CDS 110a

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Fall 2008   Due: 17 Nov 08

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. Consider a closed loop system with the loop transfer function

$$L(s) = \frac{k}{(s+a)(s^2 + 2\zeta\omega_0 s + \omega_0^2)}.$$

   (a) Assuming that $a \ll \omega_0$ and $\zeta = 1$, sketch the Bode and Nyquist plots for the system, labeling they key features (in terms of k, a and ω_0).

   (b) For each of the following parameter sets, use the Nyquist criterion to determine if the closed loop system is stable and, if so, what the gain, phase and stability margins are:

   i. $k = 200$, $a = 1$, $\zeta = 1$, $\omega_n = 10$
   ii. $k = 100$, $a = 1$, $\zeta = 0.1$, $\omega_n = 10$
   iii. $k = 100$, $a = 0$, $\zeta = 1$, $\omega_n = 10$
   iv. $k = 80$, $a = -1$, $\zeta = 1$, $\omega_n = 10$

   Be sure to show the Nyquist plot for each case and show the gain and phase margins on the Nyquist plots.

2. In this problem we will design a PI controller for a cruise control system, building on the example shown in class. Use the following transfer function to represent the vehicle and engine dynamics:

$$P(s) = \frac{Tba/m}{(s+a)(s+c/m)}$$

   where $b = 25$ is the transmission gain, $T = 200$ is the conversion factor between the throttle input and steady state torque, $a = 0.2$ is the engine lag coefficient, $m = 1000$ kg is the mass of the car, and $c = 50$ N s/m is the viscous damping coefficient.

   (a) Consider a proportional controller for the car, $u = k_p(r - y)$. Assuming a unity gain feedback controller, this gives

   $$C(s) = k_p.$$

   Set $k_p = 0.1$ and compute the steady state error, gain and phase margins, rise time, overshoot and poles/zeros for the system. Remember that the gain and phase margins are computed based on the loop transfer function $L(s) = P(s)C(s)$; the remaining quantities should be computed for the closed loop system.

   (b) Consider a proportional + integral controller for the car,

   $$C(s) = k_p + \frac{k_i}{s}.$$

   Fill in the following table (make sure to show your work):

k_p	k_i	Stable?	g_m	φ_m	T_r	M_p
0.5	0.1					
0.05	1					
0.05	0.001					
0.005	0.001					

For each entry in the table, plot the pole zero diagram (pzmap) for the *closed loop* system and the step response. (Note that the steady state error is zero in each stable case, due to the integral term in the control law.)

(Suggestion: look for relationships between the various quantities you are computing and plotting. This problem should give you some insight into the relationship between some of the quantities.)

3. Continuing the previous problem, we will now insert a small amount of time delay into the feedback path of the system. A pure time delay of τ seconds satisfies the equation

$$y(t) = u(t - \tau)$$

This system is a linear input/output system and it can be shown that its transfer function is

$$G(s) = e^{-s\tau}.$$

Unfortunately, MATLAB is not able to perfectly represent a time delay in this form, and so we have to use a "Padé approximation", which gives a constant gain transfer function with phase that approximates a time delay. Using a 2nd order Padé approximation, we can approximate our time delay as

$$G(s) = \frac{1 - \tau s/2 + (\tau s)^2/12}{1 + \tau s/2 + (\tau s)^2/12}$$

This function can be computed using the `pade` function in MATLAB (although the numerator and denominator are scaled slightly differently).

Assume that there is a time delay of τ seconds, which we will insert between the output of the plant and the controller (as we did in Monday's lecture).

(a) For the case $k_p = 0.05$, $k_i = 0.001$, insert time delays of $\tau = 0.25$ s and $\tau = 0.75$ s. Using a Padé approximation, compute the resulting gain and phase margin for each case and compute the overshoot and settling time (2%) for the step responses.

(b) Repeat part (a) using $k_p = 0.02$, $k_i = 0.0005$ and time delays of 0.75 s and 1.5 s.

(c) *Optional:* Plot the Nyquist plot for $k_p = 0.02$, $K_i = 0.0005$, $\tau = 0.75$ (with the exact time delay, not the Pade approximation).