Lehre 10.1 - Ducted Fan example

Example: balloon systems

- All have roughly similar dynamics: balance center of mass above a "pivot" while controlling rotation & translation.
- Goal: track reference trajectories in the presence of disturbances

Ducted fan dynamics

\[ m \ddot{x} = \cos \theta f_1 - \sin \theta f_2 \]
\[ m \dot{y} = \cos \theta f_2 + \sin \theta f_1 - mg \]
\[ J \ddot{\theta} = rf_1 - mgd \sin \theta - c \dot{\theta} \quad c \ll 1 \]

Linearize around hover

\[ x = 0 \quad \dot{x} = 0 \quad f_1 = 0 \]
\[ y = 0 \quad \dot{y} = 0 \quad f_2 = mg \]
\[ \theta = 0 \quad \dot{\theta} = 0 \]

\[ H_{xu_1} = \frac{J s^2 + mg \dot{\theta} + mg (x-r) \dot{x}}{m s^2 (J s^2 + cs + mgd)} \quad H_{yu_2} = \frac{1}{s^2} \]
Labral dynamics

\[
p(s) = \frac{J s^2 + cs + mg(J-r)}{ms^2 \cdot (J s^2 + cs + mg l)} \approx \frac{J s^2 + cs - mg r}{ms^2 \cdot (J s^2 + cs + mg l)}
\]

Remarks

Poles: 0, 0, -σ + jω

ω \approx \sqrt{\frac{mg^2}{J}}

σ depends on c

Zeros: -σ - α₂

α \approx \sqrt{\frac{mg r}{J}}

α₂ \gg σ \Rightarrow \text{RHP zero}

Parameter values:

\begin{align*}
m & = 4 \text{ kg} \\
J & = 0.0475 \text{ kg m}^2 \\
r & = 0.26 \text{ m} \\
g & = 9.8 \text{ m/s}^2 \\
 l & = 0.05 \text{ m}
\end{align*}

Poles: 0, 0, -0.0105 + j3.974

Zeros: -8.896, +8.875

Control approach: inner/outer design

Split design to simplify approach. Designing for single process model is very tricky.
Performance specification

- Zero steady state error
- Bandwidth ($\approx \omega_{nc}$) of at least 1 rad/sec
- $45^\circ$ phase margin

Inner loop: $BW \approx 10x$ outer loop, $\leq 5\%$ error

$$P_i(s) = \frac{1}{s^2 + mg} = \frac{0.25}{0.05s^2 + 0.75} = \frac{5}{s^2 + 16} \quad \text{(assume low damping)}$$

Use lead compensator to add phase margin 40 rad/sec

$$C_i(s) = K \frac{s+25}{s+400}$$

$$K =$$

See L10-12_decomp.m for exact calculations

Remarks

- Requires some iteration between inner & outer loop to set inner loop BW, error volves
- In practice: be careful about saturation
Lateral control

Pretend that pitch controller is perfect \( \Rightarrow \) control directly

\[
\begin{align*}
\dot{m}x &= u_i - mg \\
J\dot{\theta} &= ru_i - mgd\theta \\
SS \quad u_i &= \frac{\Theta_d mgd}{c} = 0.2 mg \cdot \Theta_d
\end{align*}
\]

So, in steady state we can assume

\[
\dot{m}x' = (0.2 mg - mg)\Theta_d = -0.8 mg \cdot \Theta_d
\]

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Q: How good an approximation is this?
A: Look at \( H_i \)

\[
H_i = \frac{C_i}{1 + C_i P_i} - mg \frac{C_e P_e}{1 + C_i P_i} = \frac{C_i (1 - mg P_e)}{1 + C_i P_e}
\]
Outer loop design goals
- 0\% steady state error
- \( BW = 1 \) rad/sec
- \( |L_o| < \frac{1}{10} \) for \( w > 10 \) rad/sec

\( L_o \) depends on controller

\( \frac{1}{s^2} \) (due to \( P_0 \))
Outer loop design

\[ H_2(0) P_0(s) = -0.8 \frac{mg}{s^2} \]

\[ C_0(s) = -K \frac{s+a_0}{s+b_0} \]

\( \Rightarrow \) to get sign of gain correct

Choose crossover at \( \omega_c = \frac{b}{10} = 1 \text{ rad/sec} \Rightarrow b = 10 \)

Choose zero at \( \frac{b}{10} < 10a < b \Rightarrow \frac{1}{10} < a < 1 \)

Try \( a = 0.3 \)

Set gain at \( \omega_c = \frac{b}{10} \) to give \( |H_2(0) P_c(j\omega_c) C(j\omega_c)| = 1 \)

\[ + 0.8 \frac{mg}{s^2} \cdot \frac{1}{1 - K \left| \frac{j + 0.3}{j + 10} \right|} = 1 \Rightarrow K = 0.8 \]
Final design check

\[ \tilde{L}_0 = C_0 H_c P_0 \]

Use Nyquist to verify stability

Poles: \(-194 \pm 216j\), \(-11.41\), \(-10\), \(-0.51 \pm 0.23j\)

Zeros: \(7.94\), \(-10\), \(-7.56\), \(-0.3\)

Note: some PI controllers in MATLAB