low overshoot

Designing Controllers

- Quick Guidelines:
  - Figure out the behavior of the current loop transfer function (the one without controller)
  - Figure out based on the bounds what behavior it SHOULD have
  - Determine the values of k, a and b based on the loop constraints

Example

› Say we have a process

$$P(s)=1/(1+s)^3$$

and we want to design a compensator such that we have 10% tracking error from 0 to 1 rad/s and 1% steady state error.

Example

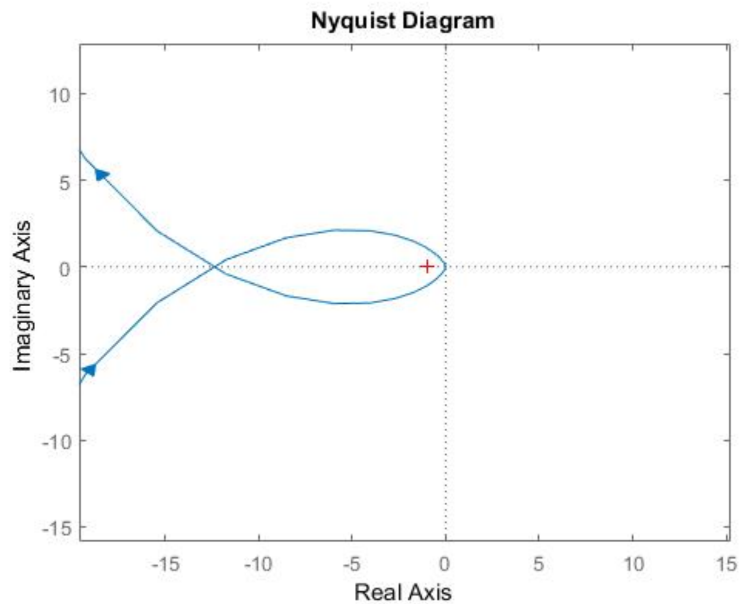
› We first achieve 1% steady state error (assuming $r=1$).
  – Recall the steady error is
  $e_{ss} = \left| \frac{1}{1+L(0)} \right|$
  – Thus we need to require $L(0) \geq 100$.

Example

- 10% tracking error from 0 to 1 rad/s
  - 10% tracking error for a frequency s translates to
  $|1/1+L(s)| \leq 10\% = 0.1$
  - Approximately, we have a bound
  $|L(s)| \geq 10$
  - These requirements tell you how to pick k.

Example

› Since $|P(1)| \approx 0.35$, let's try $k=100$ without adding zero/pole then….

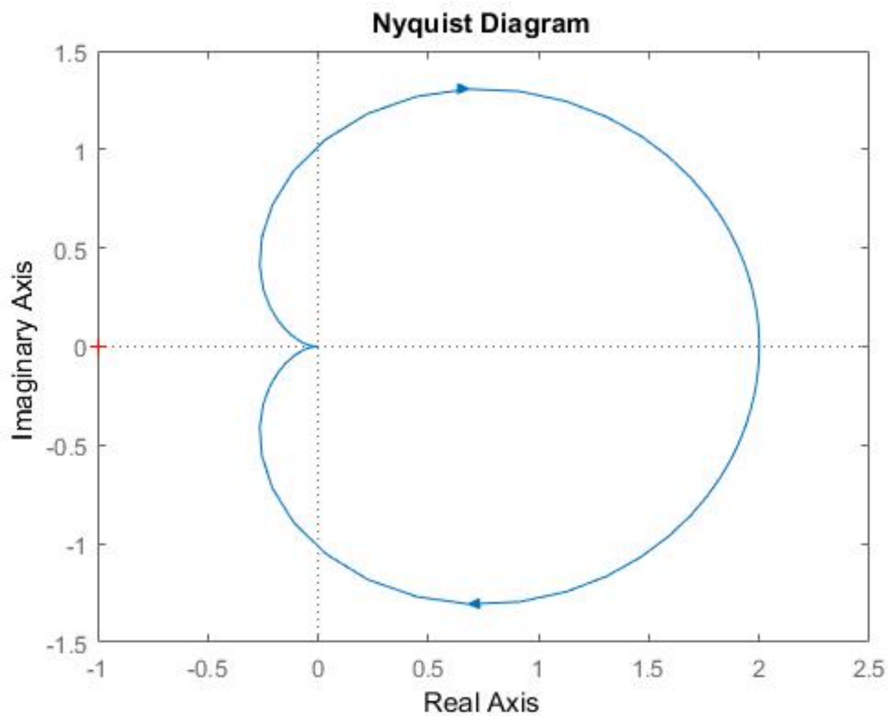


› System unstable!

Example

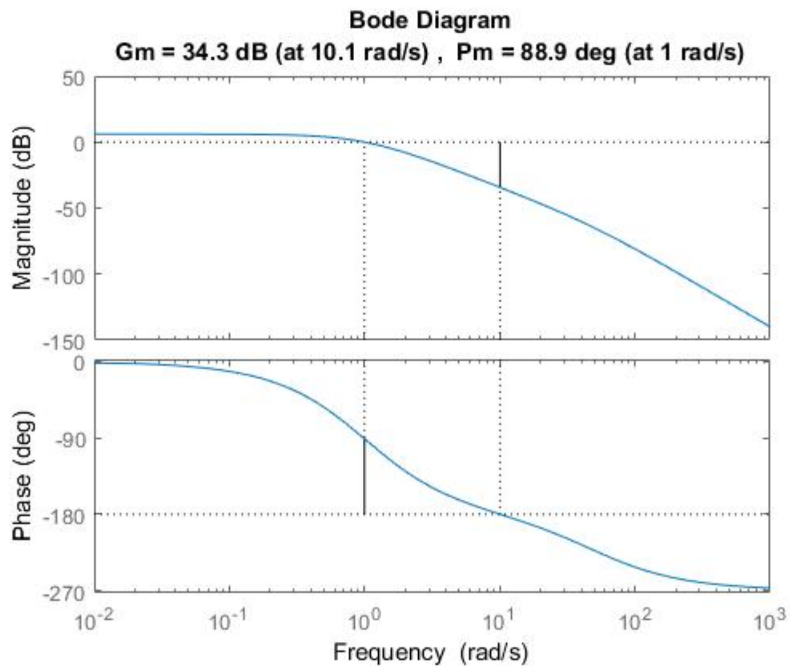
› Try shallow the slope at gain crossover frequency

› Choose a=1, $b = 50$



Example

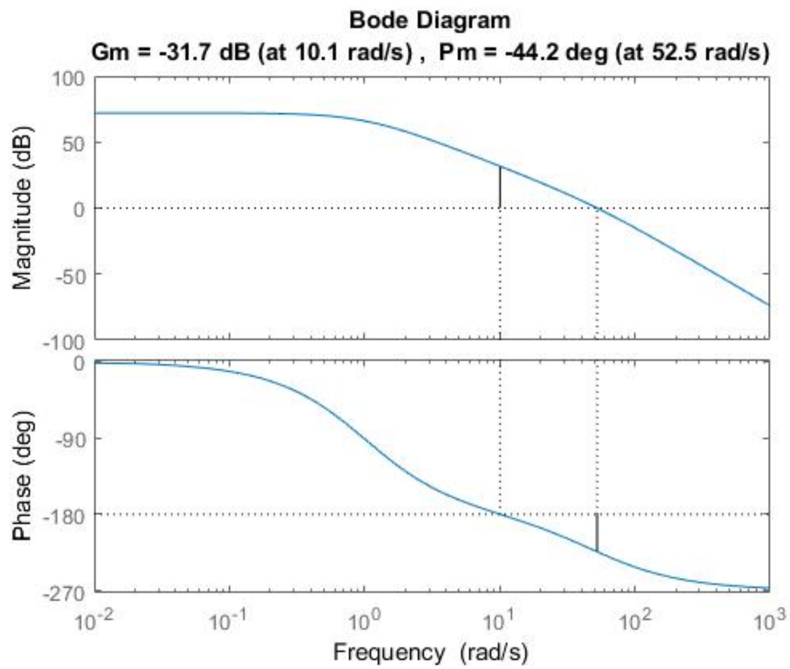
› However….



› Magnitude not right

Example

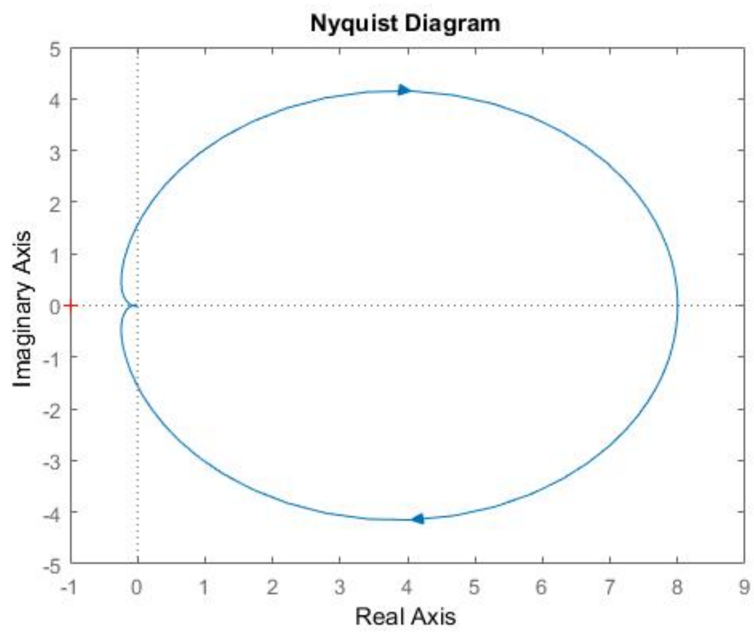
› We then choose a larger gain, say $k=200000$



› Seems ok, but system unstable again….

Example

› Let's try a second order compensator then…



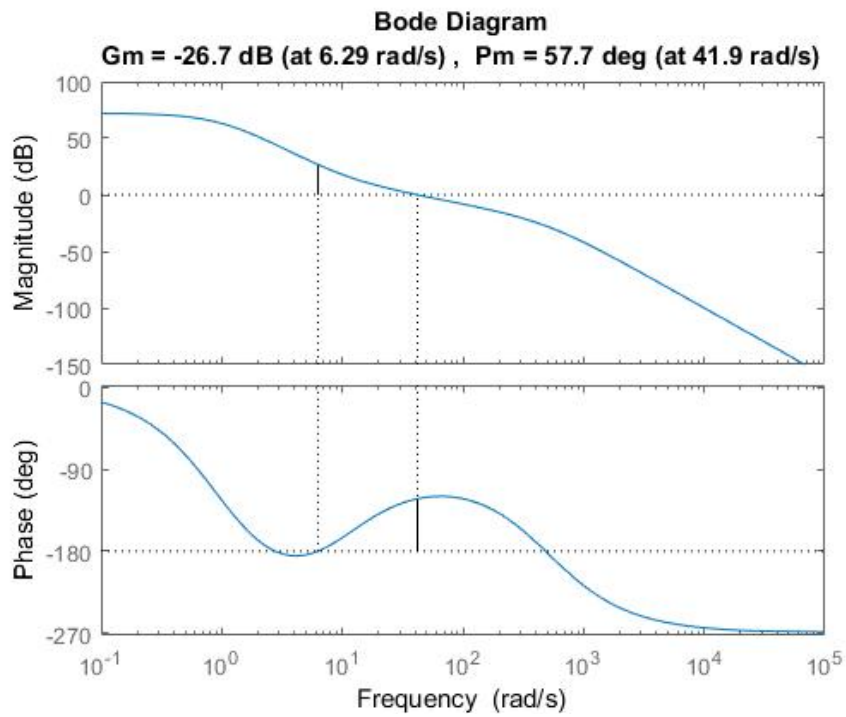
› Zero frequency magnitude not right…

Example

- We want to increase k. But this shifts the crossover frequency
- Need to choose other values of a and b

Example

› After several trials, we find k=100000000, a = 10, b=500 works



Example

- After choosing k, a and b, you should always verify the compensator really works by
  - Looking at the Nyquist plot
  - Inspect the step/frequency responses of Gang of Four
- This might be a little bit of trial-and-error. BE SURE YOU EXPLAIN WHY YOU CHOSE THE VALUES.

Questions?