CDS 101/110a: Lecture 8-1
Frequency Domain Design

Richard M. Murray
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Goals:

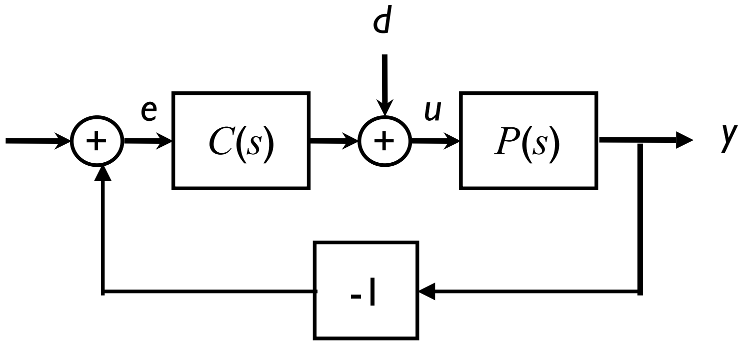
- Describe canonical control design problem and standard performance measures
- Show how to use “loop shaping” to achieve a performance specification
- Work through a detailed example of a control design problem

Reading:

- Åström and Murray, *Feedback Systems,* Ch 11
- *Advanced*: Lewis, Chapter 12
- CDS 210: DFT, Chapters 4 and 6

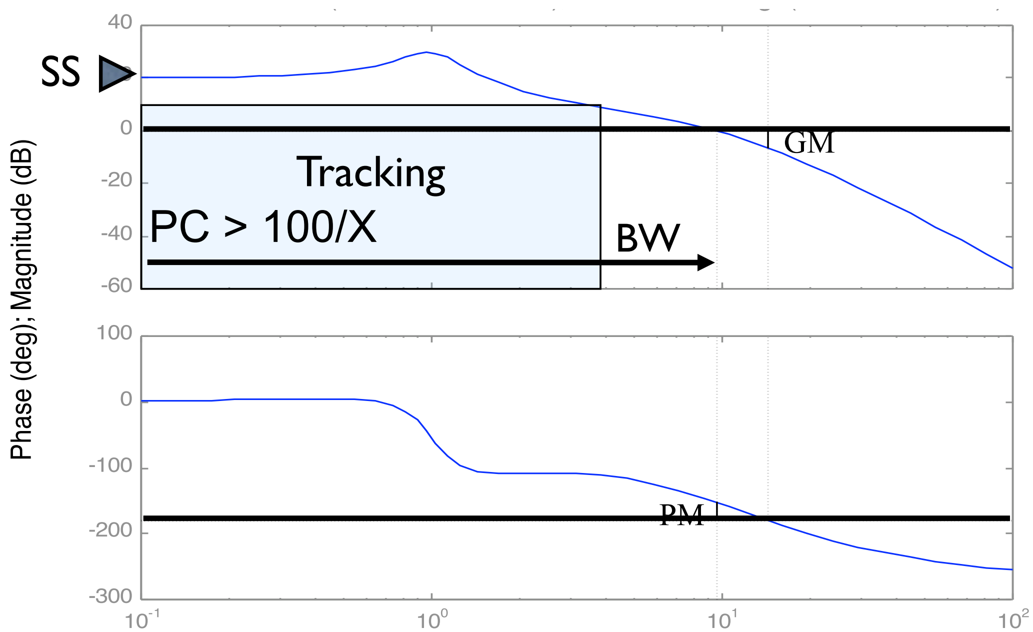
Frequency Domain Performance Specifications

Specify bounds on the loop transfer function to guarantee desired performance



$$L(s) = P(s)C(s)$$

$$H_{er} = \frac{1}{1+L} \qquad H_{yr} = \frac{L}{1+L}$$



- Steady state error:

  $$H_{er}(0) = 1/\big(1+L(0)\big) \approx 1/L(0)$$

  $\Rightarrow$ zero frequency ("DC") gain

- Bandwidth: assuming ~90° phase margin

  $$\frac{L}{1+L}(j\omega_c) \approx \left|\frac{1}{1+j}\right| = \frac{1}{\sqrt{2}}$$

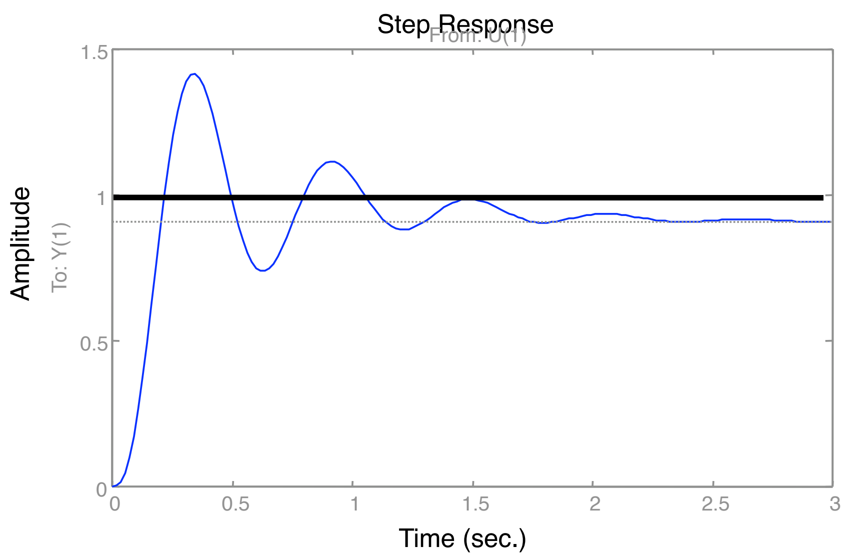
  $\Rightarrow$ sets crossover freq

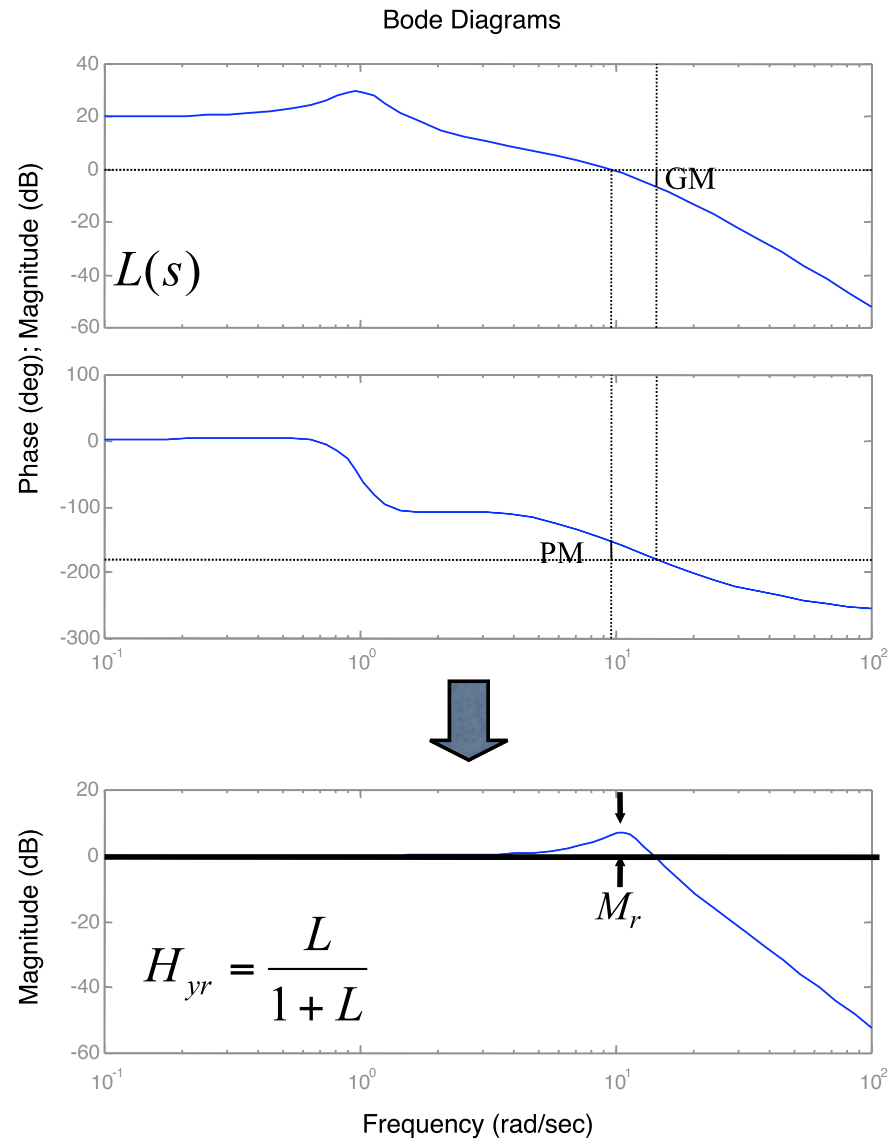
- Tracking: X% error up to frequency $\omega_t \Rightarrow$ determines gain bound ($1 + PC > 100/X$)

Relative Stability

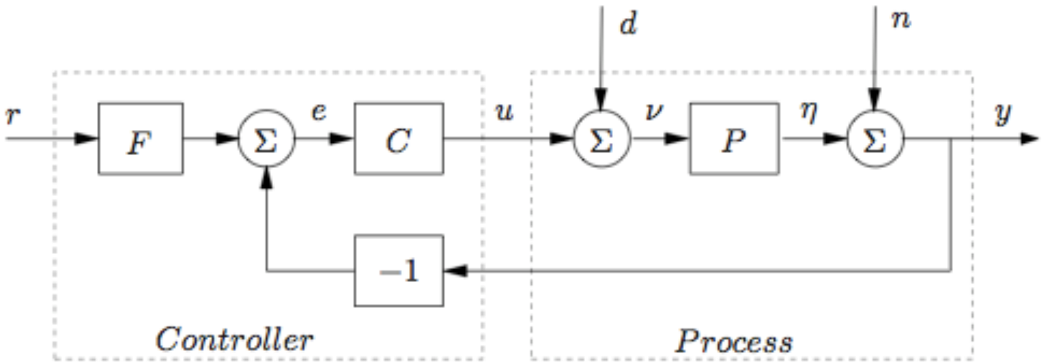
Relative stability: how stable is system to disturbances at certain frequencies?

- System can be stable but still have bad response at certain frequencies
- Typically occurs if system has low phase margin ⇒ get resonant peak in closed loop (M_r) + poor step response
- Solution: specify minimum phase margin. Typically 45˚ or more





Canonical Control Design Problem



$$\begin{bmatrix} \eta \\ y \\ u \end{bmatrix} = \begin{bmatrix} \frac{P}{1+PC} & -\frac{PC}{1+PC} & \frac{PCF}{1+PC} \\ \frac{P}{1+PC} & \frac{1}{1+PC} & \frac{PCF}{1+PC} \\ -\frac{PC}{1+PC} & -\frac{C}{1+PC} & \frac{CF}{1+PC} \end{bmatrix} \begin{bmatrix} d \\ n \\ r \end{bmatrix}$$

Noise and disturbances

- *d* = process disturbances
- *n* = sensor noise
- Keep track of transfer functions between all possible inputs and outputs

Design represents a tradeoff between the quantities

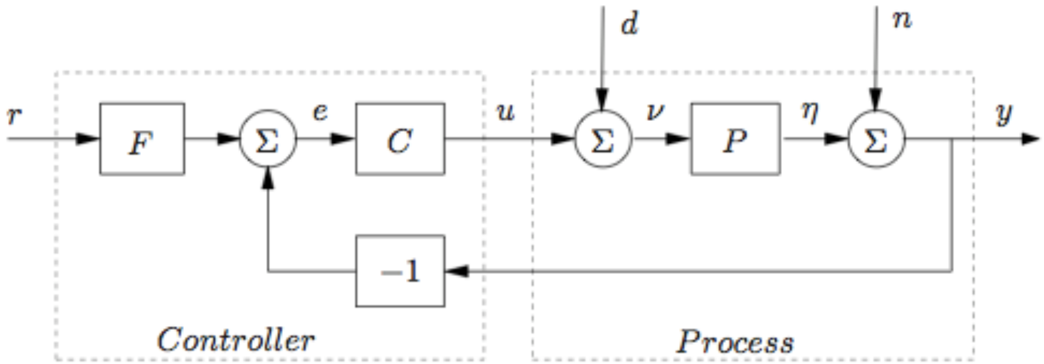
- Keep L=PC large for good performance (H_{er} << 1)
- Keep L=PC small for good noise rejection (H_{yn} << 1)

F = 1: Four unique transfer functions define performance ("Gang of Four")

- Stability is always determined by 1/(1+PC) *assuming stable process & controller*
- Numerator determined by forward path between input and output

More generally: 6 primary transfer functions; simultaneous design of each

Two Degree of Freedom Design



Sensitivity Function

$$S = \frac{1}{1+PC}$$ Sensitivity function

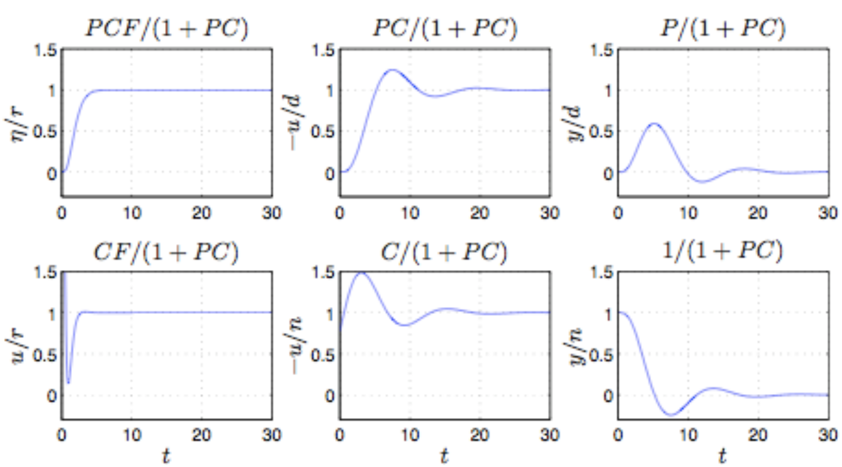
$$T = \frac{PC}{1+PC}$$ Complementary sensitivity

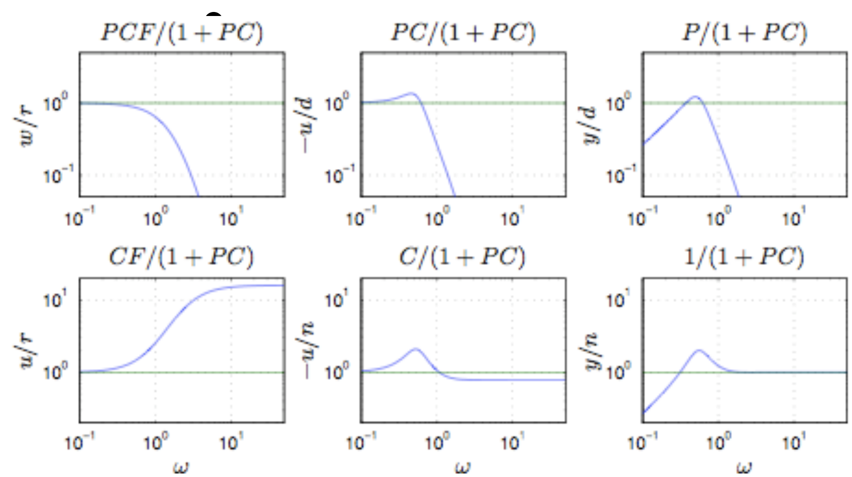
$$PS = \frac{P}{1+PC}$$ Load sensitivity

$$CS = \frac{C}{1+PC}$$ Noise sensitivity

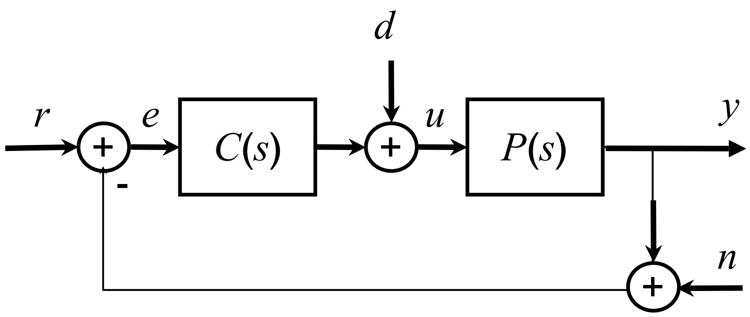
Typical design procedure

- Design *C* to provide good load/noise response
- Design *F* to provide good response to reference





Algebraic Constraints on Performance



$$H_{er} = \frac{1}{1+PC} =: S$$

Sensitivity function

$$H_{yn} = \frac{PC}{1+PC} =: T$$

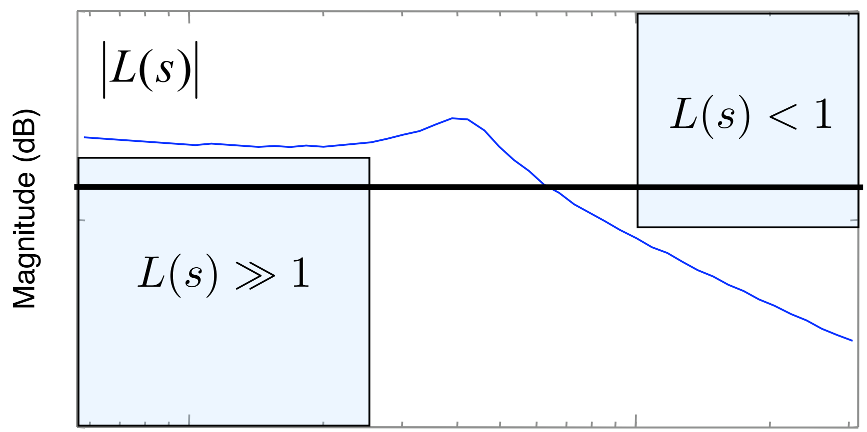
Complementary sensitivity function

Goal: keep S & T small

- S small ⇒ low tracking error
- T small ⇒ good noise rejection (and robustness [CDS 110b])

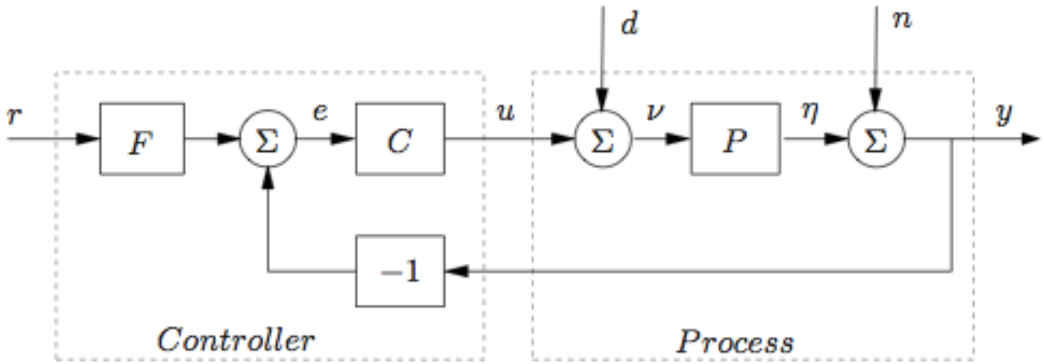
Problem: S + T = 1

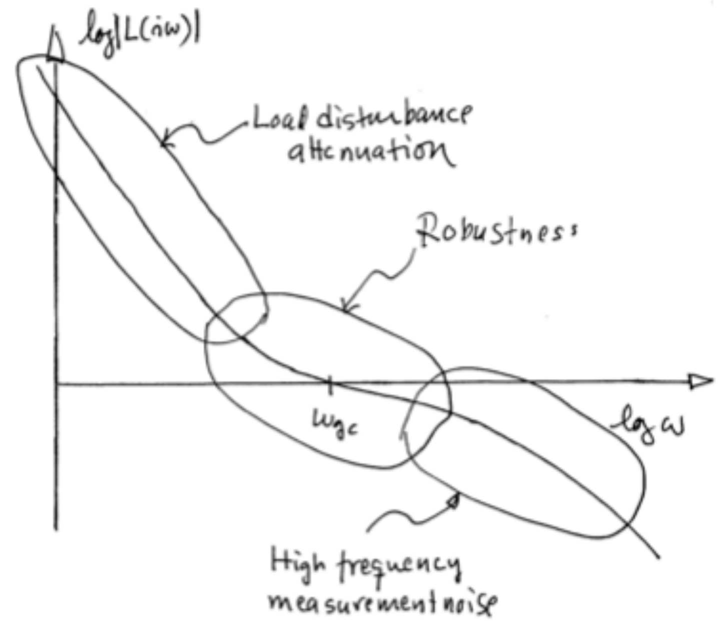
- Can't make *both* S & T small at the same frequency
- Solution: keep S small at low frequency and T small at high frequency
- Loop gain interpretation: keep L large at low frequency, and small at high frequency



- Transition between large gain and small gain complicated by stability (phase margin)

Loop Shaping Revisited





Disturbance rejection $H_{ed} = \dfrac{P}{1+L}$

- Would like H_{ed} to be small make ⇒ large $L(s)$
- Typically require this in low frequency range

High frequency measurement noise $H_{un} = \dfrac{L}{P(1+L)}$

- Want to make sure that H_{un} is small (avoid amplifying noise) ⇒ small $L(s)$
- Typically generates constraints in high frequency range

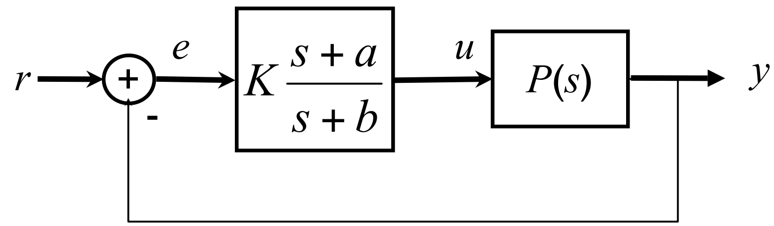
Robustness: gain and phase margin

- Focus on gain crossover region: make sure the slope is “gentle” at gain crossover
- Fundamental tradeoff: transition from high gain to low gain through crossover

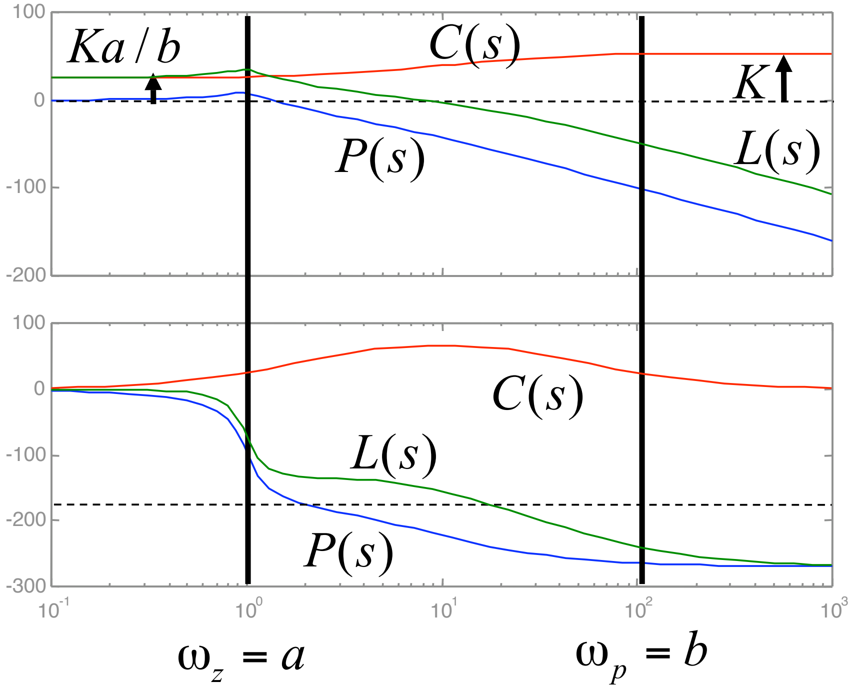
Lead compensation

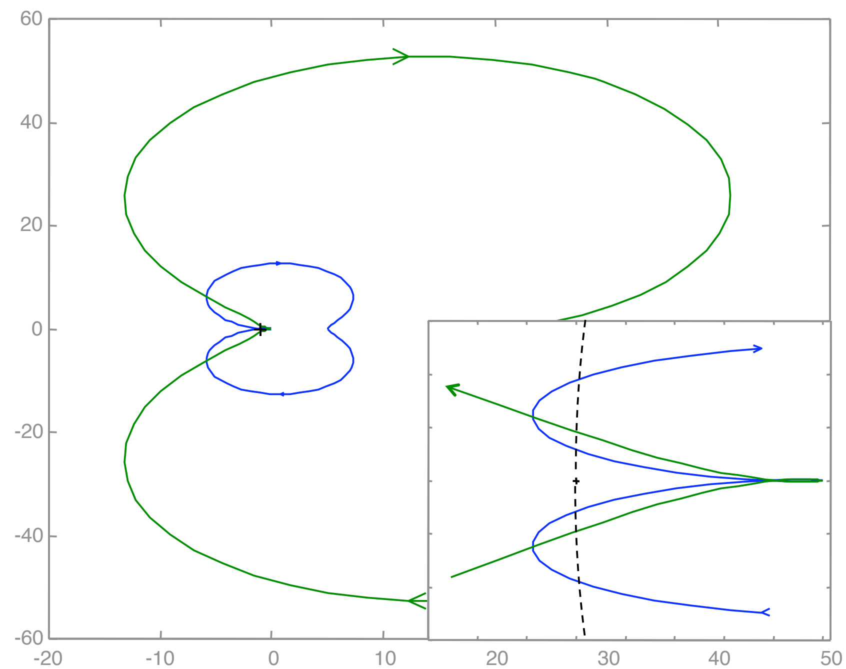
Use to increase phase in frequency band

- Effect: lifts phase by increasing gain at high frequency
- *Very* useful controller; increases PM
- Bode: add phase between zero and pole
- Nyquist: increase phase margin



$a < b \qquad K > 0$



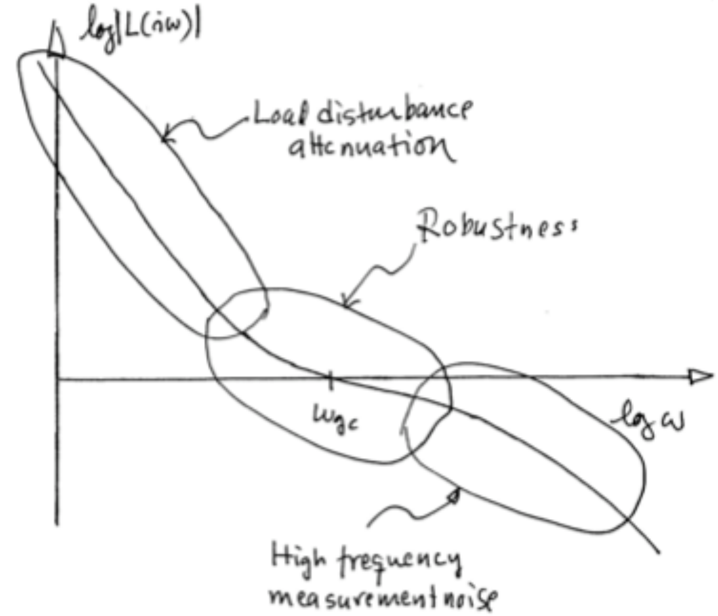


Process Inversion

Simple trick: invert out process

- Write all performance specs in terms of the desired loop transfer function
- Choose $L(s)$ that satisfies specfiications
- Choose controller by inverting $P(s)$

$$C(s) = L(s)/P(s)$$



Pros

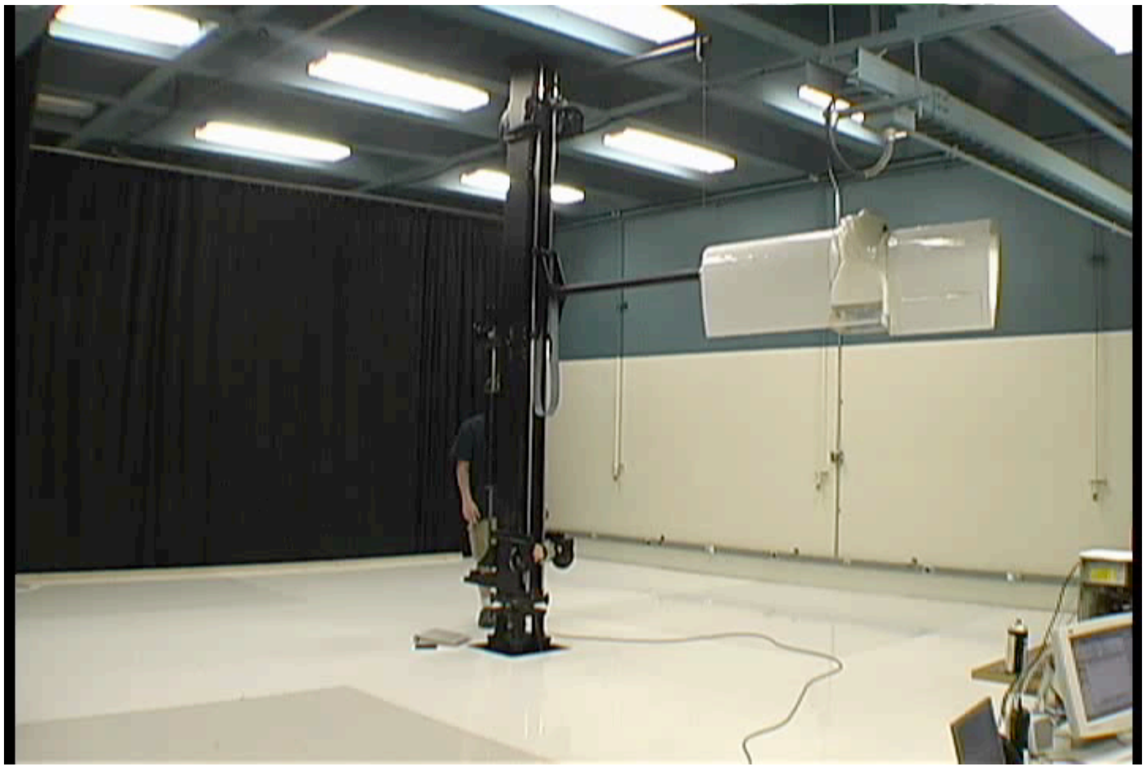
- Very easy design process
- $L(s) = 1/s$ often works very well
- Can be used as a first cut, with additional shaping to tune design

Cons

- High order controllers (at least same order as the process you are controlling)
- Requires "perfect" model of your process (since you are inverting it)
- *Does not work if you have right half plane poles or zeros* (get internal instability)

$$S = \frac{1}{1+PC} \qquad T = \frac{PC}{1+PC} \qquad PS = \frac{P}{1+PC} \qquad CS = \frac{C}{1+PC}$$

Example: Control of Vectored Thrust Aircraft



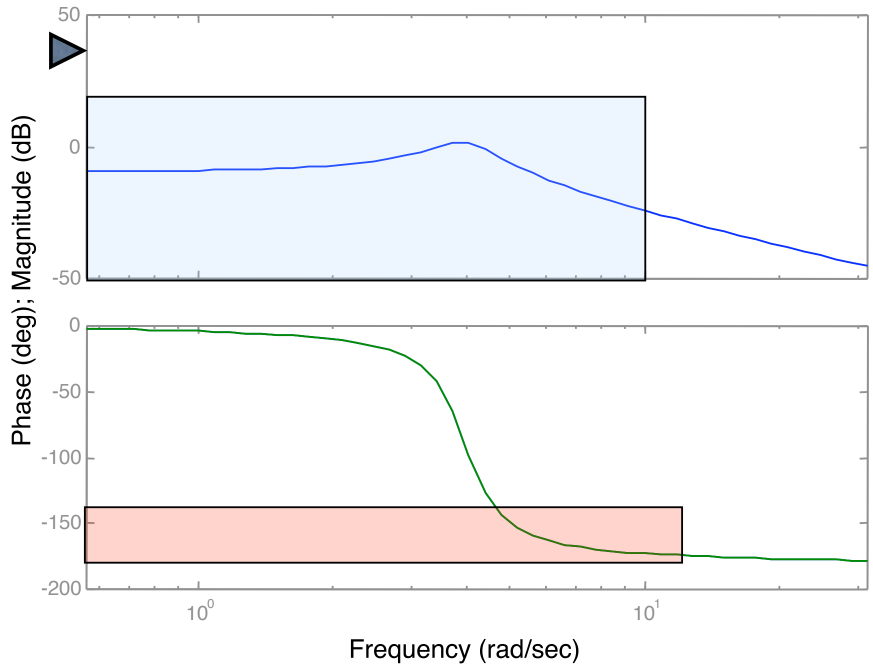
System description

- Vector thrust engine attached to wing
- Inputs: fan thrust, thrust angle (vectored)
- Outputs: position and orientation
- States: x, y, θ + derivatives
- Dynamics: flight aerodynamics

Control approach

- Design "inner loop" control law to regulate pitch (θ) using thrust vectoring
- Second "outer loop" controller regulates the position and altitude by commanding the pitch and thrust
- Basically the same approach as aircraft control laws

Performance Specification and Design Approach



Performance Specification

- ≤ 1% steady state error
  - Zero frequency gain > 100
- ≤ 10% tracking error up to 10 rad/sec
  - Gain > 10 from 0-10 rad/sec
- ≥ 45˚ phase margin
  - Gives good relative stability
  - Provides robustness to uncertainty

Design approach

- Open loop plant has poor phase margin
- Add phase lead in 5-50 rad/sec range
- Increase the gain to achieve steady state and tracking performance specs
- Avoid integrator to minimize phase

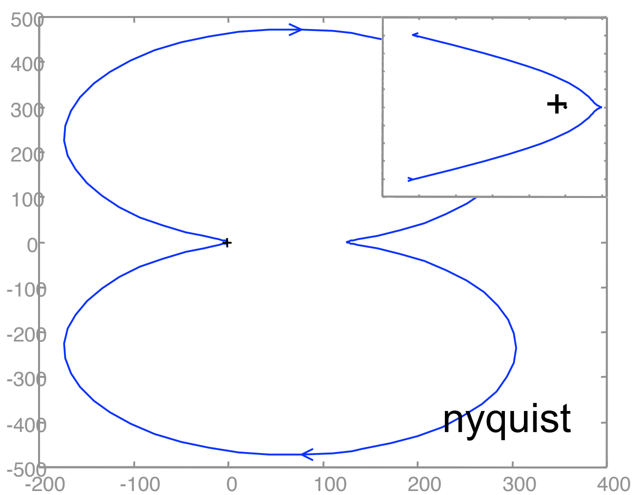
$$P(s) = \frac{r}{Js^2 + ds + mgl}$$

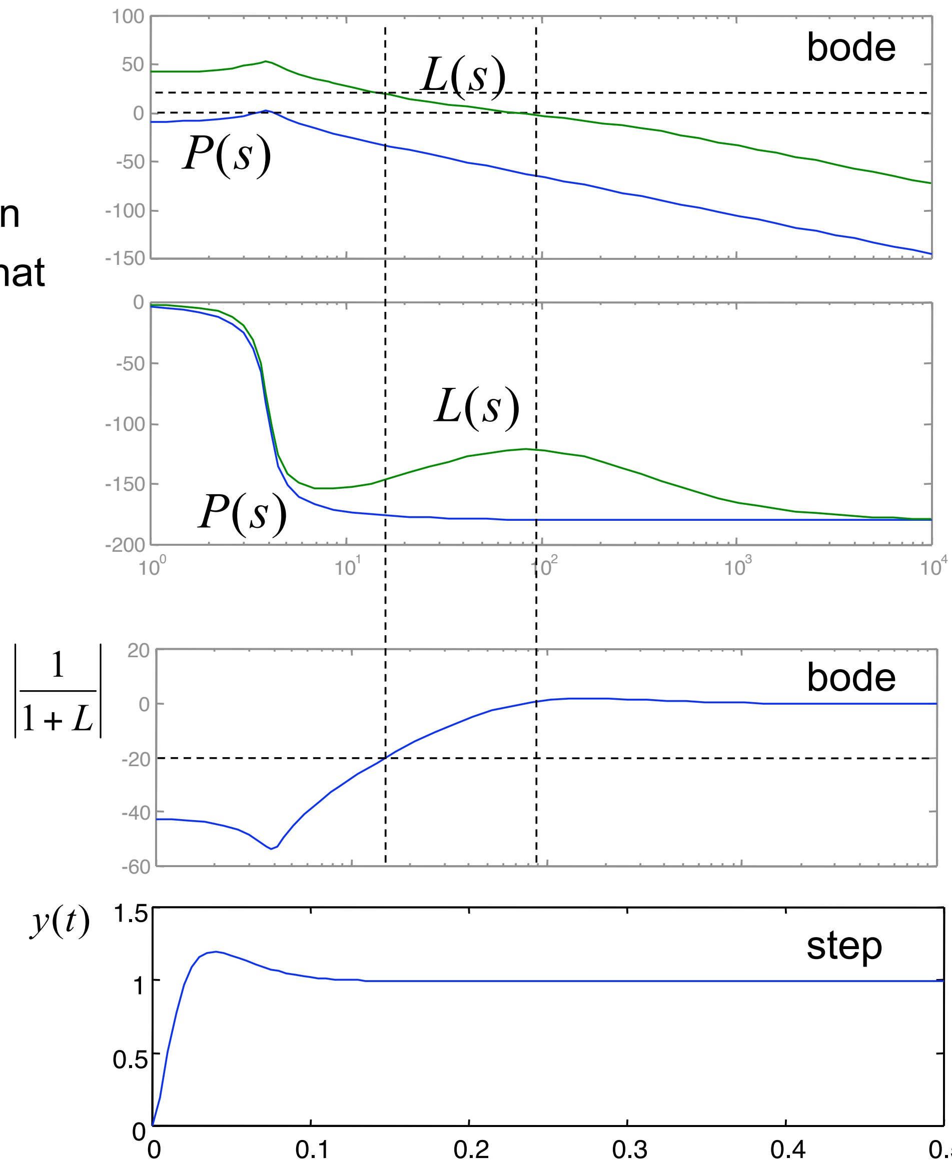
$$C(s) = K\frac{s+a}{s+b} \qquad a = 25 \quad b = 300 \quad K = 15 \times 300$$

Control Design and Analysis

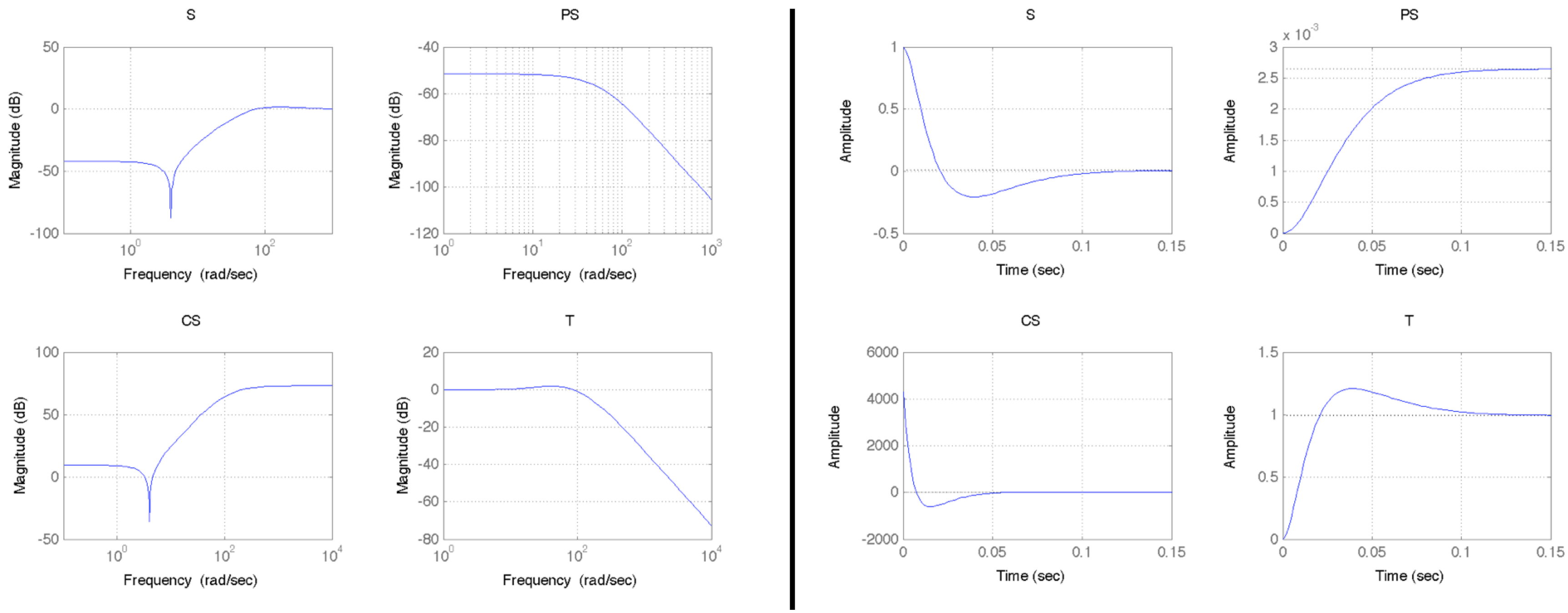
Select parameters to satisfy specs

- Place phase lead in desired crossover region (given by desired BW)
- Phase lead peaks at 10X of zero location
- Place pole sufficiently far out to insure that phase does not decrease too soon
- Set gain as needed for tracking + BW
- Verify controller using Nyquist plot, etc





Control Verification: Gang of 4



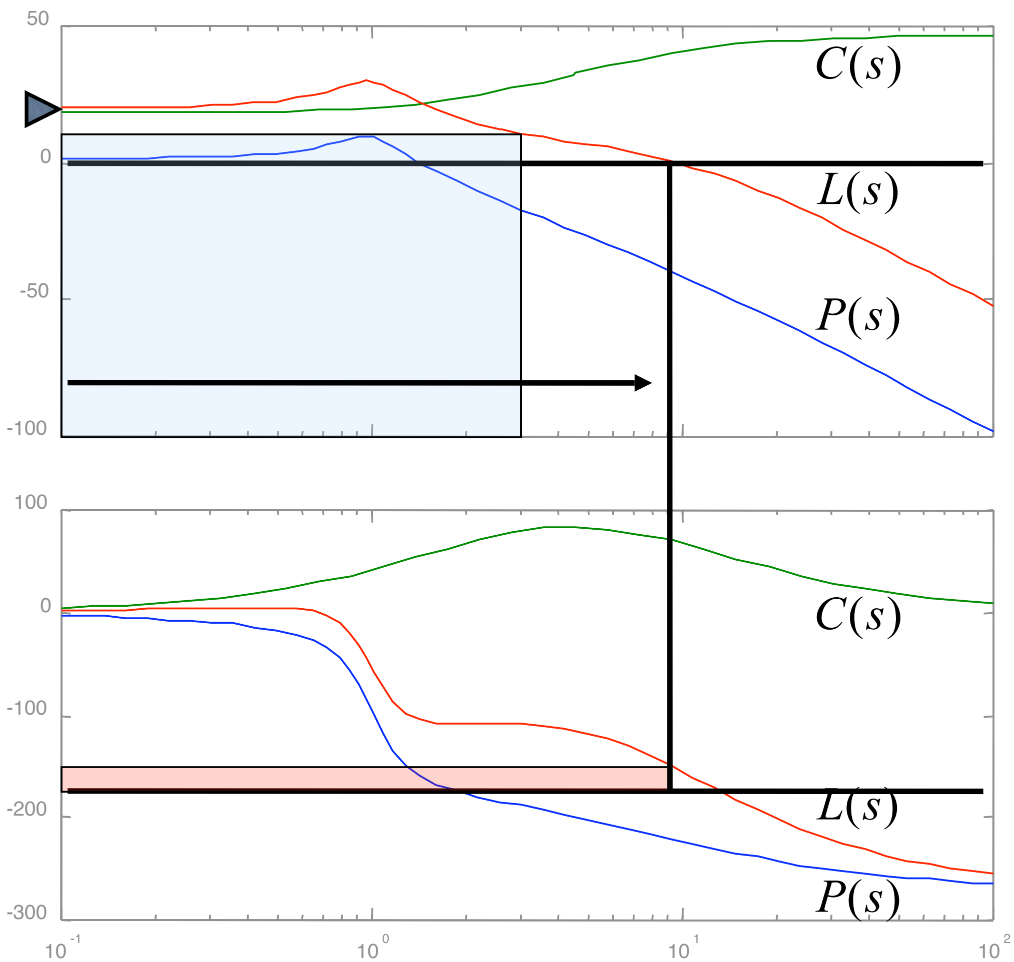
Remarks

- Check each transfer function to look for peaks, large magnitude, etc
- Example: Noise sensitivity function (CS) has very high gain; step response verifies poor step response
- Implication: controller amplifies noise at high frequency ⇒ will generate *lots* of motion of control actuators (flaps)
- Fix: roll off the loop transfer function faster (high frequency pole)

Summary: Loop Shaping

Loop Shaping for Stability & Performance

- Steady state error, bandwidth, tracking



Main ideas

- Performance specs give bounds on loop transfer function
- Use controller to shape response
- Gain/phase relationships constrain design approach
- Standard compensators: proportional, lead, PI