CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101

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Fall 2008		Due: 17 Nov 08

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. Plot the (open loop) Nyquist and Bode plots for the following systems and compute the gain and phase margin of each. You should annotate your plots to show the gain and phase margin computations. For the Nyquist plot, mark the branches corresponding to the following sections of the Nyquist "D" contour: negative imaginary axis, positive imaginary axis, semicircle at infinity (the curved part of the "D").

   (a) Disk drive read head positioning system, using a lead compensator (described in Chapter 11):
   $$P(s) = \frac{1}{s^3 + 10s^2 + 3s + 10}, \qquad C(s) = 1000\frac{s+1}{s+10}.$$

   (b) Second-order system with PD compensator:
   $$P(s) = \frac{100}{(100s+1)(s+1)}, \qquad C(s) = s + 10.$$

   Note: you may find it easier to sketch the Nyquist plot from the Bode plot (taking some liberties with the scale) rather than relying on MATLAB.

2. In this problem we will design a PI controller for a cruise control system, building on the example shown in class. Use the following transfer function to represent the vehicle and engine dynamics:
   $$P(s) = \frac{Tba/m}{(s+a)(s+c/m)}$$
   where $b = 25$ is the transmission gain, $T = 200$ is the conversion factor between the throttle input and steady state torque, $a = 0.2$ is the engine lag coefficient, $m = 1000$ kg is the mass of the car, and $c = 50$ N s/m is the viscous damping coefficient.

   (a) Consider a proportional controller for the car, $u = k_p(r - y)$. Assuming a unity gain feedback controller, this gives
   $$C(s) = k_p.$$
   Set $k_p = 0.1$ and compute the steady state error, gain and phase margins, rise time, overshoot and poles/zeros for the system. Remember that the gain and phase margins are computed based on the loop transfer function $L(s) = P(s)C(s)$; the remaining quantities should be computed for the closed loop system.

   (b) Consider a proportional + integral controller for the car,
   $$C(s) = k_p + \frac{k_i}{s}.$$
   Fill in the following table (make sure to show your work):

k_p	k_i	Stable?	g_m	φ_m	T_r	M_p
0.5	0.1					
0.05	1					
0.05	0.001					
0.005	0.001					

For each entry in the table, plot the pole zero diagram (pzmap) for the *closed loop* system and the step response. (Note that the steady state error is zero in each stable case, due to the integral term in the control law.)

(Suggestion: look for relationships between the various quantities you are computing and plotting. This problem should give you some insight into the relationship between some of the quantities.)