CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101

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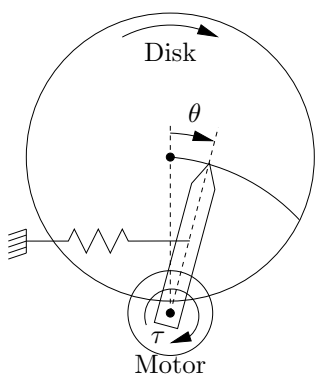
Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. Åström and Murray, Exercise 11.2 ("Time response" refers to the step response of the system.)

2. The figure below shows a simple mechanism for positioning a disk drive read head and the associated equations of motion:



$$J\ddot{\theta} = -b\dot{\theta} - kr\sin\theta + \tau_m$$

$$\dot{\tau}_m = -a(\tau_m - u)$$

The system consists of a spring-loaded arm that is driven by a small motor. The motor applies a force against the spring and pulls the head across the platter. The input to the system is the desired motor torque, u. In the diagram above, the force exerted by the spring is a nonlinear function of the head position due to the way it is attached. All constants are positive.

We wish to design a controller that holds the drive head at a given location θ_d.

(a) Show that the transfer function of the process can be written as

$$P(s) = \frac{a}{a+s} \cdot \frac{k}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

(b) Assume that the system parameters are such that $K = 0.001$, $\zeta = 0.5$, $\omega_n = 0.1$ and $a = 1$. Design a compensator that provides tracking with less than 10% error up to 1 rad/s and has a phase margin of 60°.

(c) Plot the Nyquist plot for the (open loop) system corresponding to your control design and compute the gain margin, phase margin and stability margin.

(d) Compute and plot the Gang of Four for your system. Comment on any of the transfer functions that might lead to large errors or control signals and indicate the conditions under which this might occur.