CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

# CDS 210

R. M. Murray
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## Problem Set #7

Issued: 17 Nov 08
Due: 24 Nov 08

**Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).**

1. Åström and Murray, Exercise 11.12

2. Åström and Murray, Exercise 11.17

3. [DFT 4.4, page 63] Suppose that

$$P(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)} \qquad C(s) = 1$$

   with $\omega_n, \zeta > 0$. Plot the phase margin as a function of $\zeta$.

4. [DFT 4.8, page 64] Assume that the nominal plant transfer function is a double integrator,

$$P(s) = \frac{1}{s^2}.$$

   The performance requirement is that the plant output should track reference inputs over the frequency range $[0,\ 1]$. Approximatele this requirement by choosing a performance weight $W_1$ whose magnitude is roughly constant over this frequency range and then rolls off at higher frequencies. Take the weight $W_2$ to be

$$W_2(s) = \frac{0.21s}{0.1s + 1}.$$

   (a) Design a proper $C$ to achieve internal stability for the nominal plant.

   (b) Check the robust stability condition $\|W_2T\|_\infty < 1$. If this does not hold, redesign $C$ until it does. It is not necessary to get a $C$ that yields good performance.

   (c) Compute the robust performance level $\alpha$ for your controller from (4.6).

5. [DFT 4.10, page 64] Suppose that the plant transfer function is

$$\tilde{P}(s) = [1 + \Delta(s)W_2(s)]P(s),$$

   where

$$W_2(s) = \frac{2}{s + 10},\ P(s) = \frac{1}{s - 1},$$

   and the stable perturbation $\Delta$ satisfies $\|\Delta\|_\infty \leq 2$. Suppose that the controller is the pure gain $C(s) = k$. We want the feedback system to be internally stable for all such perturbations. Determine over what range of $k$ this is true.