CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101

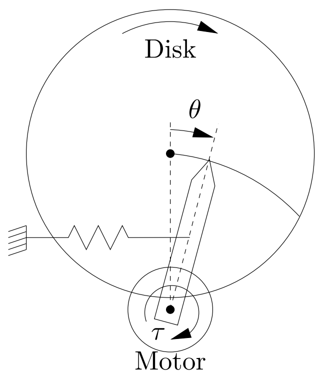
R. M. Murray — **Problem Set #7** — Issued: 17 Nov 08

Fall 2008 — Due: 24 Nov 08

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. Åström and Murray, Exercise 11.2 ("Time response" refers to the step response of the system.)

2. The figure below shows a simple mechanism for positioning a disk drive read head and the associated equations of motion:



$$J\ddot{\theta} = -b\dot{\theta} - kr\sin\theta + \tau_m$$
$$\dot{\tau}_m = -a(\tau_m - u)$$

The system consists of a spring-loaded arm that is driven by a small motor. The motor applies a force against the spring and pulls the head across the platter. The input to the system is the desired motor torque, u. In the diagram above, the force exerted by the spring is a nonlinear function of the head position due to the way it is attached. All constants are positive.

We wish to design a controller that holds the drive head at a given location θ_d.

(a) Show that the transfer function of the process can be written as

$$P(s) = \frac{a}{a+s} \cdot \frac{k}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

(b) Assume that the system parameters are such that $K = 0.001$, $\zeta = 0.5$, $\omega_n = 0.1$ and $a = 1$. Design a compensator that provides tracking with less than 10% error up to 1 rad/s and has a phase margin of 60°.

(c) Plot the Nyquist plot for the (open loop) system corresponding to your control design and compute the gain margin, phase margin and stability margin.

(d) Compute and plot the Gang of Four for your system. Comment on any of the transfer functions that might lead to large errors or control signals and indicate the conditions under which this might occur.

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CDS 110a

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Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. Consider a control system with with process and controller dynamics given by

$$P(s) = \frac{1}{s(s+c)} \qquad C(s) = k.$$

where $k > 0$.

(a) Show that the closed loop response of the system can be written as

$$G(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

and give formulas for ζ and ω_0 in terms of c and k.

(b) Show that the phase margin for the system is given by

$$\varphi_m = \tan^{-1}\left(\frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4} - 2\zeta^2}}\right)$$

(Hint: compute the frequency at which $|L(i\omega)| = 1$ and then find the phase at that frequency.)

(c) Show that the overshoot for the closed loop step response is given by

$$M_p = \begin{cases} e^{-\pi\zeta/\sqrt{1-\zeta^2}} & \text{for } |\zeta| < 1 \\ 0 & \text{for } \zeta \geq 1. \end{cases}$$

(Hint: use the form of the solution from equation (6.24) and search for the shortest time when $\dot{y}(t) = 0$.)

(d) Use the formulas from parts 1b and 1c to plot M_p as a function of φ_m for ζ in the range $0 < \zeta \leq 1$.

2. Consider the problem of stabilizing the orientation of a flying insect, modeled as a rigid body with moment of inertia $J = 0.41$ and damping constant $D = 1$.[1] We assume there is a small delay $\tau = 0.01$ sgiven by the neural circuitry that implements the control system. The resulting transfer function for the system is taken to be

$$P(s) = \frac{1}{Js^2 + Ds} e^{-\tau s}.$$

[1] Based loosely on "Biologically Inspired Feedback Design for Drosophila Flight", M. Epstein, S. Waydo, S. B. Fuller, W. Dickson, A. Straw, M. H. Dickinson and R. M. Murray, 2007 American Control Conference.

(a) Suppose that we can measure the orientation of the insect relative to its environment and we wish to design a control law that that gives zero steady state error, less than 10% tracking error from 0 to 0.5 Hz and has an overshoot of no more than 10%. Convert these specifications to appropriate bounds on the loop transfer function and sketch the resulting constraints on a Bode plot. (Hint: Try using problem 1 to convert the overshoot requirement to a phase margin requirement.)

(b) Using a lead compensator, design a controller that meets the specifications in part (a). Provide whatever plots are required to verify that the specification is met. You may use a Padé approximation for the time delay, but make sure that it is a good approximation over a frequency range that includes your gain crossover frequency.

(c) Plot or sketch the Nyquist plot corresponding to your controller and the process. You can again use a Padé approximation for the time delay.

(d) Extra credit: genetically modify a fly to implement your controller, using the fly visual system as your input.

3. Consider the dynamics of the magnetic levitation system from lecture. The transfer function from the electromagnet input voltage to the IR sensor output voltage is given by

$$P(s) = \frac{k}{s^2 - r^2}$$

with $k = 4000$ and $r = 25$ (these parameters are slightly different than those used in the MATLAB files distributed with the lecture).

(a) Design a stabilizing compensator for the process, assuming unity feedback. Compute the poles and zeros for the loop transfer function and for the closed loop transfer function between the reference input and measured output.

(b) Plot the Nyquist plot corresponding to your compensator and the process dynamics, and verify that the Nyquist criterion is satisfied.

(c) Plot the log of the magnitude of the sensitivity function, $\log |S(j\omega)|$, versus ω on a *linear* scale and numerically verify that the Bode integral formula is (approximately) satisfied. (Hint: you can do the integration numerically in MATLAB, using the `trapz` function. Make sure to choose your frequency range sufficiently large.)

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1. Åström and Murray, Exercise 11.12

2. Åström and Murray, Exercise 11.17

3. [DFT 4.4, page 63] Suppose that

$$P(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)} \qquad C(s) = 1$$

with $\omega_n, \zeta > 0$. Plot the phase margin as a function of ζ.

4. [DFT 4.8, page 64] Assume that the nominal plant transfer function is a double integrator,

$$P(s) = \frac{1}{s^2}.$$

The performance requirement is that the plant output should track reference inputs over the frequency range [0, 1]. Approximatele this requirement by choosing a performance weight W_1 whose magnitude is roughly constant over this frequency range and then rolls off at higher frequencies. Take the weight W_2 to be

$$W_2(s) = \frac{0.21s}{0.1s + 1}.$$

(a) Design a proper C to achieve internal stability for the nominal plant.

(b) Check the robust stability condition $\|W_2 T\|_\infty < 1$. If this does not hold, redesign C until it does. It is not necessary to get a C that yields good performance.

(c) Compute the robust performance level α for your controller from (4.6).

5. [DFT 4.10, page 64] Suppose that the plant transfer function is

$$\tilde{P}(s) = [1 + \Delta(s) W_2(s)] P(s),$$

where

$$W_2(s) = \frac{2}{s + 10}, \ P(s) = \frac{1}{s - 1},$$

and the stable perturbation Δ satisfies $\|\Delta\|_\infty \leq 2$. Suppose that the controller is the pure gain $C(s) = k$. We want the feedback system to be internally stable for all such perturbations. Determine over what range of k this is true.