CDS 101/110a Final Review

12/05/2008

* 1st half of the course (<u>time-domain</u>)
  - System modeling (ODE, state space)
  - Stability
  - Reachability & Observability
  - Controller design via state feedback (eigenvalue/pole placement)

* 2nd half (<u>frequency domain</u>)
  - System modeling (transfer function)
  - Frequency domain analysis
    - Bode plot
    - Nyquist plot / Nyquist criterion
  - Frequency domain design (aka loop shaping)
    - Performance specs
    - Design approaches: lead compensator, PID, ..

1. Transfer function

① Describing sys in freq domain:

$$e^{st} \longrightarrow \boxed{G} \longrightarrow G(s) \cdot e^{st}$$

ONLY works for <u>linear systems</u>

2 State space model $\leftrightarrow$ transfer fcn

i) State space $\rightarrow$ tr. fcn.

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \longrightarrow \quad Y(s) = G(s)\,U(s), \quad G = C(sI-A)^{-1}B + D$$

$\downarrow \mathcal{L} \quad \frac{d}{dt} \rightarrow s$

$$\begin{cases} sX = Ax + BU \\ Y = CX + DU \end{cases}$$

ii) Transfer fcn $\rightarrow$ s.s.:

$\boxed{\text{NOT UNIQUE!}}$ (see p.223 for details)

One way: use reachable/observable canonical form & match coeff's.

$G(s) = \frac{d(s)}{n(s)}$, $\quad n = \deg(n(s))$

$$A = \begin{pmatrix} -a_1 & -a_2 & \cdots & -a_n \\ 1 & & & \vdots \\ & 1 & & \vdots \\ & & \ddots & \\ & & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$C = (b_1 \ b_2 \ \cdots \ b_n)$, $\quad D = d$ (usually $= 0$ in this course)

iii) Relationship:

If p is a pole of $G(s) \Rightarrow p$ is an eigenvalue of A

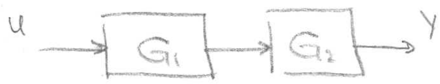
<u>Proof</u>: By $G(s) = C(sI-A)^{-1}B + D$

$\Rightarrow$ Can "determine" stability from freq. domain

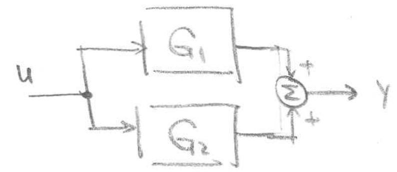
3 Block diagram algebra

* Calculation; label the signals and do algebra.

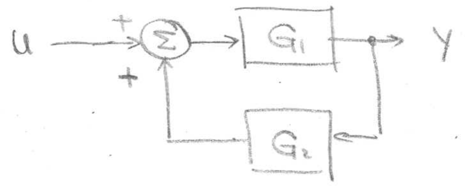
* Typical configurations:



$G(s) = G_2 G_1$



$G(s) = G_1 + G_2$



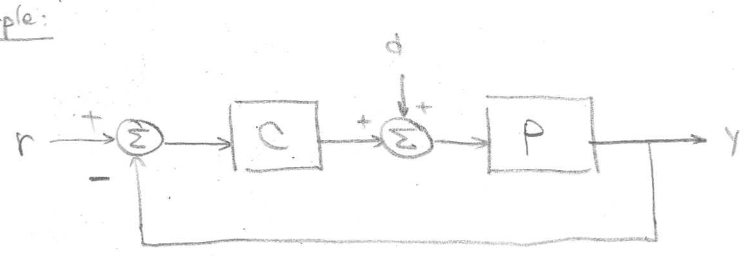
$$G(s) = \frac{G_1}{1 - G_1 G_2}$$ (Note the signs)

MATLAB commands (always use them)

- series
- parallel
- feedback

* Multiple inputs:
  - Multiple transfer fcns
  - When computing, ignore the rest inputs

Example:



$H_{yd} = ?$

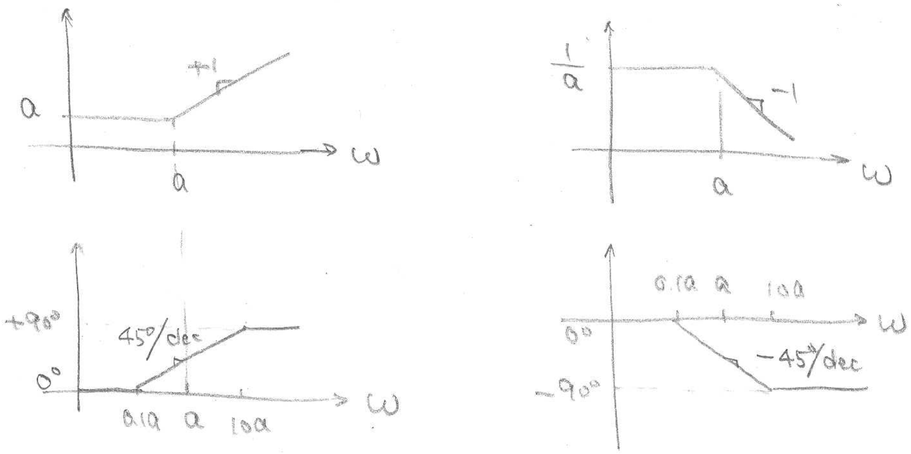
↓ Let $r = 0$

$$H_{yd} = \frac{P}{1 + PC}$$

2. Freq. domain analysis:

① Bode plots: should be able to draw w/ straight line approx.
($s \to i\omega$, $\omega \in \mathbb{R}$)

$s+a \quad (a>0)$ $\qquad\qquad$ $\frac{1}{s+a} \quad (a>0)$



Need to understand rather than simply remember!

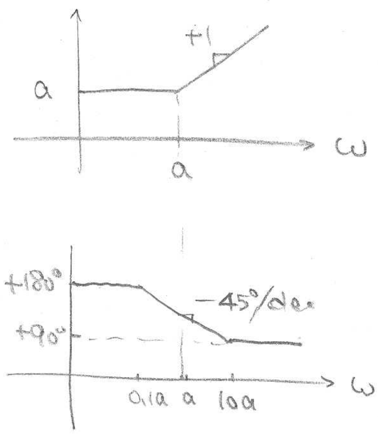
Example: (RHP zero)

$G(s) = s-a \quad (a>0)$

① $s \to i\omega$: $G(i\omega) = i\omega - a$

② $\omega \ll a$: $G(i\omega) \approx -a$ $\quad |G| = a$, $\angle G \doteq 180°$

$\omega \gg a$: $G(i\omega) \approx i\omega$ $\quad |G| = \omega$, $\angle G \doteq 90°$



Always mark the Bode plot!

- DC gain/phase (gain/phase @ $\omega=0$)
- Slope
- Important freq's (poles, zeros, 0.1×, 10×)
- Phase as $\omega \to \infty$

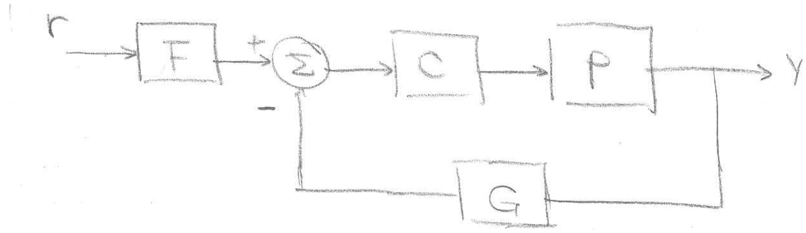
$\boxed{\text{Remember:}}$ ω is in rad/sec, $\omega = 2\pi f$. (f in Hz)

② Nyquist plot

- Loop transfer fcn (loop gain)

Closed loop: $\dfrac{\Sigma}{1+\bigcirc}$

loop gain

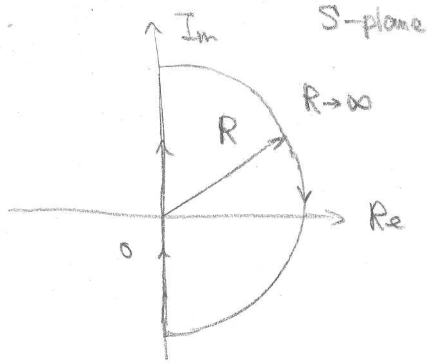


Loop gain : $L(s) = PCG$

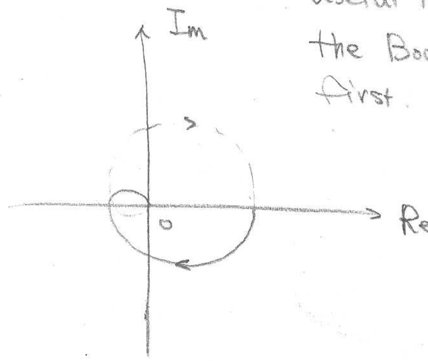
- $(1+L(s))$ in the complex plane : Nyquist plot

Tip:

Useful to draw the Bode plot first.



$1+L(s)$



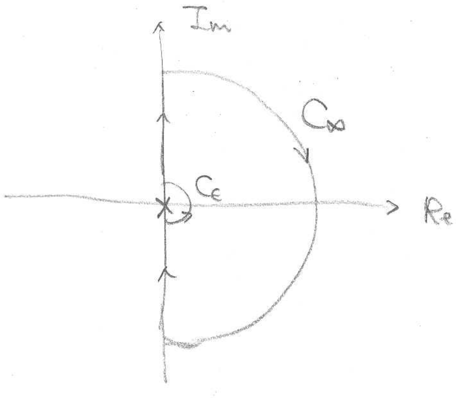
D-contour

Image of the D-contour

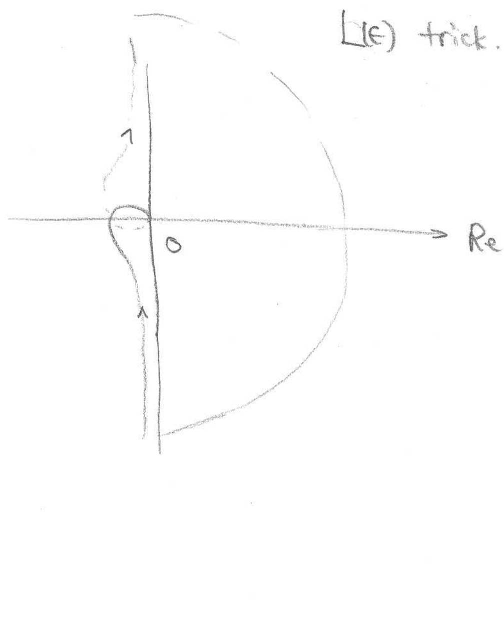
- Poles on the imaginary axis

← → Which side?

$L(\epsilon)$ trick.



$1+L(s)$



Modified D-contour

Mark the Nyquist plot:

- DC gain
- Gain at cross-over ($\angle L(i\omega) = -180^\circ$)
- "-1" point
- Directions.

③ Stability analysis in the freq. domain

Methods:

- Compute the closed loop trans. fcn. (Gang of Four)
- Bode plot (of $L(s)$)
- Nyquist plot (...)

i) "Gang of Four" : check closed loop poles

ii) Bode plot : gain at $\angle L(i\omega) = -180^\circ$

iii) Nyquist : $\boxed{\text{Nyquist criterion: } Z = N + P}$

Stable $\Leftrightarrow$ $Z = 0$.

From plot (N) Directly from $L(s)$ (P)

Summary

	Open/Closed loop	Validity
"Gang of Four"	Closed	always works
Bode	open	with certain restrictions
Nyquist	Open	always works

3. Freq domain design

① Stability (very first thing):

Example: balanced ball system (RHP pole)

Often intuitive to use the Nyquist plot

② Performance

Specs:

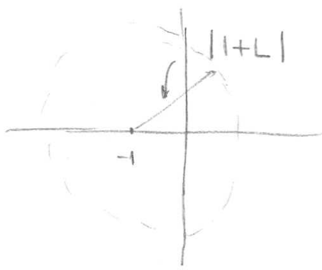
- Steady-state error < $x\%$
- Tracking error (up to ω_0) < $Y\%$
- Overshoot < M_0
- BW > ω_M
- - - -

First thing to do: convert everything into freq. domain

i) Steady-state error < $x\%$:

$$|S(0)| < x\% \Rightarrow \frac{1}{|1+L(0)|} < x\%$$

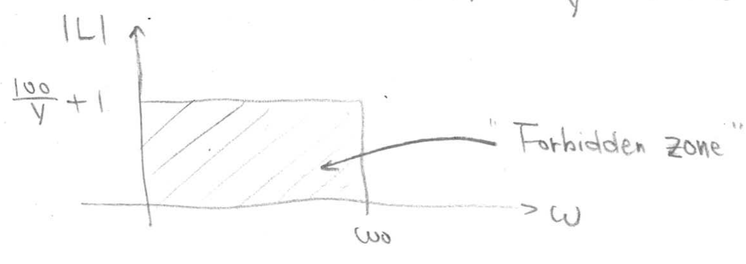
$$\Rightarrow |1+L(0)| > \frac{100}{x}$$



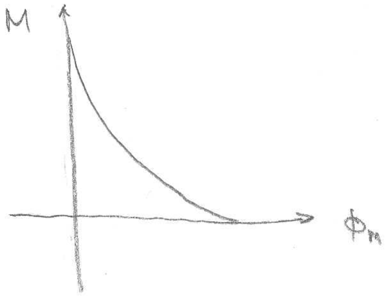
A sufficient cond.: $|L(0)| > \frac{100}{x} + 1 \approx \frac{100}{x}$ (when $\frac{100}{x} >> 1$)

ii) Tracking error < $Y\%$

Similar to ①: $|L(i\omega)| > \frac{100}{Y} + 1$ (up to ω_0)



iii) Overshoot $< M_0$

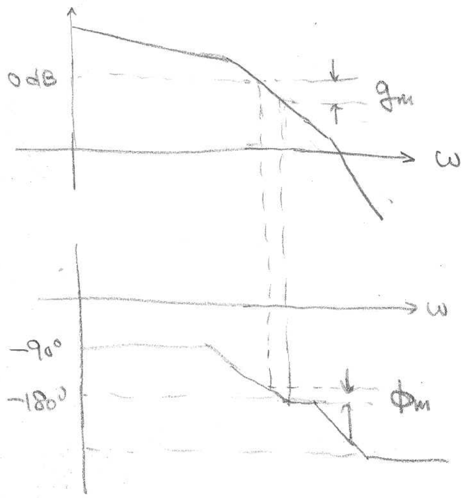


e.g. $M < 10\% \Rightarrow \phi_m > 60$ deg.

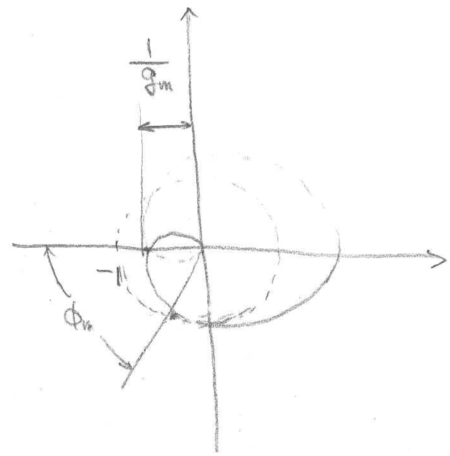
$M < 20\% \Rightarrow \phi_m > 45$ deg.

Gain / phase margin :

* From Bode (not always valid) :



* From Nyquist



<u>Meaning of gain / phase margin</u>: maximum gain / phase lag that can be applied while maintaining stability.

iv) Gang of Four (revisited)

S : $\dfrac{1}{1+PC}$: steady-state error, tracking error $\rightarrow$ small

T : $\dfrac{PC}{1+PC}$: robustness $\left(\delta < \frac{1}{|T|} \right) \rightarrow$ small

$\left.\right\}$ $|S+T| = 1$

PS : $\dfrac{P}{1+PC}$: load disturbance sensitivity ($|PS| < x\%$ up to ω_0)
(H_{yd})

CS : $\dfrac{C}{1+PC}$: Sensor noise sensitivity ($|CS| < y\%$ from $\omega_1 \rightarrow \omega_2$)
(H_{un})

③ Design approaches

Proportional	k		Simple, use if PM is good
Lead	$k \cdot \frac{s+a}{s+b}$	$(0<a<b)$	add PM, note low freq gain
Lag	$k \frac{s+a}{s+b}$	$(a>b>0)$	improve low freq gain note $-90°$ lag.
PD (similar to lead)	$k_p + k_d s$		add PM, note high freq. CS and ω_{gc}.
PI (special case of lag, b=0)	$k_p + k_i \frac{1}{s}$		$L(0) = \infty$, note $-90°$ lag.
PID	$k_p + k_i \frac{1}{s} + k_d s$		$L(0) = \infty$, add PM. note high freq.

④ Open loop vs. closed loop

Open Loop	Closed Loop
GM/PM	Stability
ω_{gc}	Step response (M_p, T_r ...)
Nyquist plot	
"Loop shaping"	

Suggestions:

① Review the materials before midterm if you don't feel comfortable with them.

② Have MATLAB available when taking the exam.