CDS 110
L10.2: Motion Control Systems

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4 December 2002

Announcements

- Final exam available at 3 pm (during break); due 5 pm, Friday, 13 Dec 02

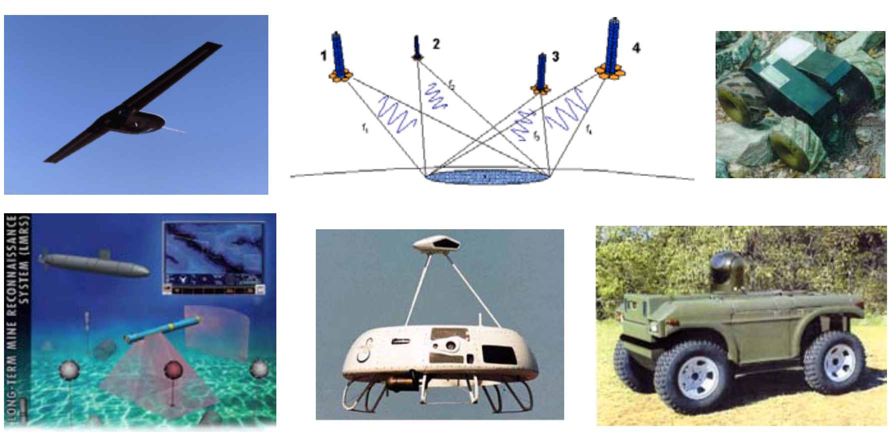
Outline:

- Motion Control Systems
  - Two degree of freedom design
  - Control design for the "Kelly II" vehicle
- Cooperative Control of Multi-Vehicle Systems (optional)

Reading:

- Optional:

Motion Control Systems

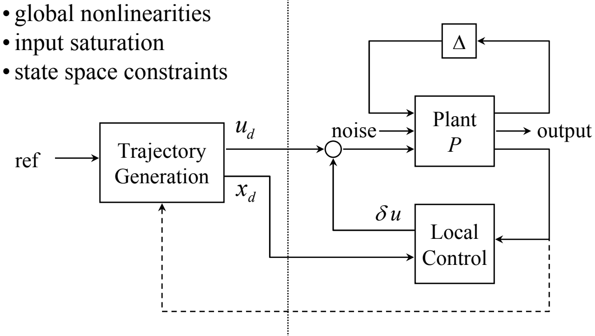


- Control objective: move the vehicle to a give point or along a given path
- Special features: second order dynamics, input constraints are important, often have a human in the loop (someplace)

Approach: Two Degree of Freedom Design

Nonlinear design
- global nonlinearities
- input saturation
- state space constraints

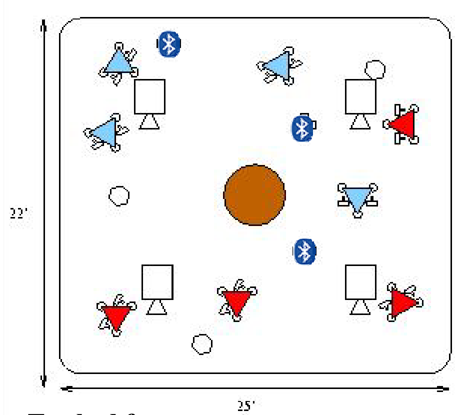
Linear design



- Use *real-time* trajectory generation to construct (suboptimal) feasible trajectories
- Use local linear/nonlinear control for tracking & robust performance

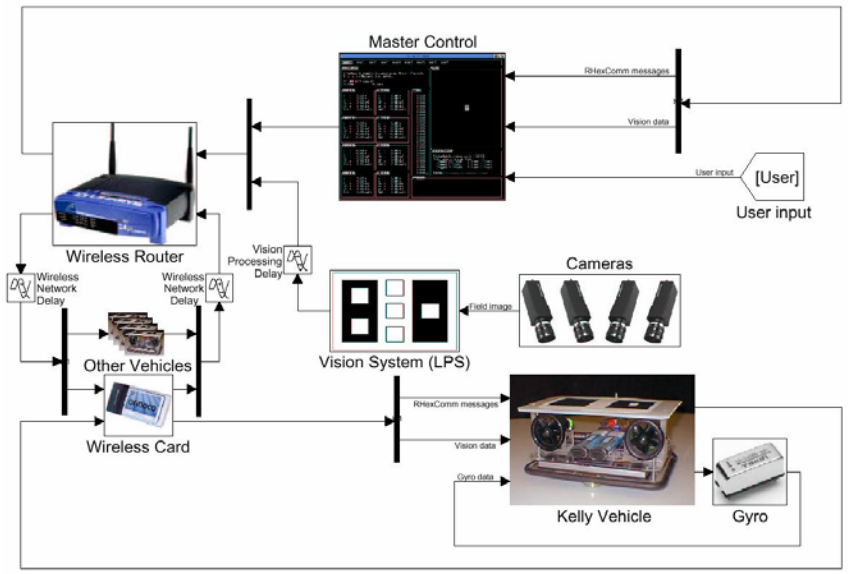
Cremean et al
CDC 2002

Multi-Vehicle Wireless Testbed for Integrated Control, Communications and Computation (DURIP)

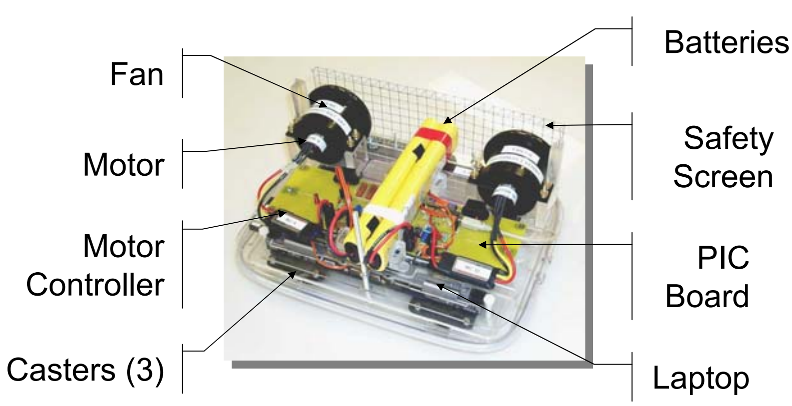


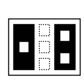
Testbed features
- Distributed computation on vehicles + command and control console
- Point to point networking (bluetooth) + local area networking (802.11)
- Overhead vision system provides global position data (LPS)

MVWT Block diagram

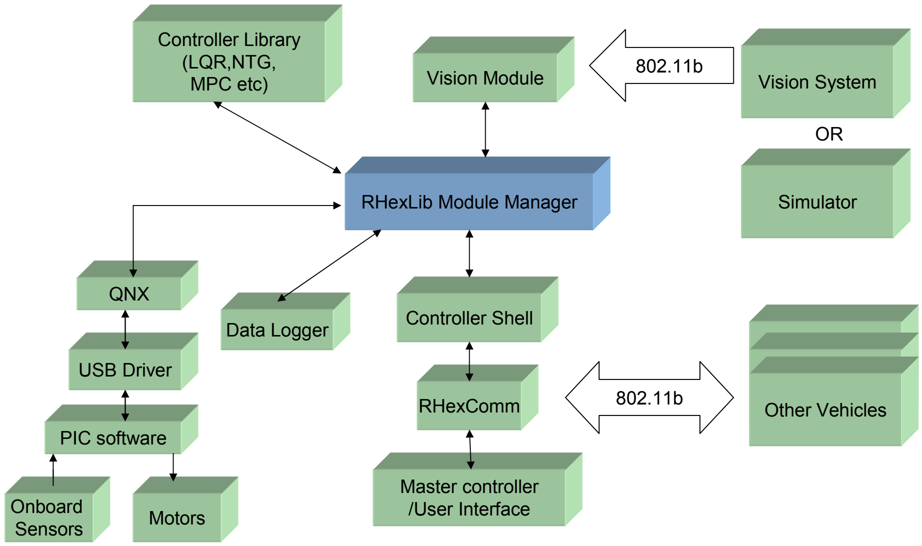


Vehicle Hardware

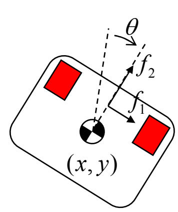



“Hat” for vision system not shown.

Software Architecture

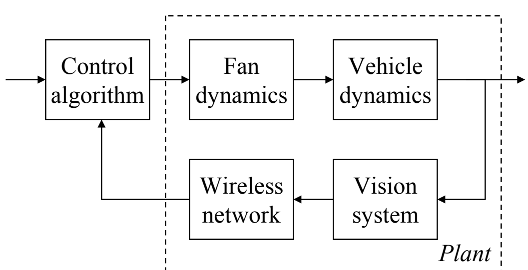


Dynamics



Vehicle dynamics:

$$m\ddot{x} = f_1 \cos\theta - f_2 \sin\theta - d\dot{x}$$
$$m\ddot{y} = f_1 \sin\theta + f_2 \cos\theta - d\dot{y}$$
$$J\ddot{\theta} = r f_1$$



Fan dynamics

$$m\dot{\omega}_1 = -\tau\omega_1 + \alpha(i_1) \qquad f_1 = N\omega_1$$
$$m\dot{\omega}_2 = -\tau\omega_2 + \alpha(i_2) \qquad f_2 = N\omega_2$$

Nonlinear map

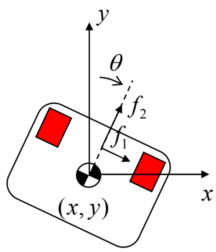
Vision System

- Time delay + noise (pixelation)

Wireless network

- Variable time delay

Linearization



$$m\dot{v} = f_1 \cos\theta - f_2 \sin\theta - dv \dot{x}$$
$$m\ddot{x} = f_1 \sin\theta + f_2 \cos\theta - d\dot{x}\,\dot{v}$$
$$J\ddot{\theta} = rf_1$$

Linearize around a constant velocity along *y*

$$v = v_0 \qquad f_1 = dv_0$$
$$x = 0, \dot{x} = 0 \qquad f_2 = 0$$
$$\theta = 0, \dot{\theta} = 0$$

Shift coordinates to the origin and write in state space form:

$$\tilde{v} = v - v_0$$
$$u_1 = f_2$$
$$u_2 = f_1 - dv_0$$

$$\frac{d}{dt}\begin{bmatrix}\tilde{v}\\ y\\ \theta\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix} -d/m & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & -dv_0/m & -d/m & 0\\ 0 & 0 & 0 & 0 & -b/J \end{bmatrix}\begin{bmatrix}\tilde{v}\\ y\\ \theta\\ \dot{y}\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix} 1/m & 0\\ 0 & 0\\ 0 & 0\\ 0 & 1/m\\ 0 & -r/J\end{bmatrix} u$$

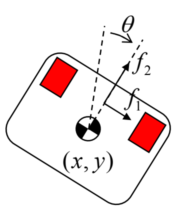
Remarks

- Ignores actuator dynamics (assume fast)
- Ignores time delays (for now)

Parameters

m = 5.5 b = 0.1
J = 0.047 r = 0.123
d = 0.5

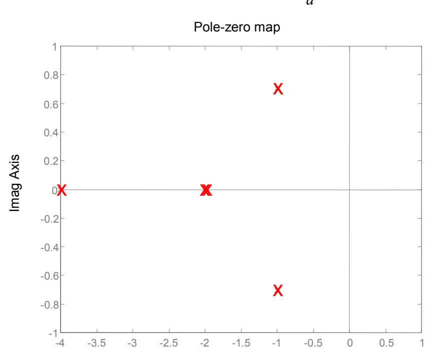
Control Design: State Space

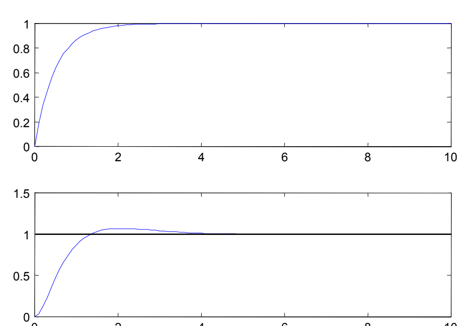


$$\frac{d}{dt}\begin{bmatrix}\tilde{v}\\ y\\ \theta\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix} -d/m & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & -dv_0/m & -d/m & 0\\ 0 & 0 & 0 & 0 & -b/J \end{bmatrix}\begin{bmatrix}\tilde{v}\\ y\\ \theta\\ \dot{y}\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix} 1/m & 0\\ 0 & 0\\ 0 & 0\\ 0 & 1/m\\ 0 & -r/J\end{bmatrix} u$$

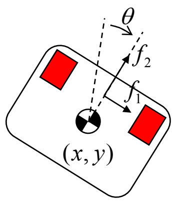
Choose control to stabilize error, $e = x - x_d \rightarrow u = K(x - x_d) + u_d$

- x_d = desired state
- u_d = nominal force





Control Design: Frequency Domain

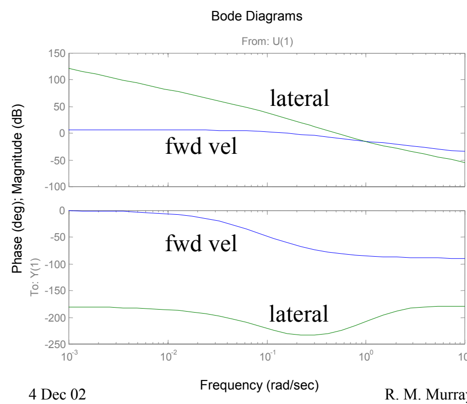


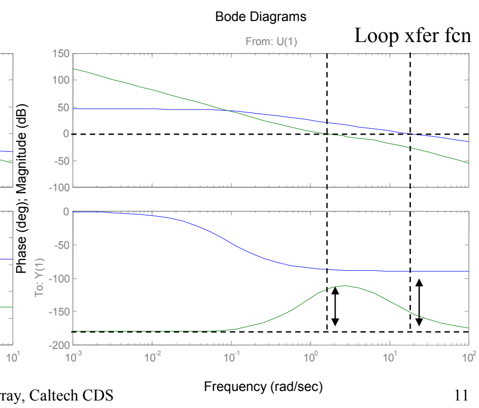
$$P_{\tilde{v}u_1} = \frac{1}{s+d} \qquad P_{yu_2} = \frac{Js^2 + bs + Jdv_0/m}{(ms^2+ds)(Js^2+bs)}$$

Compute transfer functions using $H = C(sI-A)^{-1}B$

Use loop shaping to design compensator

- Forward velocity: simple proportional gain
- Lateral position: use lead compensator





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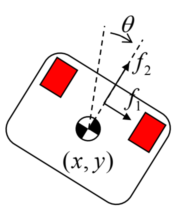
Control Design: Frequency Domain



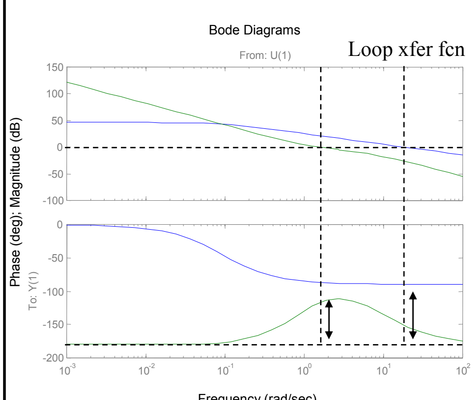
$$P_{\tilde{v}u_1} = \frac{1}{s+d} \qquad P_{yu_2} = \frac{Js^2 + bs + Jdv_0/m}{(ms^2+ds)(Js^2+bs)}$$

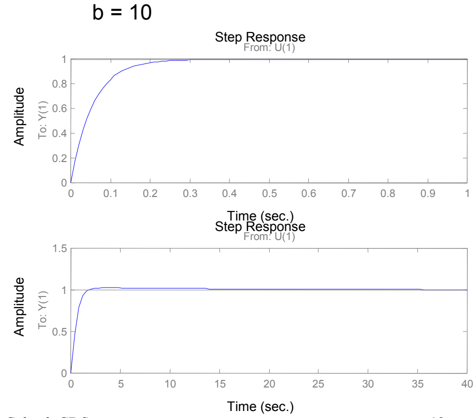
$$C_v = K_p \qquad C_l = K\frac{s+a}{s+b}$$

Kp = 100

a = 0.1 K = 100

b = 10





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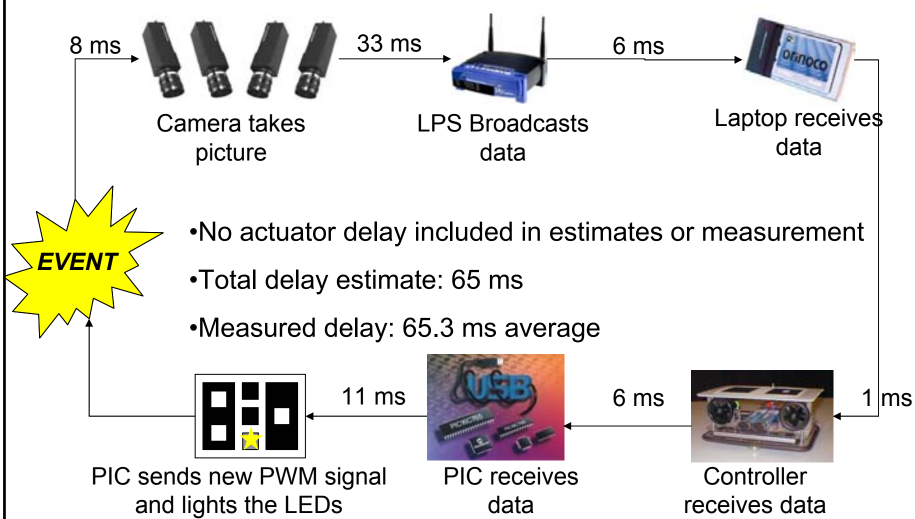
Time Delay Estimates



- No actuator delay included in estimates or measurement
- Total delay estimate: 65 ms
- Measured delay: 65.3 ms average

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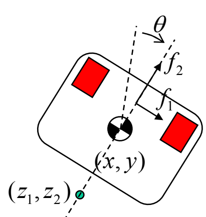
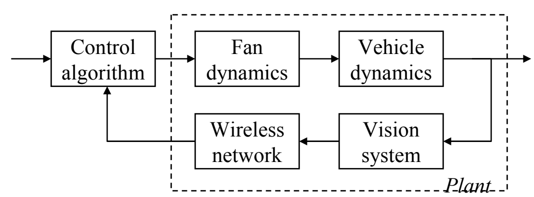
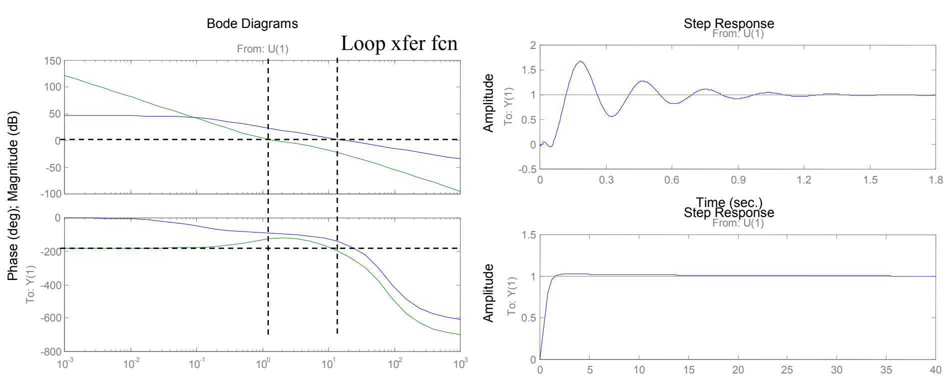
Unmodeled Dynamics: Sensor Delay







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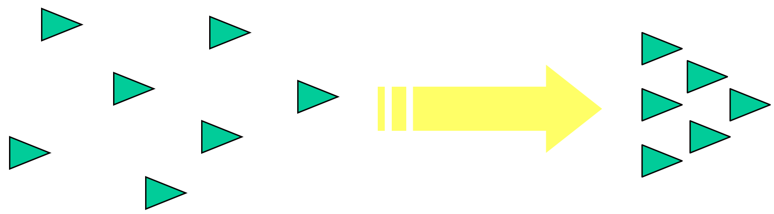
Next Steps: Multi-Vehicle Operations



Applications

- Cooperative control in dynamic, uncertain, adversarial environments (RoboCup)
- Formation flight of micro-satellite clusters (TechSat 21, TPF)

Questions

- How do we *coordinate* motion between multiple vehicles?
- How do we provide redundancy and failure tolerance?
- What should we communicate between vehicles and how often?

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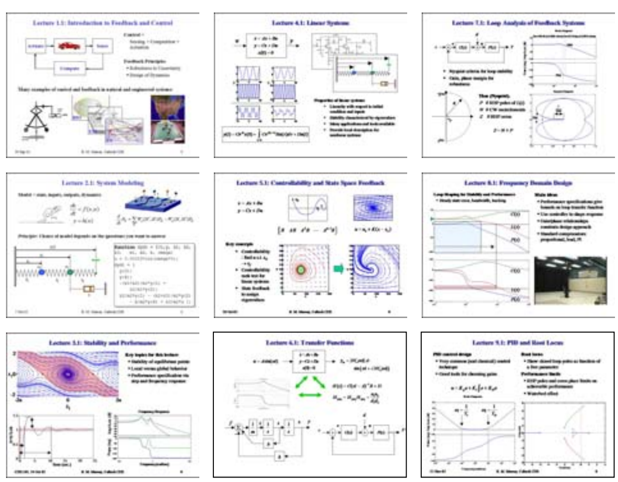
What you should know for the final

Basic concepts + analytical tools from entire course

- See summary slides on web for overview of main concepts



Course Topics

1. **Feedback concepts**
2. **System modeling**
3. **Stability/performance**
   - Step response
   - Frequency response
4. **Linear systems**
5. **Controllability, state space feedback**
6. **Transfer functions**
7. **Loop analysis**
   - Nyquist criterion
   - Gain/phase margin
8. **Loop shaping**
   - Loop xfer specs
9. **PID + root locus**

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