RMM 27 Nov 06

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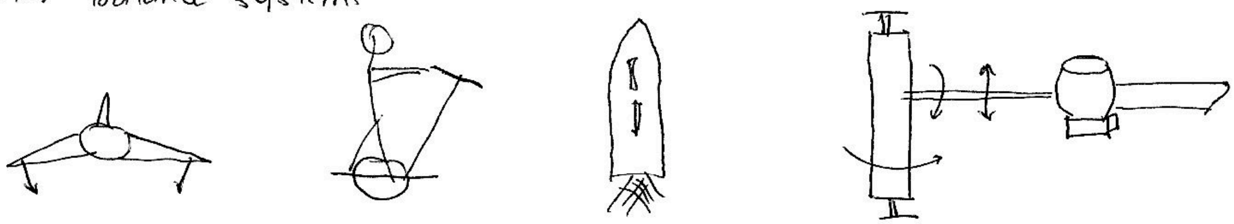
Leture 10.1 - Ducted fan example

Example: balance systems

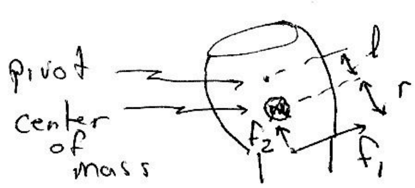


VTOL aircraft   Segway   Rocket   Caltech ducted fan

- All have roughly similar dynamics: balance center of mass above a "pivot" while controlling rotation & translation.

- Goal: track reference trajectories in the presence of disturbances

Ducted fan dynamics



$\ell \ll r$

$$m\ddot{x} = \cos\theta\, f_1 - \sin\theta\, f_2$$

$$m\ddot{y} = \cos\theta\, f_2 + \sin\theta\, f_1 - mg$$

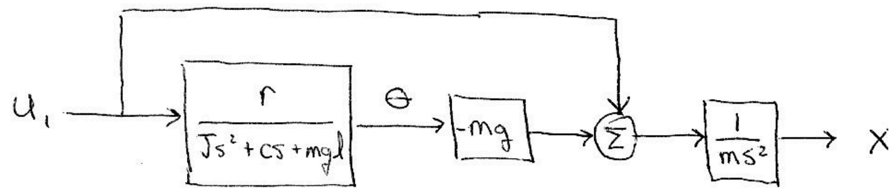
$$J\ddot{\theta} = r f_1 - mg\ell \sin\theta - c\dot{\theta} \qquad c \ll 1$$

Linearize around <u>hover</u>

$$\begin{array}{lll} x = 0 & \dot{x} = 0 & f_1 = 0 \\ y = 0 & \dot{y} = 0 & f_2 = mg \\ \theta = 0 & \dot{\theta} = 0 & \end{array} \longrightarrow \begin{array}{l} m\ddot{x} = u_1 - mg\theta \\ m\ddot{y} = u_2 \\ J\ddot{\theta} = r u_1 - mg\ell\theta - c\dot{\theta} \end{array}$$

$$H_{xu_1} = \frac{Js^2 + \cancel{mg\ell}\ cs + mg(\ell - r)}{ms^2(Js^2 + cs + mg\ell)} \qquad H_{yu_2} = \frac{1}{s^2}$$

Lateral dynamics



$$P(s) = \frac{Js^2 + cs + mg(l-r)}{ms^2(Js^2+cs+mgl)} \approx \frac{Js^2 + cs - mgr}{ms^2(Js^2+cs+mgl)}$$

Remarks

Poles: $0, 0, -\sigma_1 \pm j\omega_1$ $\quad \omega_1 \approx \sqrt{mgl/J}$ $\quad \sigma$ depends on c

Zeros: $-\sigma_2 \pm \alpha_2$ $\quad \alpha \approx \sqrt{mgr/J}$ $\quad \alpha_2 \gg \sigma_2 \Rightarrow$ RHP zero

Parameter values:

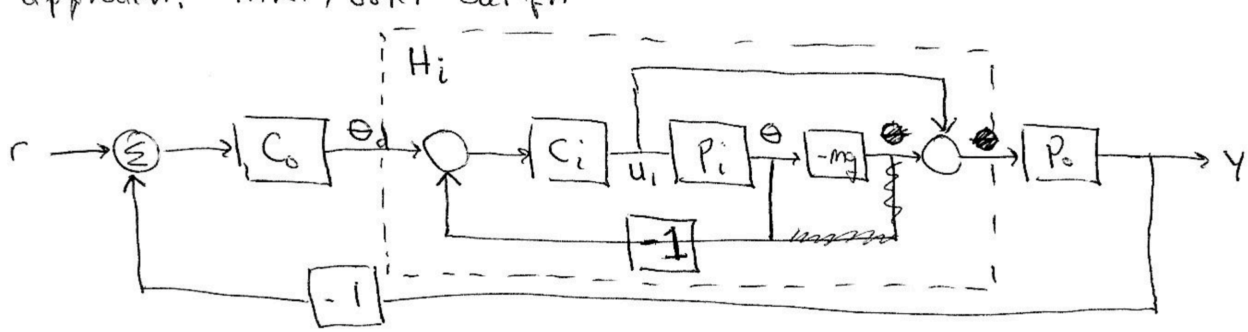
$m = \cancel{4}\ 1.5$ kg $\quad r = 0.26$ m $\quad g = 9.8$ m/s^2

$J = 0.0475$ kg m^2 $\quad l \approx 0.05$ m $\quad c = 0.001$ kg m/s

Poles: $0, 0, -0.0105 \pm j3.974$

Zeros: $-8.896, +8.875$

Control approach: inner/outer design



Split design to simply approach. Designing for single process model is very tricky.

Quickly

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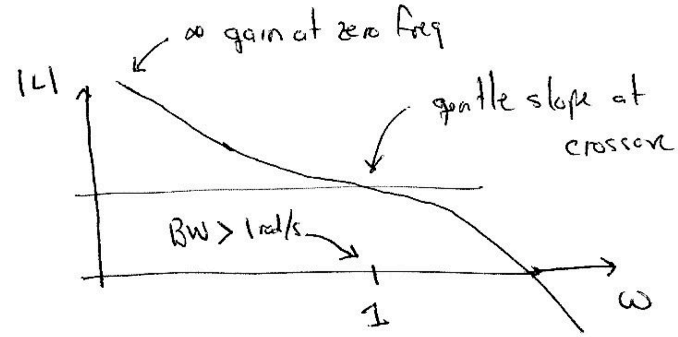
(3)

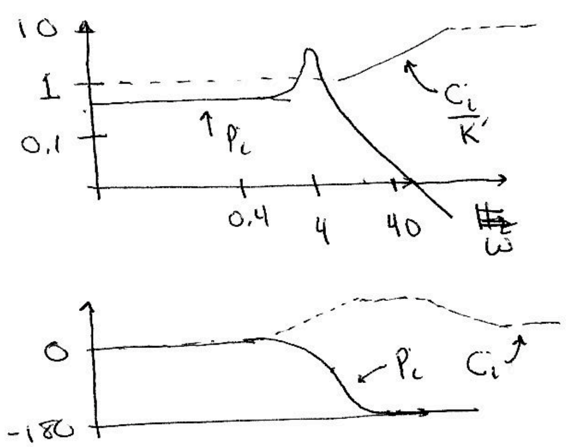
Performance specification

- Zero steady state error
- Bandwidth ($\approx \omega_{gc}$) of at least 1 rad/sec
- 45° phase margin



Inner loop: BW $\approx$ 10x outer loop, $\leq$ 5% error

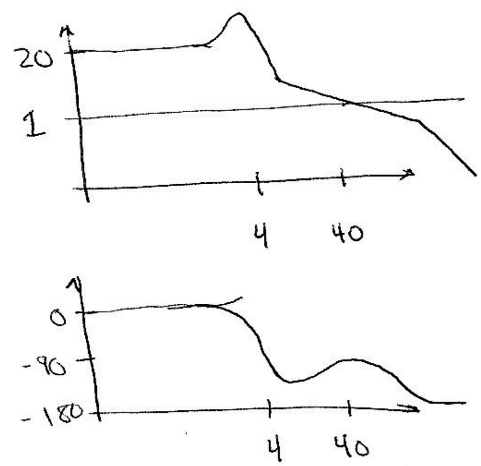
$$P_i(s) \approx \frac{r}{Js^2 + mgl} \approx \frac{0.25}{0.05s^2 + 0.75} = \frac{5}{s^2+16} \quad \text{(assume low damping)}$$



Use lead compensator to add phase around 40 rad/sec

$$C_i(s) = K \frac{s+25}{s+400}$$

$K =$

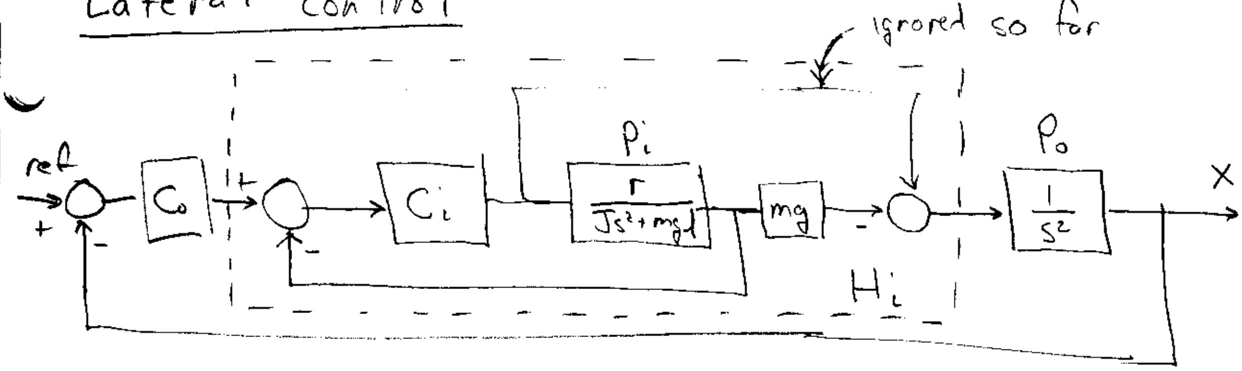


See L10-1-dfen.m for exact calculations

Remarks

- Requires some iteration between inner & outer loop to set inner loop BW, error values
- In practice: be careful about saturation

Lateral control



Pretend that pitch controller is <u>perfect</u> $\Rightarrow$ control θ directly

$$m\ddot{x} = u_1 - mg\theta$$

$$J\ddot{\theta} = ru_1 - mgl\theta \longrightarrow \theta_d \text{ given}$$

$$SS \quad u_1 = \theta_d \frac{mgl}{r} = 0.2\, mg \cdot \theta_d$$

<u>So</u>, in steady state we can assume

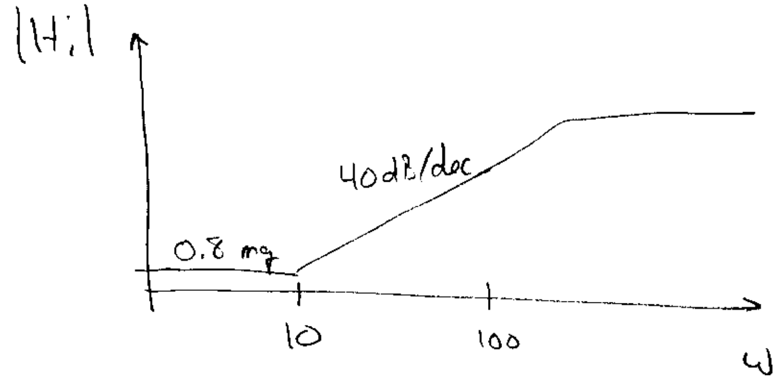
$$m\ddot{x} = (0.2\, mg - mg)\,\theta_d = \underbrace{-0.8\, mg\,\theta_d}_{v_1}$$

v_1 ← pretend we control this

Q: How good an approximation is this?

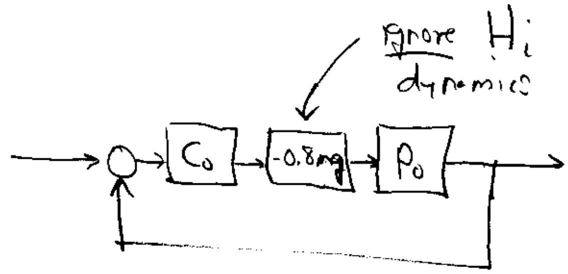
A: Look at H_i

$$H_i = \frac{C_i}{1+C_iP_i} - mg\frac{C_iP_i}{1+C_iP_i} = \frac{C_i(1-mgP_i)}{1+C_iP_i}$$



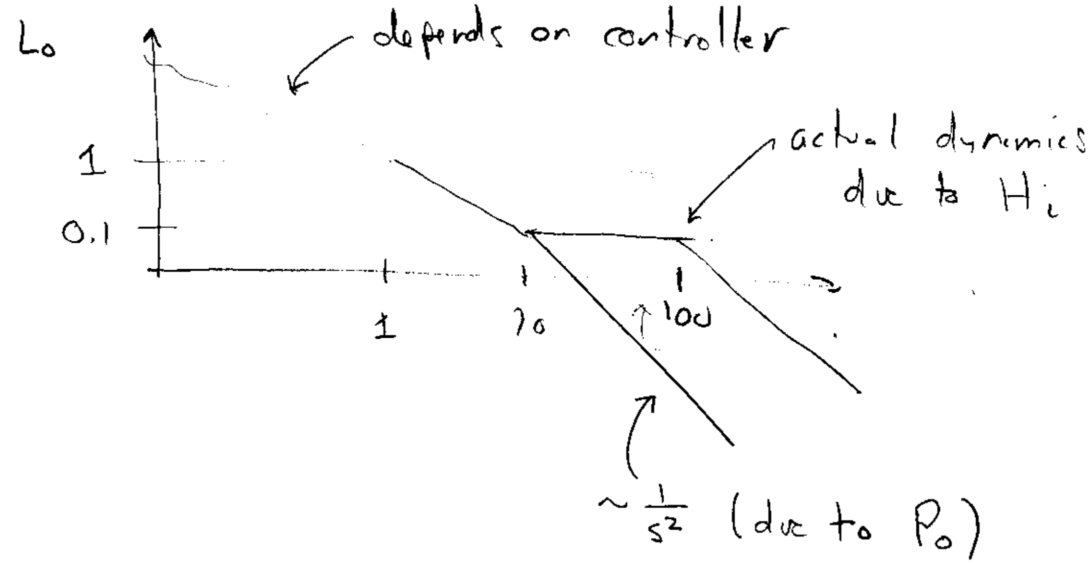
⇒ good approx up to 10 rad/sec ≈ 2 Hz

Outer loop design goals



- 0% steady state error
- BW = 1 rad/sec
- $|L_o| < \frac{1}{10}$ for $\omega > 10$ rad/sec

⇒ roll off gain so that H_i dynamics are not a factor

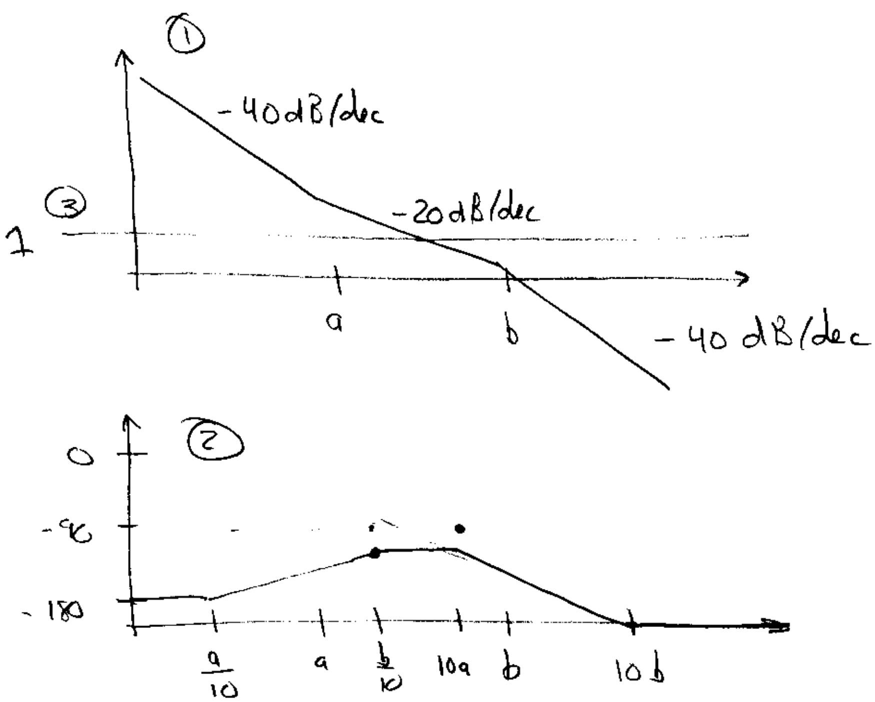


Outer loop design

$$H_2(0)\,P_o(s) = \frac{-0.8\,mg}{s^2}$$

$$C_o(s) = -K_o\,\frac{s+a_o}{s+b_o}$$

to get sign of gain correct



Choose crossover at $\sim \frac{b}{10} = 1$ rad/sec $\Rightarrow b = 10$

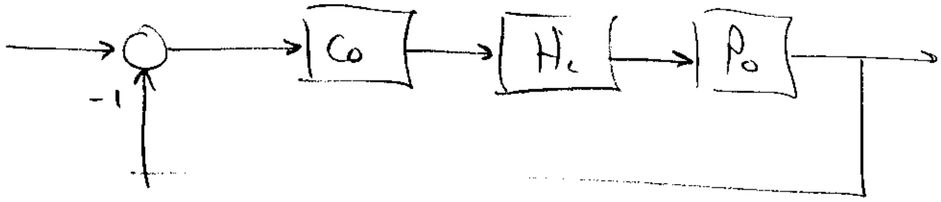
Choose zero at $\frac{b}{10} < 10a < b \;\Rightarrow\; \frac{1}{10} < a < 1$

Try $a = 0.3$

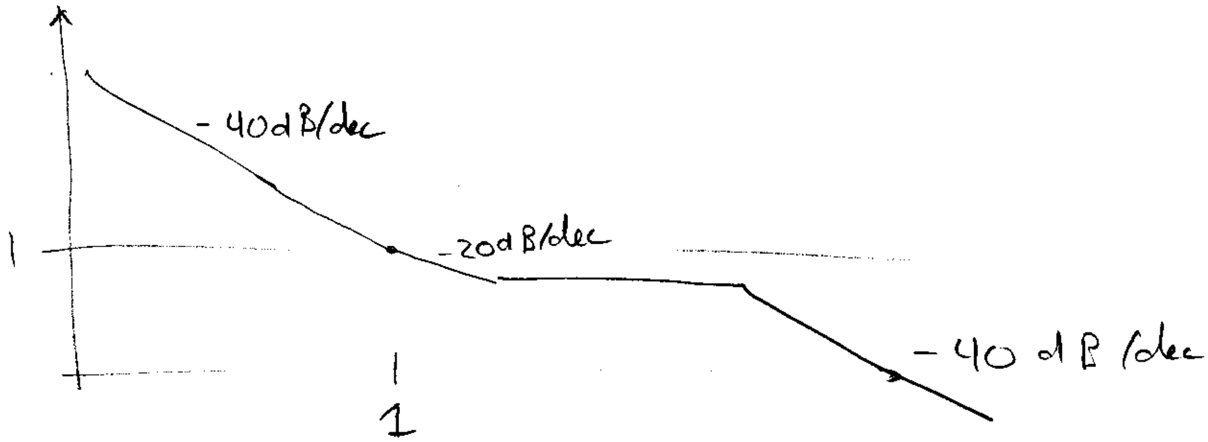
Set gain at $\omega_c = \frac{b}{10}$ to give $\left|H_2(0)\,P_o(j\omega_c)\,C(j\omega_c)\right| = 1$

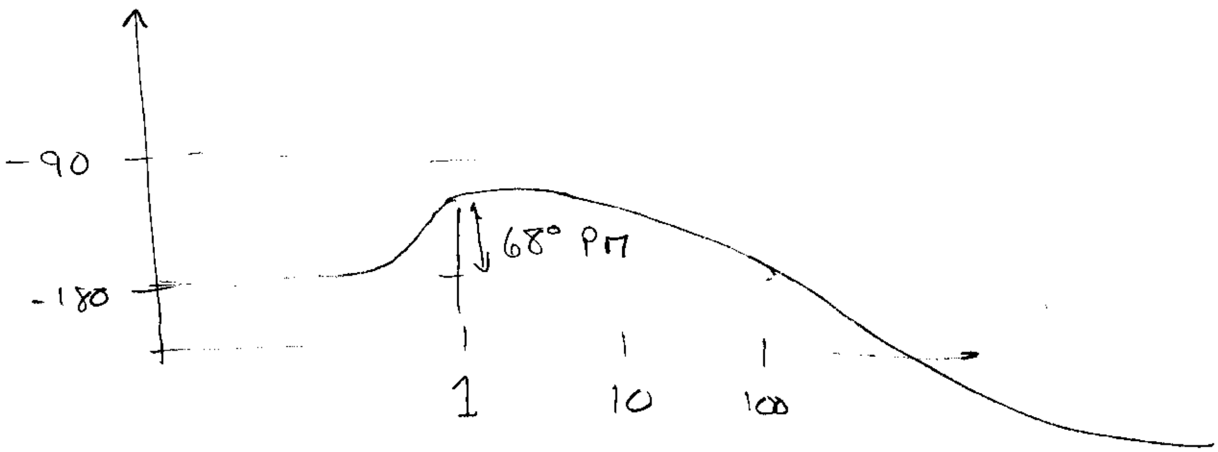
$$+0.8\,mg\,\frac{1}{1}\cdot K\left|\frac{j+0.3}{j+10}\right| = 1 \;\Rightarrow\; K = 0.8$$

Final design check



$$\tilde{L}_o = C_o H_c P_o$$





Use Nyquist to verify stability

Poles: $-194 \pm 216j$, -11.41, -10 $-0.51 \pm 0.23j$

Zeros: 7.94, -10, -7.96, -0.3

Note: some pz cancellations in MATLAB