CALIFORNIA INSTITUTE OF TECHNOLOGY
Control and Dynamical Systems

CDS 101

D. G. MacMynowski

Problem Set #4

Fall 2008

Issued: 20 Oct 08
Due: 27 Oct 08

Note: In the upper left hand corner of the *second* page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. Åström and Murray, Exercise 6.1, modified as follows: Choose constant inputs u_1 over the interval from $t = 0$ to $t = T/2$ and u_2 over the interval from $t = T/2$ to $t = T$, and find u_1, u_2 that drive the system from the origin to the point $(1, -1)$.

2. Åström and Murray, Exercise 6.12

   Download the file `bike_linmod.m` from the course web-site, which contains the parameters for the bicycle and generates the matrices M, C, K_0 and K_2 in Eq. (3.7) of the text.

   Find the controller gains corresponding to choosing the final pair of complex poles at $-1 \pm i$ as stated in the text, and also with these poles at $-2 \pm 2i$ and $-5 \pm 5i$. Remember to solve for the reference gain k_r as well! For each case, simulate the response to a step change in the steering reference of $0.002\,\text{rad}$ and plot both the steering angle output δ and the torque command.

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CDS 110a

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1. Åström and Murray, Exercise 6.3
2. Åström and Murray, Exercise 6.9
3. Åström and Murray, Exercise 6.12

   Download the file `bike_linmod.m` from the course web-site, which contains the parameters for the bicycle and generates the matrices M, C, K_0 and K_2 in Eq. (3.7) of the text.

   Find the controller gains corresponding to choosing the final pair of complex poles at $-1 \pm i$ as stated in the text, and also with these poles at $-2 \pm 2i$ and $-5 \pm 5i$. Remember to solve for the reference gain k_r as well! For each case, simulate the response to a step change in the steering reference of 0.002 rad and plot both the steering angle output δ and the torque command.

4. Consider the normalized, linearized inverted pendulum which is described by

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u = Ax + Bu, \qquad y = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = Cx$$

   Determine a state feedback and reference gain $u = Kx + k_r r$ that gives a closed loop system with unit static gain (steady-state output $y = r$) and with the characteristic polynomial $s^2 + 2\zeta_0\omega_0 s + \omega_0^2$.

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CDS 210

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1. Åström and Murray, Exercise 6.3
2. Åström and Murray, Exercise 6.9
3. Åström and Murray, Exercise 6.10, assuming A is diagonalizable
4. Dullerud & Paganini, problem 2.9 part (a)
5. Dullerud & Paganini, problem 4.4