R. M. Murray   **Problem Set #6**   Issued: 10 Nov 08
Fall 2008   Due: 17 Nov 08

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. Plot the (open loop) Nyquist and Bode plots for the following systems and compute the gain and phase margin of each. You should annotate your plots to show the gain and phase margin computations. For the Nyquist plot, mark the branches corresponding to the following sections of the Nyquist "D" contour: negative imaginary axis, positive imaginary axis, semi-circle at infinity (the curved part of the "D").

   (a) Disk drive read head positioning system, using a lead compensator (described in Chapter 11):
   $$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10}, \qquad C(s) = 1000\frac{s+1}{s+10}.$$

   (b) Second-order system with PD compensator:
   $$P(s) = \frac{100}{(100s+1)(s+1)}, \qquad C(s) = s + 10.$$

   Note: you may find it easier to sketch the Nyquist plot from the Bode plot (taking some liberties with the scale) rather than relying on MATLAB.

2. In this problem we will design a PI controller for a cruise control system, building on the example shown in class. Use the following transfer function to represent the vehicle and engine dynamics:
   $$P(s) = \frac{Tba/m}{(s+a)(s+c/m)}$$
   where $b = 25$ is the transmission gain, $T = 200$ is the conversion factor between the throttle input and steady state torque, $a = 0.2$ is the engine lag coefficient, $m = 1000$ kg is the mass of the car, and $c = 50$ N s/m is the viscous damping coefficient.

   (a) Consider a proportional controller for the car, $u = k_p(r - y)$. Assuming a unity gain feedback controller, this gives
   $$C(s) = k_p.$$
   Set $k_p = 0.1$ and compute the steady state error, gain and phase margins, rise time, overshoot and poles/zeros for the system. Remember that the gain and phase margins are computed based on the loop transfer function $L(s) = P(s)C(s)$; the remaining quantities should be computed for the closed loop system.

   (b) Consider a proportional + integral controller for the car,
   $$C(s) = k_p + \frac{k_i}{s}.$$
   Fill in the following table (make sure to show your work):

k_p	k_i	Stable?	g_m	φ_m	T_r	M_p
0.5	0.1					
0.05	1					
0.05	0.001					
0.005	0.001					

For each entry in the table, plot the pole zero diagram (pzmap) for the *closed loop* system and the step response. (Note that the steady state error is zero in each stable case, due to the integral term in the control law.)

(Suggestion: look for relationships between the various quantities you are computing and plotting. This problem should give you some insight into the relationship between some of the quantities.)

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1. Consider a closed loop system with the loop transfer function

$$L(s) = \frac{k}{(s+a)(s^2 + 2\zeta\omega_0 s + \omega_0^2)}.$$

   (a) Assuming that $a \ll \omega_0$ and $\zeta = 1$, sketch the Bode and Nyquist plots for the system, labeling they key features (in terms of k, a and ω_0).

   (b) For each of the following parameter sets, use the Nyquist criterion to determine if the closed loop system is stable and, if so, what the gain, phase and stability margins are:

      i. $k = 200$, $a = 1$, $\zeta = 1$, $\omega_n = 10$
      ii. $k = 100$, $a = 1$, $\zeta = 0.1$, $\omega_n = 10$
      iii. $k = 100$, $a = 0$, $\zeta = 1$, $\omega_n = 10$
      iv. $k = 80$, $a = -1$, $\zeta = 1$, $\omega_n = 10$

      Be sure to show the Nyquist plot for each case and show the gain and phase margins on the Nyquist plots.

2. In this problem we will design a PI controller for a cruise control system, building on the example shown in class. Use the following transfer function to represent the vehicle and engine dynamics:

$$P(s) = \frac{Tba/m}{(s+a)(s+c/m)}$$

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   (a) Consider a proportional controller for the car, $u = k_p(r - y)$. Assuming a unity gain feedback controller, this gives

$$C(s) = k_p.$$

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      Fill in the following table (make sure to show your work):

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(Suggestion: look for relationships between the various quantities you are computing and plotting. This problem should give you some insight into the relationship between some of the quantities.)

3. Continuing the previous problem, we will now insert a small amount of time delay into the feedback path of the system. A pure time delay of τ seconds satisfies the equation

$$y(t) = u(t - \tau)$$

This system is a linear input/output system and it can be shown that its transfer function is

$$G(s) = e^{-s\tau}.$$

Unfortunately, MATLAB is not able to perfectly represent a time delay in this form, and so we have to use a "Padé approximation", which gives a constant gain transfer function with phase that approximates a time delay. Using a 2nd order Padé approximation, we can approximate our time delay as

$$G(s) = \frac{1 - \tau s/2 + (\tau s)^2/12}{1 + \tau s/2 + (\tau s)^2/12}$$

This function can be computed using the `pade` function in MATLAB (although the numerator and denominator are scaled slightly differently).

Assume that there is a time delay of τ seconds, which we will insert between the output of the plant and the controller (as we did in Monday's lecture).

(a) For the case $k_p = 0.05$, $k_i = 0.001$, insert time delays of $\tau = 0.25$ s and $\tau = 0.75$ s. Using a Padé approximation, compute the resulting gain and phase margin for each case and compute the overshoot and settling time (2%) for the step responses.

(b) Repeat part (a) using $k_p = 0.02$, $k_i = 0.0005$ and time delays of 0.75 s and 1.5 s.

(c) *Optional:* Plot the Nyquist plot for $k_p = 0.02$, $K_i = 0.0005$, $\tau = 0.75$ (with the exact time delay, not the Pade approximation).

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   (a) Assuming that $a \ll \omega_0$ and $\zeta = 1$, sketch the Bode and Nyquist plots for the system, labeling they key features (in terms of k, a and ω_0).

   (b) For each of the following parameter sets, use the Nyquist criterion to determine if the closed loop system is stable and, if so, what the gain, phase and stability margins are:

      i. $k = 200$, $a = 1$, $\zeta = 1$, $\omega_n = 10$
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      Be sure to show the Nyquist plot for each case and show the gain and phase margins on the Nyquist plots.

2. [DFT 3.1, page 44] Show that for a unity feedback system it suffices to check only two transfer functions to determine internal stability.

3. [DFT 3.2, page 44] Let

$$\widehat{P}(s) = \frac{1}{10s + 1} \qquad \widehat{C}(s) = k \qquad \widehat{F}(s) = 1.$$

   Find the least positive gain k such that the following are all true:

   (a) The feedback system is internally stable
   (b) $|e(\infty)| \leq 0.1$ when $r(t)$ is the unit step and $n = d = 0$.
   (c) $\|y\|_\infty \leq 0.1$ for all $d(t)$ such that $\|d\|_2 \leq 1$ when $r = n = 0$.

4. [DFT 3.3, page 44] Consider a unity gain feedback system with $r = n = 0$ and $d(t) = \sin(\omega(t)1(t)$. Prove that if the feedback system is internally stable then $y(t) \to 0$ as $t \to \infty$ if and only if $\widehat{P}$ has a zero at $s = i\omega$ or $\widehat{C}$ has a pole at $s = i\omega$.

5. (ÅM08, Exercise 9.10) Consider a system whose input/output response is modeled by $G(s) = 6(-s+1)/(s^2 + 5s + 6)$, which has a zero in the right half-plane.

   (a) Sketch the Bode plot for the system. (Hint: try sketching these by hand first and use MATLAB only if you get stuck.)

(b) Compute the step response for the system, and show that the output goes in the wrong direction initially, which is also referred to as an *inverse-response.*

(c) Compare the response to a minimum phase system by replacing the zero at $s = 1$ with a zero at $s = -1$. Show that the gain curve on the Bode plot is unchanged, but that the phase curve and step response are (significantly) different.