CDS 101/110: Lecture 9-1
Frequency Domain Design

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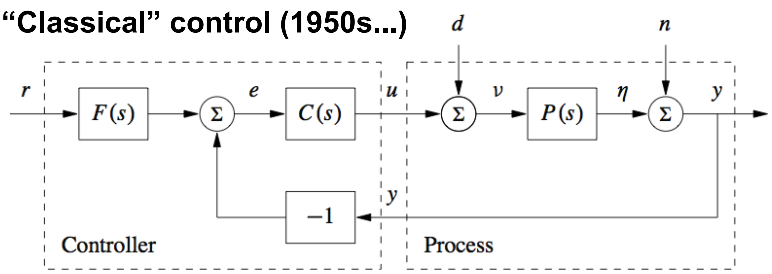
Goals:
- Review canonical control design problem / std performance measures
- Show how to use “loop shaping” to achieve a performance specification
- Work through a simple example of a control design problem

Reading:
- Åström and Murray, Feedback Systems, Ch 12

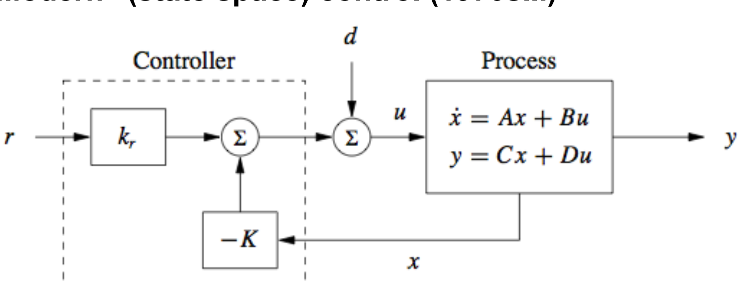
Design Patterns for Control Systems

“Classical” control (1950s...)



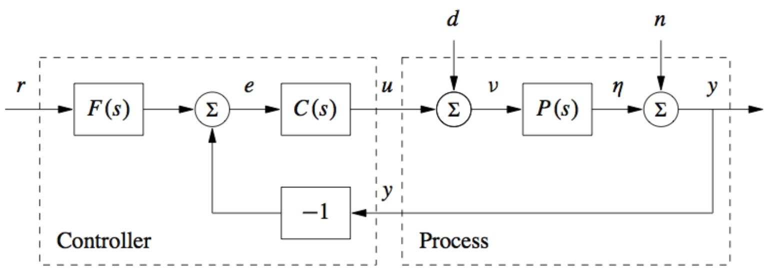
- Reference input shaping
- Feedback on output error
- Compensator dynamics shape closed loop response
- *Uncertainty* in process dynamics $P(s)$ + external disturbances (d) & noise (n)
- Goal: output y(t) should track reference trajectory r(t)
- Design typically done in “frequency domain” (second half of CDS 101/110)

“Modern” (state space) control (1970s...)



- Assume dynamics are given by linear system, with known A, B, C, D matrices
- Measure the state of the system and use this to modify the input
- $u = -K x + k_r r$
- Goal unchanged: output y(t) should track reference trajectory r(t) [often constant]

Input/Output Control Design Specifications



Keep track all input/output transfer functions

- Keep error small for all reference signals r
- Attenuate effect of sensor noise n and disturbances d
- Avoid large input cmds u

$$\begin{bmatrix}\eta \\ y \\ u\end{bmatrix} = \begin{bmatrix} \frac{P}{1+PC} & -\frac{PC}{1+PC} & \frac{PCF}{1+PC} \\ \frac{P}{1+PC} & \frac{1}{1+PC} & \frac{PCF}{1+PC} \\ -\frac{PC}{1+PC} & -\frac{C}{1+PC} & \frac{CF}{1+PC} \end{bmatrix} \begin{bmatrix} d \\ n \\ r \end{bmatrix}$$

Design represents a tradeoff between the quantities

- Keep L=PC large for good performance (H_{er} << 1)
- Keep L=PC small for good noise rejection ($H_{\eta n}$ < 1)

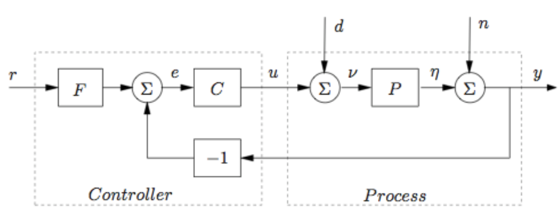
***F(s)* = 1: Four unique transfer functions define performance ("Gang of Four")**

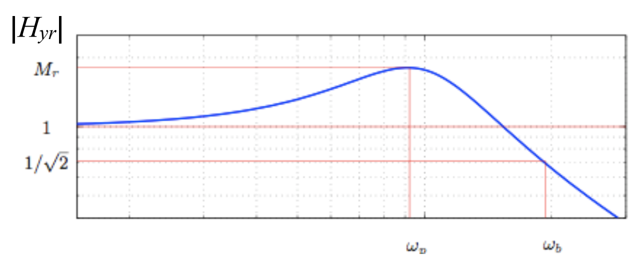
- Stability is always determined by 1/(1+PC) assuming stable process & controller
- Numerator determined by forward path between input and output

More generally: 6 primary transfer functions; simultaneous design of each

- Controller C(s) enters in multiple places ⇒ hard to understand tradeoffs

Frequency Domain Specifications





Specifications on the open loop transfer function (L)

- Gain crossover frequency, ωgc, is the lowest frequency at which loop gain = 1
- Gain margin, gm, is the amount the loop gain can be increased before instability
- Phase margin, φm, is amount of phase lag required to generate instability

Specifications on closed loop frequency response (eg Hyr, Hyd, etc)

- Resonant peak, Mr, is the largest value of the frequency response
- Peak frequency, ωp , is the frequency where the maximum occurs
- Bandwidth, ωb, is the frequency where the gain has decreased to 1/√2

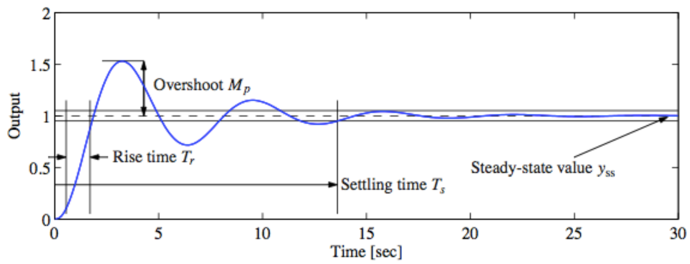
Basic idea: convert specs on closed loop to specs on open loop

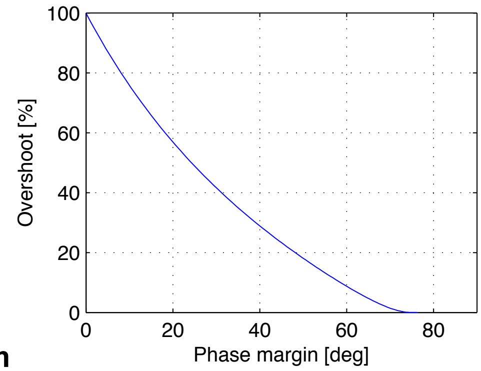
- Bandwidth ≈ value for which |L| = 1
- Resonant peak set by phase margin
- Keep L large to set Hyr ≈ 1

$$H_{yr} = \frac{L}{1+L} \qquad H_{er} = \frac{1}{1+L}$$

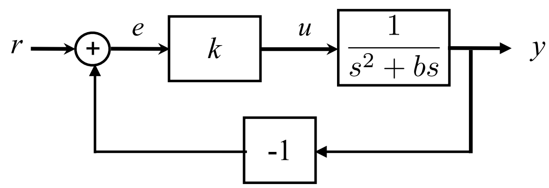
Time Domain Specs → Frequency Domain Specs

Time domain specifications



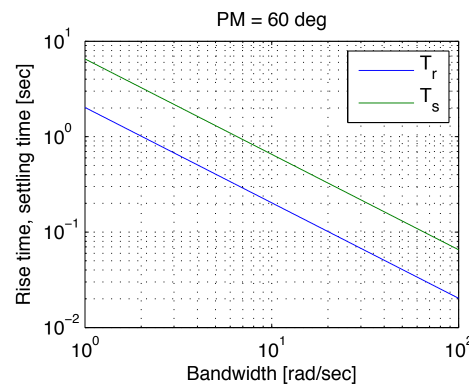


Map to frequency domain for second order system

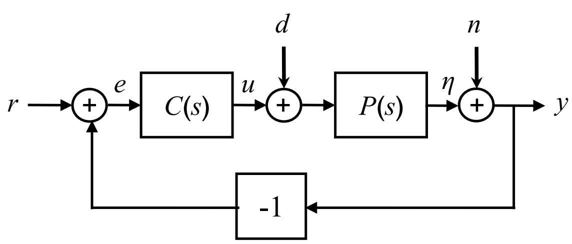


$$L(s) = \frac{k}{s^2 + bs} \qquad H_{yr} = \frac{k}{s^2 + bs + k}$$

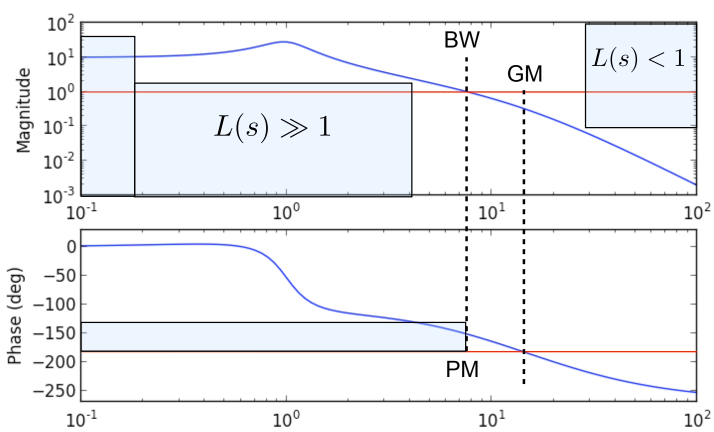
- Use properties of second order systems (Ch 7)
- HW #8, problem 1 (CDS 110 only)



"Loop Shaping": Design Loop Transfer Function



$$H_{er} = \frac{1}{1+L} \qquad H_{\eta n} = \frac{-L}{1+L}$$



Translate specs to "loop shape"

$$L(s) = P(s)C(s)$$

- Design C(s) to obey constraints

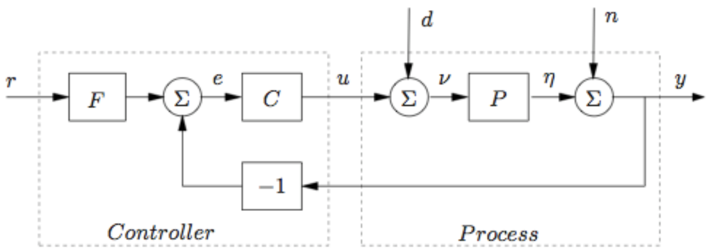
Typical loop constraints

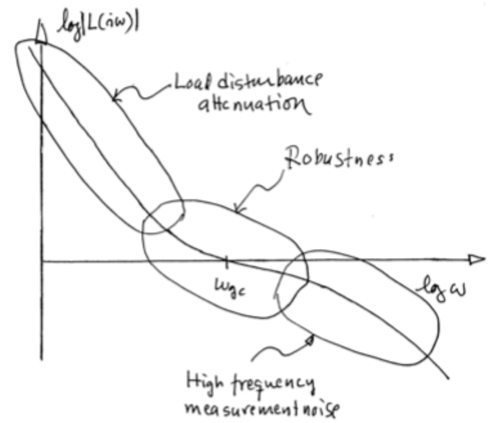
- High gain at low frequency
  - Good tracking, disturbance rejection at low freqs
- Low gain at high frequency
  - Avoid amplifying noise
- Sufficiently high bandwidth
  - Good rise/settling time
- Shallow slope at crossover
  - Sufficient phase margin for robustness, low overshoot

Key constraint: slope of gain curve determines phase curve

- Can't independently adjust
- Eg: slope at crossover sets PM

Loop Shaping: Basic Approach





Disturbance rejection $H_{ed} = \dfrac{-P}{1+L}$

- Would like Hed to be small make ⇒ large L(s)
- Typically require this in low frequency range

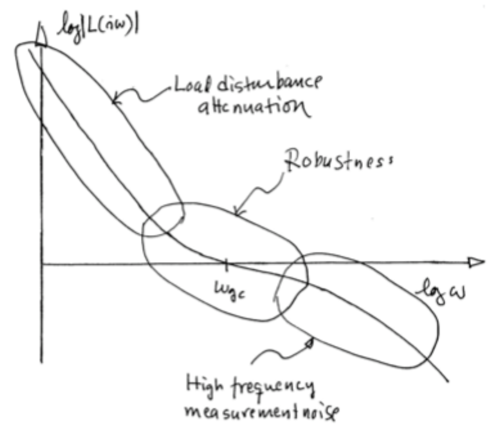
High frequency measurement noise $H_{un} = \dfrac{-L}{P(1+L)}$

- Want to make sure that Hun is small (avoid amplifying noise) ⇒ small L(s)
- Typically generates constraints in high frequency range

Robustness: gain and phase margin

- Focus on gain crossover region: make sure the slope is "gentle" at gain crossover
- Fundamental tradeoff: transition from high gain to low gain through crossover

Design Method #1: Process Inversion



Simple trick: invert out process

- Write all performance specs in terms of the desired loop transfer function
- Choose L(s) that satisfies specfiications
- Choose controller by inverting P(s)

$$C(s) = L(s)/P(s)$$

Pros

- Very easy design process
- L(s) = k/s often works very well
- Can be used as a first cut, with additional shaping to tune design

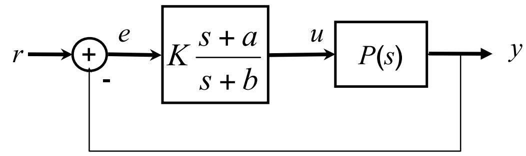
Cons

- High order controllers (at least same order as the process you are controlling)
- Requires "perfect" model of your process (since you are inverting it)
- Can generate non-proper controllers (order(num) > order(den))
  - Difficult to implement, plus amplifies noise at high frequency (C(∞) = ∞)
  - Fix by adding high frequency poles to roll off control response at high frequency
- Does not work if you have right half plane poles or zeros (get internal instability)

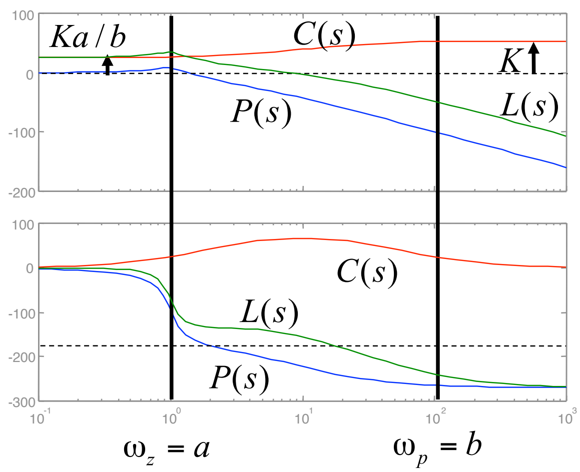
Design Method #2: Lead compensation

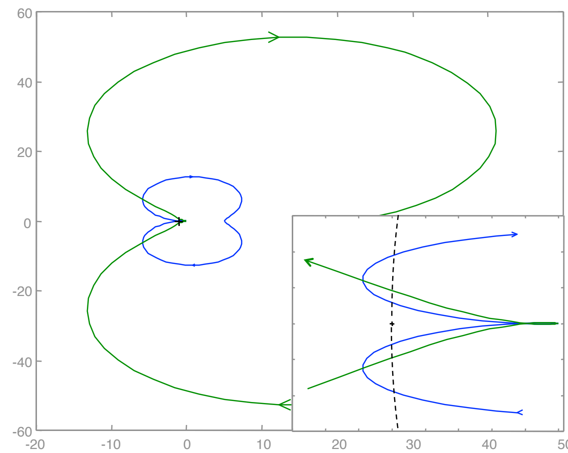
Use to increase phase in frequency band

- Effect: lifts phase by increasing gain at high frequency
- Very useful controller; increases PM
- Bode: add phase between zero and pole
- Nyquist: increase phase margin



$a < b \qquad K > 0$





Example: Lead Compensation for Second Order System

System description

$$P(s) = \frac{p_1 p_2}{(s+p_1)(s+p_2)}$$

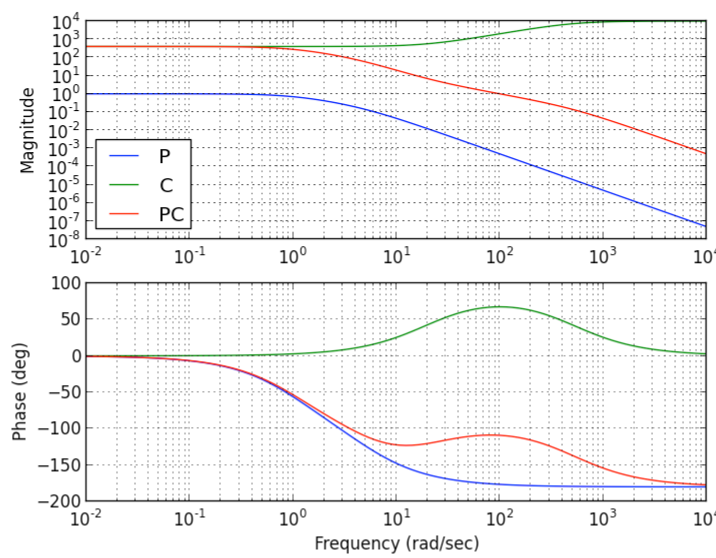
- Poles: p1 = 1, p2 = 5

Control specs

- Track constant reference with error < 1%
- Good tracking up to 100 rad/s (less than 10% error)
- Overshoot less than 10%
  - Gives PM of ~60 deg

Try a lead compensator

$$C(s) = K\frac{s+a}{s+b}$$



- Want gain crosssover at approximately 100 rad/sec => center phase gain there
- Set zero frequency gain of controller to give small error => |L(0)| > 100
- a = 20, b = 500, K = 10,000 (gives |C(0)| = |L(0)| = 400)

Safety Check: Nyquist + Gang of 4

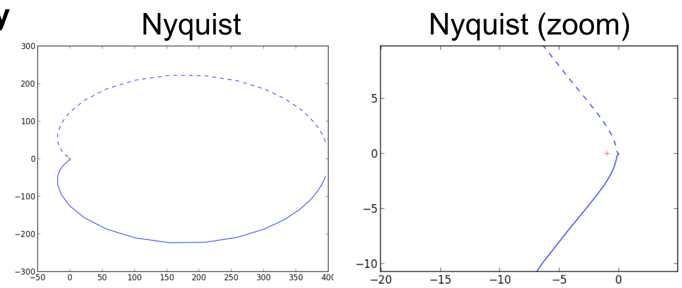
Nyquist verifies closed loop stability

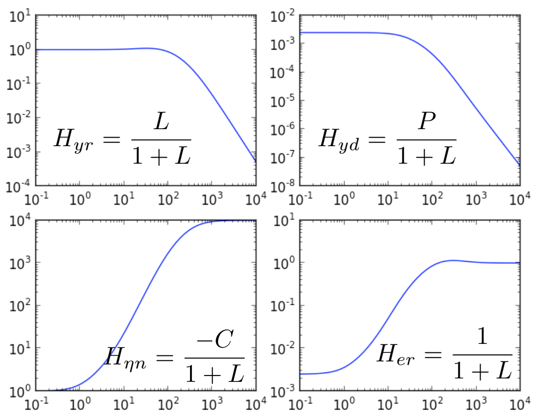
- Infinite GM; good phase margin

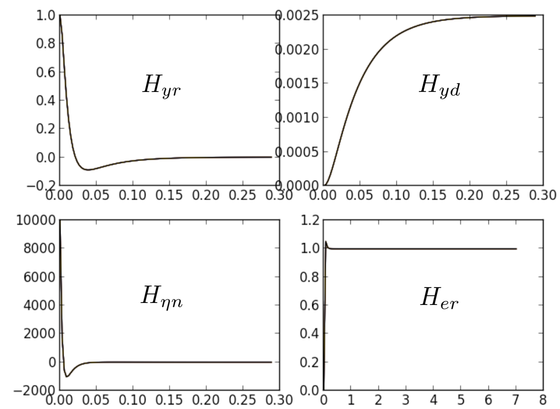
Gang of 4 shows high noise sens'y

- Factor of 10,000 gain at high freq
- Step responses show similar sensitivity

Solution? (HW #8…)



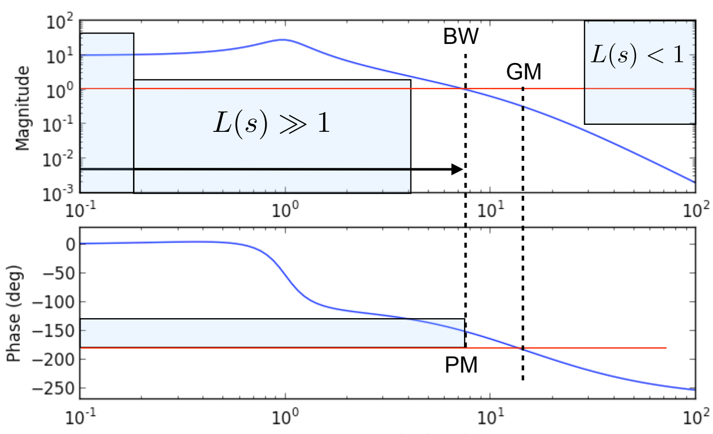




Summary: Loop Shaping

Loop Shaping for Stability & Performance

- Steady state error, bandwidth, tracking response
- Specs can be on any input/output response pair



Things to remember (for homework and exams)

- Always plot Nyquist to verify stability/robustness
- Check gang of 4 to make sure that noise and disturbance responses also look OK

Main ideas

- Performance specs give bounds on loop transfer function
- Use controller to shape response
- Gain/phase relationships constrain design approach
- Standard compensators: proportional, lead, PI