## CALIFORNIA INSTITUTE OF TECHNOLOGY

Control and Dynamical Systems

## CDS 210

D. G. MacMynowski Fall 2008 Problem Set #M

Issued: 27 Oct 08 Due: 3 Nov 08

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

- 1. Åström and Murray, Exercise 7.3
- 2. Consider the one-dimensional heat-equation with dissipation:

$$\frac{\partial h}{\partial t} = k \frac{\partial^2 h}{\partial z^2} - \epsilon h$$

with forcing at z=0 and measurement at z=1. Using a central finite-difference, we can discretize this to give  $\dot{x}=Ax$  where each row of the A-matrix has terms

$$k/\ell^2[-1 \ 2 \ -1] - \epsilon[0 \ 1 \ 0]$$

The A-matrix is generated by the matlab code heat\_pde.m on the course web-site. Compute the Reachability and Observability Gramians  $W_c$  and  $W_o$ . Is the system reachable and observable? Plot the most reachable and most observable directions. Considering the magnitudes of the eigenvalues of  $W_c$  and  $W_o$ , comment on the feasibility of designing a control system for these dynamics using this input and output.

- 3. Dullerud & Paganini, problem 2.9 part (b)
- 4. Dullerud & Paganini, problem 4.1